Radiative corrections in $K \to 3\pi$ decays

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Outline

Part I: Non-relativistic effective field theory for $K \to 3\pi$ decays

Part II: What's new? $\longrightarrow$ radiative corrections

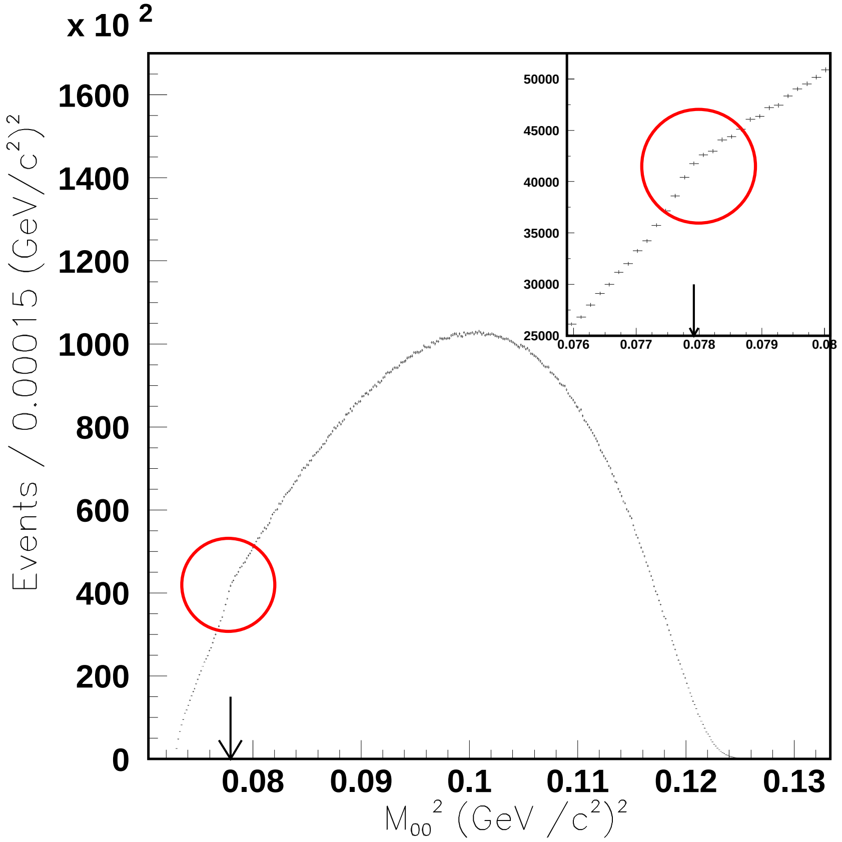
Part I

Non-relativistic effective field theory for $K \to 3\pi$ decays

Colangelo, Gasser, BK, Rusetsky, Phys. Lett. B 638 (2006) 187

Bissegger, Fuhrer, Gasser, BK, Rusetsky, Phys. Lett. B 659 (2008) 576

The cusp effect in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$



- cusp at $M_{\pi^0\pi^0} = 2M_{\pi^+}$

Batley et al., PLB 633 (2006) 173

Madigozhin, this workshop

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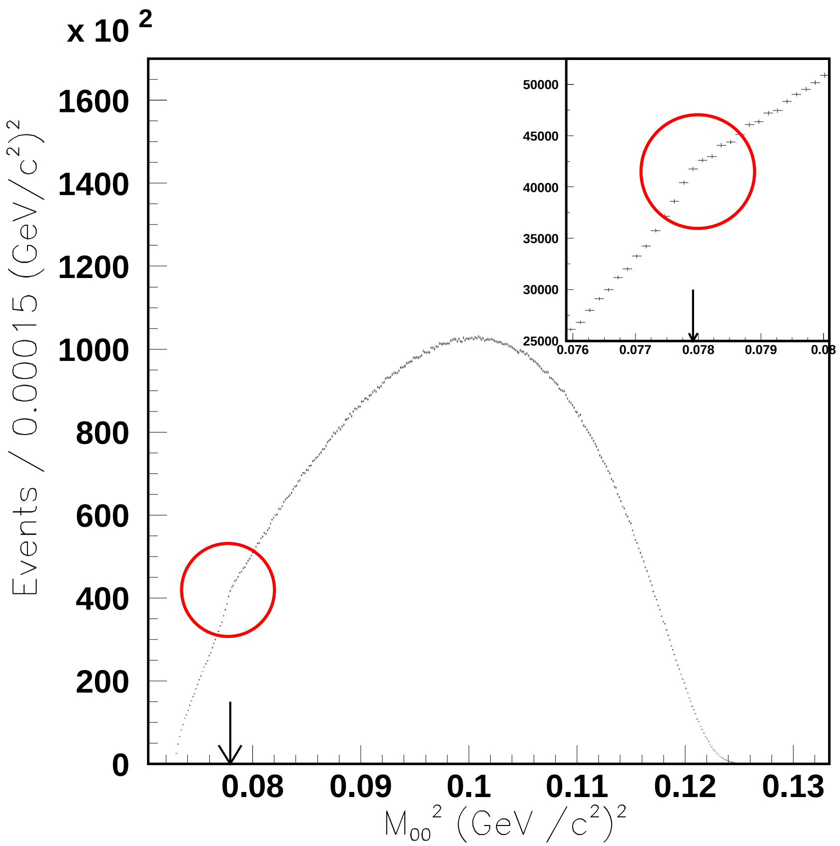

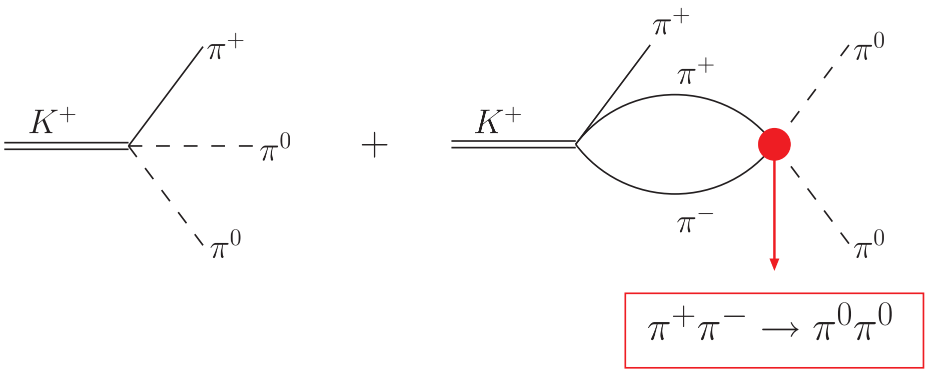

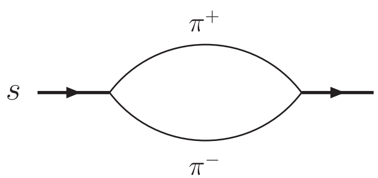

$$= \ldots + \frac{i}{16\pi}\, v_{\pm}(s)$$

$$v_{\pm}(s) = \begin{cases} i\sqrt{\dfrac{4M_{\pi^+}^2}{s} - 1}\,, & s < 4M_{\pi^+}^2 \\ \sqrt{1 - \dfrac{4M_{\pi^+}^2}{s}}\,, & s > 4M_{\pi^+}^2 \end{cases}$$

- cusp at $M_{\pi^0\pi^0} = 2M_{\pi^+}$

Batley et al., PLB 633 (2006) 173

Madigozhin, this workshop

- interference tree + 1-loop below $\pi^+\pi^-$ threshold
- square-root behaviour = cusp

Cabibbo, PRL 93 (2004) 121801

Non-relativistic EFT vs. ChPT

- Consider partial waves in $\pi\pi$ scattering:

$$\mathrm{Re}\, T = a + b\, q^2 + c\, q^4 + \ldots$$

scattering length a, effective range b etc.

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- ChPT: a, b, c expanded in powers of M_π^2,

$$a = \frac{7M_\pi^2}{32\pi F_\pi^2} + \mathcal{O}(M_\pi^4)$$

  Weinberg 1966

  contributions from tree, 1-loop, 2-loop . . .

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- NRQFT:
  $a \Leftrightarrow$ tree
  $b \Leftrightarrow$ tree + 2-loop
  $c \Leftrightarrow$ tree + 2-loop + 4-loop

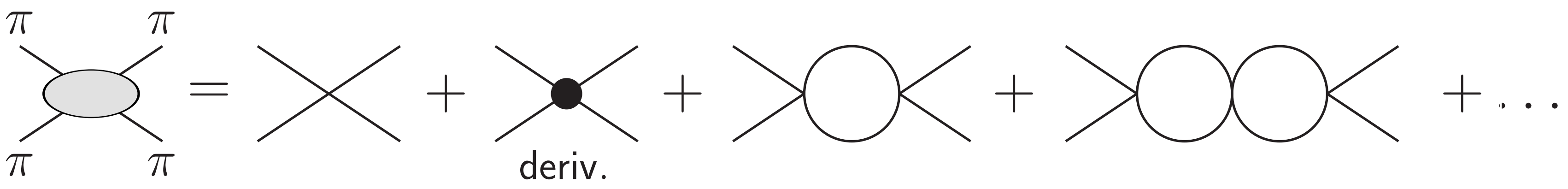
  a, b, c parameters of the theory
  $\Rightarrow$ parametrise T directly in terms of scattering lengths
  $\Rightarrow$ do not predict these, extract as parameters from data

Non-relativistic EFT (1): basics

momenta	:	$\lvert\mathbf{p}\rvert/M_\pi = \mathcal{O}(\epsilon)$
kinetic energy	:	$T = \omega(\mathbf{p}) - M_\pi = \mathcal{O}(\epsilon^2)$
in $K \to 3\pi$	:	$M_K - \sum_i M_i = \sum_i T_i = \mathcal{O}(\epsilon^2)$

where $\omega(\mathbf{p}) = \sqrt{M_\pi^2 + \mathbf{p}^2}$

- non-relativistic region = whole decay region (and slightly beyond)
- two-fold expansion in ϵ and $\pi\pi$ scattering length a
- at given order a, ϵ, only finite number of graphs contribute
  $\Rightarrow$ power counting:



- each loop $\propto i\lvert\mathbf{p}\rvert = \mathcal{O}(\epsilon)$ suppressed

Non-relativistic EFT (2): Lagrangian

- propagator: $$\underbrace{\frac{1}{M_\pi^2 - p^2}}_{\text{relativistic}} = \underbrace{\frac{1}{2\omega(\mathbf{p})}\frac{1}{\omega(\mathbf{p}) - p^0}}_{\text{“non−relativistic”}} + \underbrace{\frac{1}{2\omega(\mathbf{p})}\frac{1}{\omega(\mathbf{p}) + p^0}}_{\text{antiparticles}}$$

  generated by Lagrangian

  $$\mathcal{L}_{\text{kin}} = \Phi^\dagger(2W)(i\partial_t - W)\Phi \; , \qquad W = \sqrt{M_\pi^2 - \triangle}$$

  Note: non-local $\mathcal{L}_{\text{kin}}$ generates all relativistic corrections; manifestly Lorentz-invariant / frame-independent

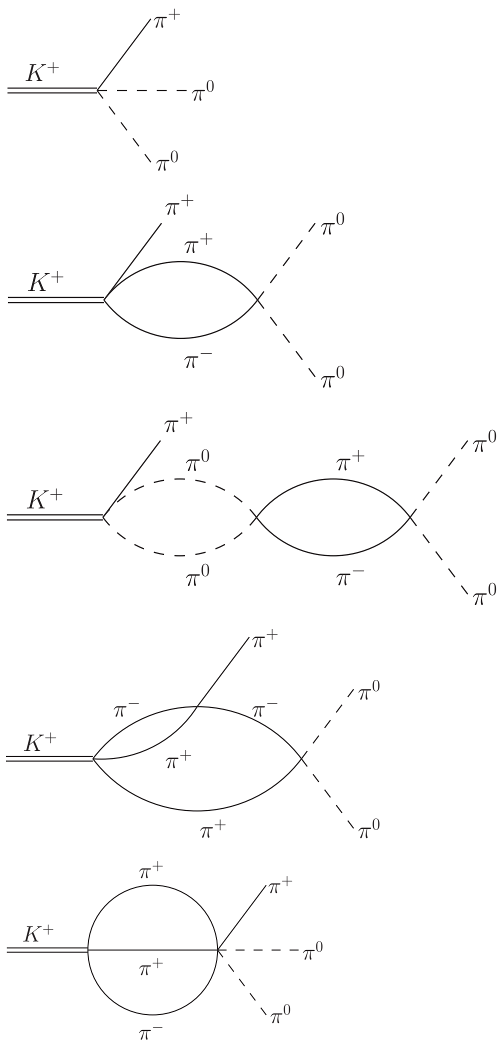
- correctly reproduces singularity structure at small momenta $|\mathbf{p}| \ll M_\pi$, subsumes far-away singularities in effective couplings

- interaction terms:

$$\begin{aligned}
\mathcal{L}_{\pi\pi} &= C_x\big(\pi_-^\dagger\pi_+^\dagger(\pi_0)^2 + h.c.\big) + \ldots \; , \qquad C_x \propto (a_0 - a_2)\{1 + \ldots\} \; , \\
\mathcal{L}_{K3\pi} &= \frac{G_0}{2}\big(K_+^\dagger\pi_+(\pi_0)^2 + h.c.\big) + \frac{H_0}{2}\big(K_+^\dagger\pi_-(\pi_+)^2 + h.c.\big) + \ldots
\end{aligned}$$

- Lagrangian-based QFT, analyticity + unitarity obeyed

Representation of $K \longrightarrow 3\pi$ amplitude up to two loops



$$\mathcal{M}^{\text{tree}} = G_0 + G_1(p_3^0 - M_\pi) + \ldots$$

$$\mathcal{M}^{1-\text{loop}} = B_1 J_{+-}(s_3) + B_2 J_{00}(s_3) + \left[B_3 J_{+0}(s_1) + (s_1 \leftrightarrow s_2)\right]$$

$$\mathcal{M}^{2-\text{loop}} = 2G_0 C_x^2 \underbrace{J_{+-}(s_3) J_{00}(s_3)}_{\text{double loops}} + \ldots$$

$$+ \quad 4 H_0 C_x C_{+-} \underbrace{F_+(\ldots; s_3)}_{\text{overlapping loops}}$$

$$+ \quad \mathcal{O}(i\,\epsilon^4) \quad [\not\propto \text{ scatt. lengths}]$$

- complete representation up to $\mathcal{O}(\epsilon^4)$, $\mathcal{O}(a\epsilon^5)$, $\mathcal{O}(a^2\epsilon^2)$
- compared to ChPT: valid to all orders in the quark masses

Part II:

What's new? $\longrightarrow$ radiative corrections

Bissegger, Fuhrer, Gasser, BK, Rusetsky, in preparation

compare also Nehme (2004), Bijnens & Borg (2005),
Gevorkyan et al. (2007), Isidori (2008)

Including photons

Aim:

$$\left.\frac{d\Gamma}{ds_3}\right|_{E_\gamma<E_{\text{max}}} = \frac{d\Gamma(K\to 3\pi)}{ds_3} + \left.\frac{d\Gamma(K\to 3\pi\gamma)}{ds_3}\right|_{E_\gamma<E_{\text{max}}} + \mathcal{O}(\alpha^2)$$

$$= \underbrace{\Omega(s_3, E_{\text{max}})}_{\text{"external"}} \underbrace{\frac{d\Gamma^{\text{int}}}{ds_3}}_{\text{"internal"}} + \mathcal{O}(\alpha^2)$$

to $\mathcal{O}(e^2 a^0 \epsilon^4)$ (all channels)
plus $\mathcal{O}(e^2 a^1 \epsilon^2)$ ("main" modes $K^+ \to \pi^0\pi^0\pi^+$, $K_L \to 3\pi^0$)

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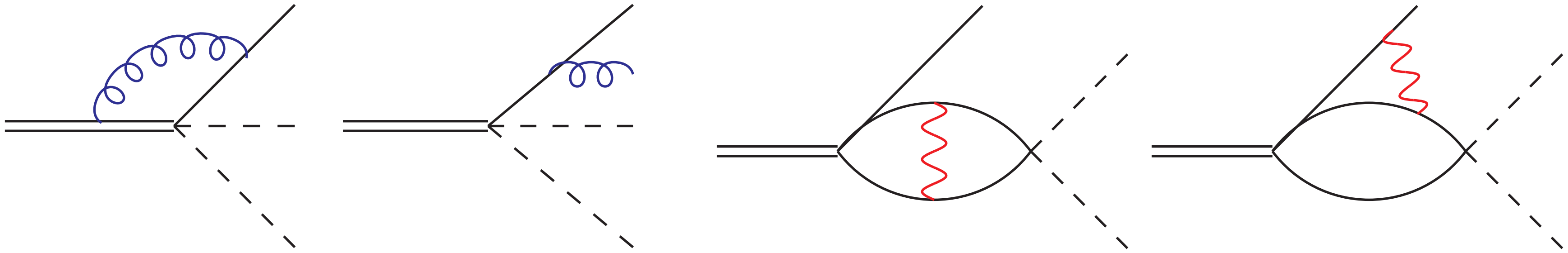
"external" (universal): "internal":



Including photons

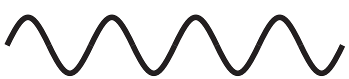
How-to:

- Lagrangian framework $\Rightarrow$ inclusion of photons straightforward via minimal substitution

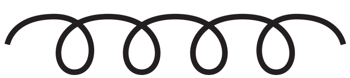
$$\partial_\mu \Phi_\pm \to (\partial_\mu \mp ieA_\mu)\Phi_\pm \,, \qquad \partial_\mu K_+ \to (\partial_\mu - ieA_\mu)K_+$$

- add all possible non-minimal gauge-invariant terms
- work in the Coulomb gauge $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$

A_0: Coulomb photons



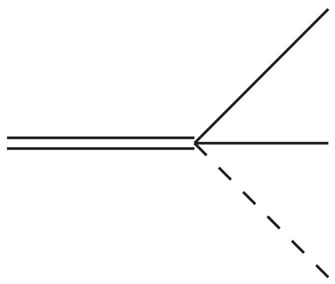
$\mathbf{A}$: transverse photons



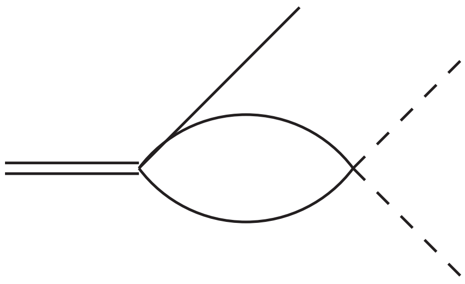
- eliminate A_0 via equation of motion $\longrightarrow$ non-local vertex

Power counting with photons (perturbative)

- addition of a Coulomb photon to a hadronic "skeleton":



$\mathcal{O}(1)$



$\mathcal{O}(a\,\epsilon)$

Power counting with photons (perturbative)

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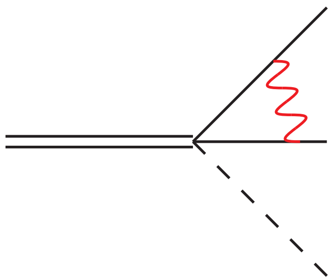

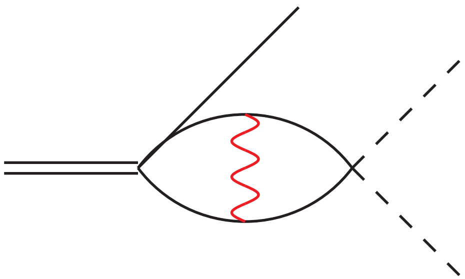

$$\mathcal{O}(1) \Rightarrow \mathcal{O}\left(\frac{e^2}{\epsilon}\right) \qquad \mathcal{O}(a\,\epsilon) \Rightarrow \mathcal{O}(a\,e^2\epsilon^0(\log\epsilon))$$

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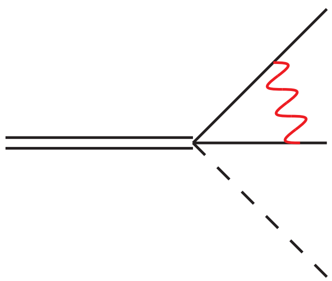

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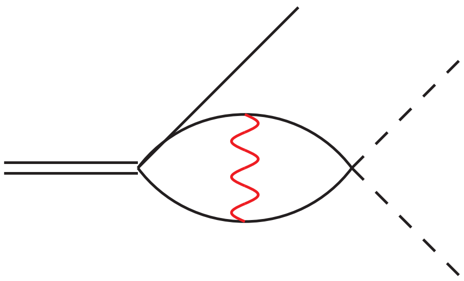
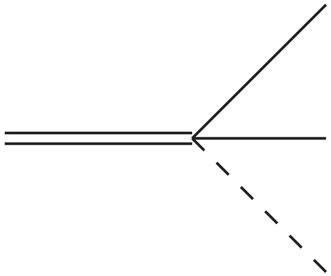
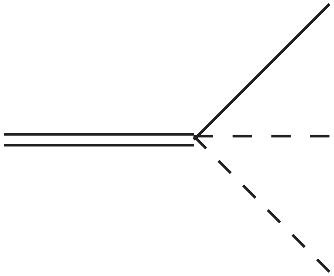


$$\mathcal{O}(a\,\epsilon) \Rightarrow \mathcal{O}(a\,e^2\epsilon^0(\log\epsilon))$$

- addition of a transverse photon to a hadronic "skeleton":



$$\mathcal{O}(1)$$



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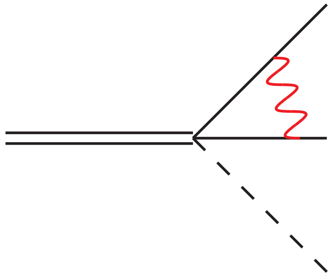

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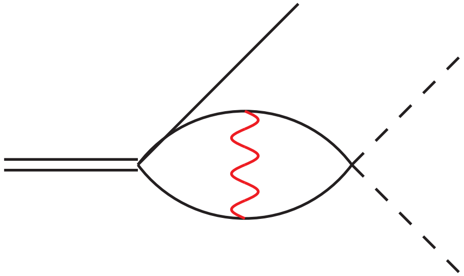

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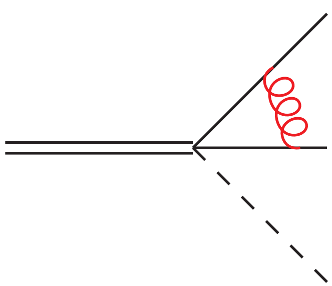
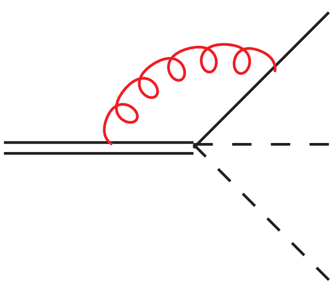


$$\mathcal{O}(1) \Rightarrow \mathcal{O}(e^2 \epsilon)$$

"soft"

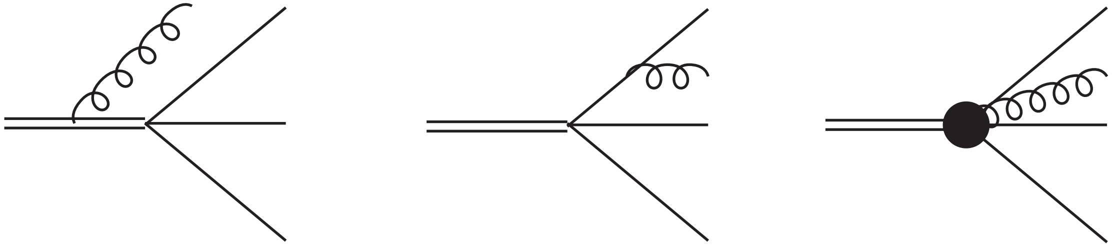


$$\mathcal{O}(1) \Rightarrow \mathcal{O}(e^2 \epsilon^2)$$

"ultrasoft"

Bremsstrahlung

- need to include radiation of real photons (cancels infrared divergences)



Bremsstrahlung

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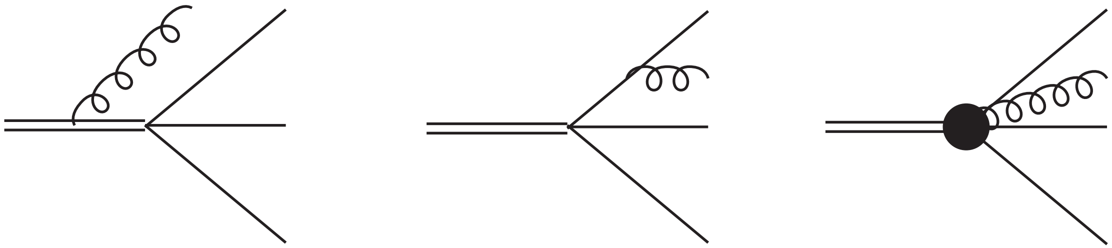

- power counting for decay spectra:

$$\frac{d\Gamma(K \to 3\pi)}{ds_3} = \mathcal{O}(\epsilon^2)$$

$$\frac{d\Gamma(K \to 3\pi\gamma)}{ds_3} = \mathcal{O}(e^2\epsilon^4)$$

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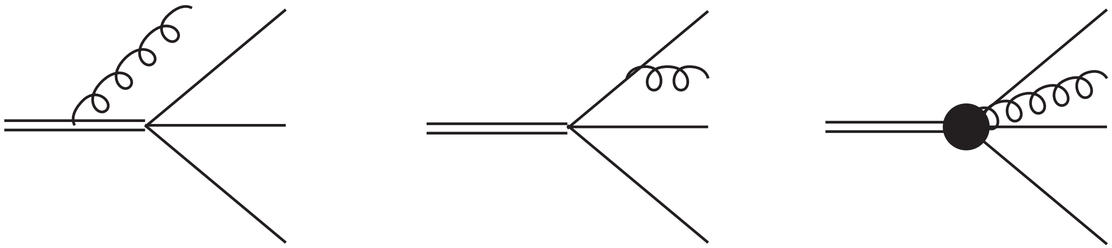

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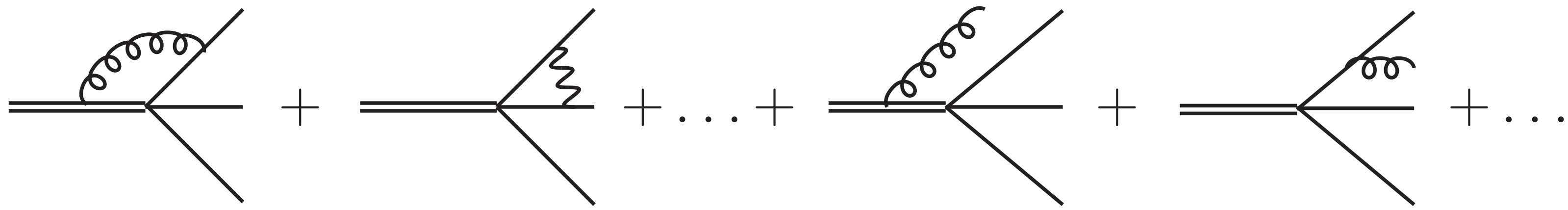
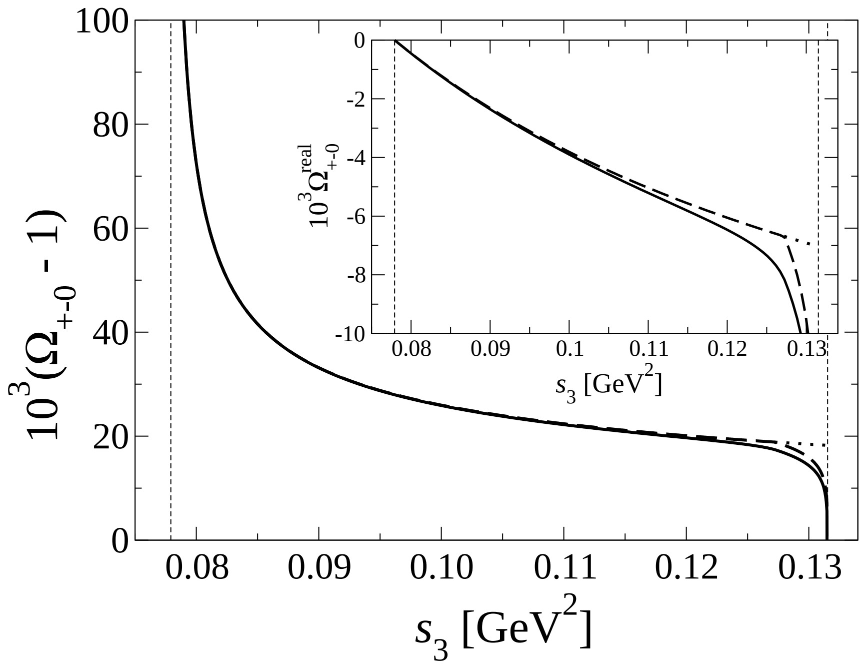
$\Rightarrow$ non-relativistic power counting shows why

- ▷ Coulomb photons are important
- ▷ (finite) Bremsstrahlung effects are very small

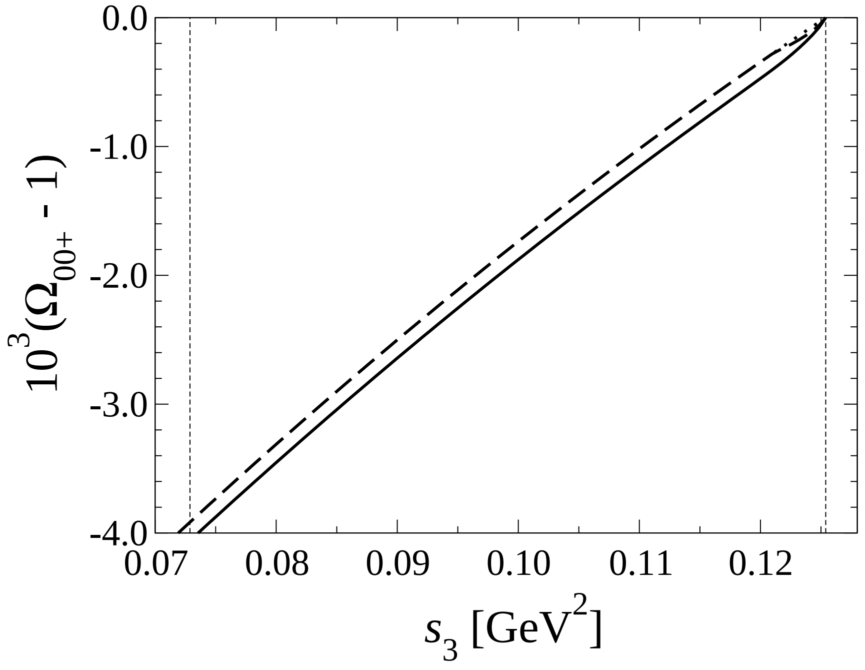
External/universal corrections: Ω_{+-0}, Ω_{00+}



$K_L \to \pi^+\pi^-\pi^0$:



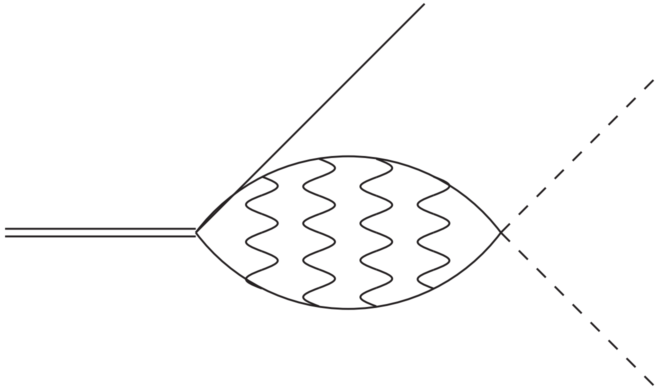
$K^+ \to \pi^0\pi^0\pi^+$:



- soft-photon approximation (dashed): small effect
- all except Coulomb pole small and smooth

Non-perturbative effects: pionium

- charged pions may get bound: pionium



ionisation energy: ~ 1.86 keV
ground state width: ~ 0.2 eV

- changes analytical structure at threshold $\Rightarrow \dfrac{\alpha}{v_\pm}$ not small

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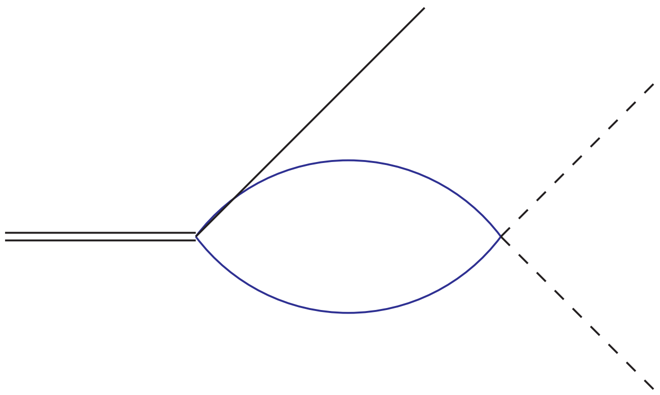

$$G(s) = \frac{i}{16\pi} v_\pm$$

$$v_\pm(s) = \begin{cases} i\sqrt{\frac{4M_{\pi^+}^2}{s} - 1}\,, & s < 4M_{\pi^+}^2\,, \\ \sqrt{1 - \frac{4M_{\pi^+}^2}{s}}\,, & s > 4M_{\pi^+}^2\,, \end{cases}$$

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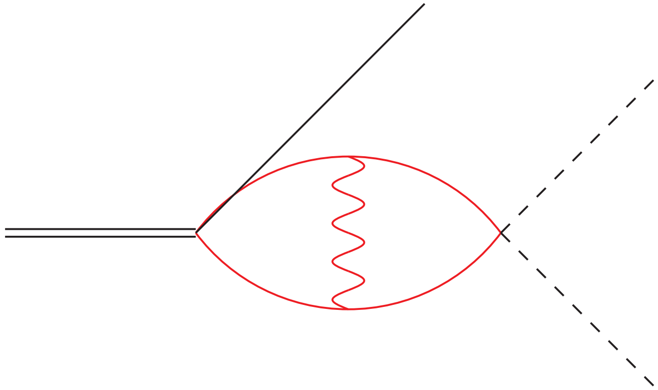

$$G(s) = \frac{i}{16\pi} v_\pm - \frac{\alpha}{32\pi}\left[\log\left(-v_\pm^2\right) + C\right]$$

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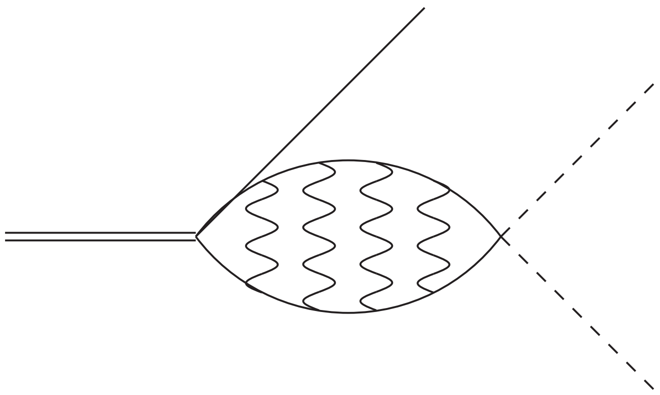

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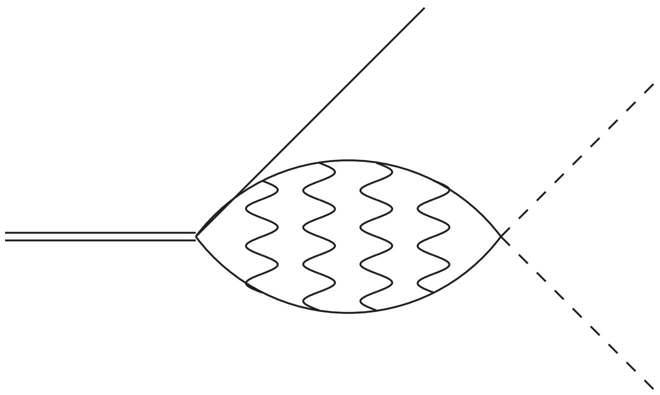

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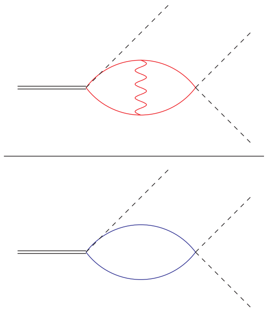
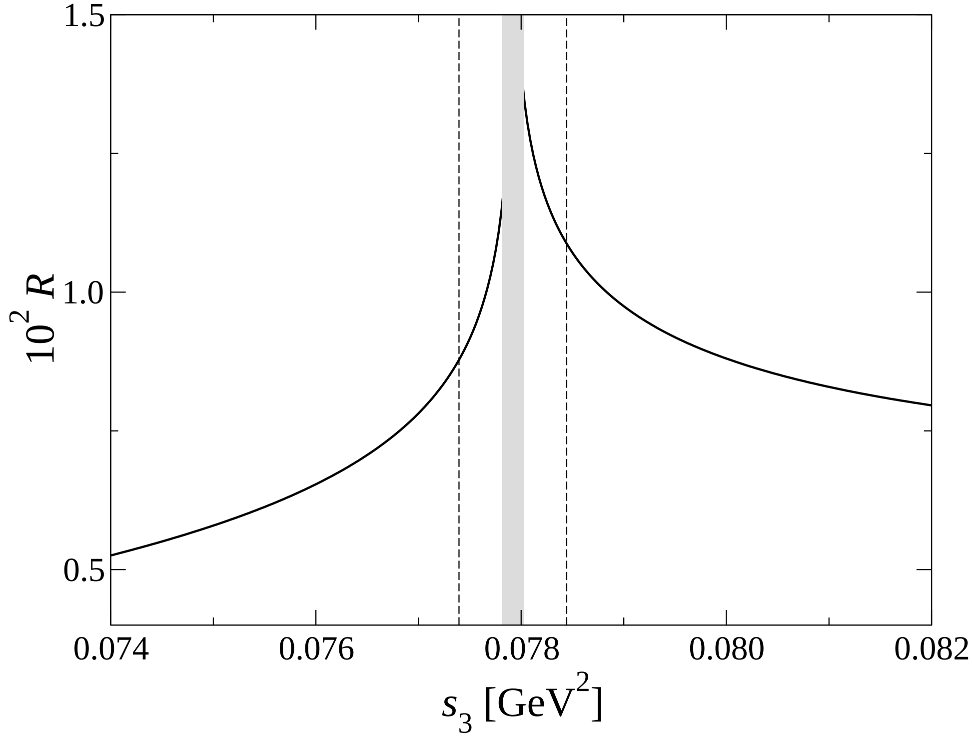
- strategy:
  - ▷ exclude region around the cusp
  - ▷ choose such that one-photon exchange $\mathcal{O}(a\,e^2)$ is sufficient

Internal corrections

- relative size of one-photon exchange:

$$R = \frac{\frac{d\Gamma}{ds_3}\Big|_C - \frac{d\Gamma}{ds_3}\Big|_0}{\frac{d\Gamma}{ds_3}\Big|_0} =$$





- "dashed" region: excluded in the NA48/2 analysis

Conclusions

- NRQFT provides systematic effective field theory framework for an analysis of cusp phenomena and $\pi\pi$ scattering lengths in $K \to 3\pi$ decays
- combined expansion in non-relativistic parameter ϵ
  and scattering lengths a
  currently performed up to $\mathcal{O}(\epsilon^4)$, $\mathcal{O}(a\epsilon^5)$, $\mathcal{O}(a^2\epsilon^2)$
- inclusion of radiative corrections:
  additional expansion parameter $e^2 = 4\pi\alpha$
- calculated decay spectra to
  $\mathcal{O}(e^2 a^0 \epsilon^4)$ for all $K \to 3\pi$ channels, plus
  $\mathcal{O}(e^2 a^1 \epsilon^2)$ for "main" channels $K^+ \to \pi^0\pi^0\pi^+$, $K_L \to 3\pi^0$
- formulae ready to be used in refined experimental analyses