Pion scattering lengths from the cusp effect analysis

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on behalf of the **NA48/2** Collaboration:

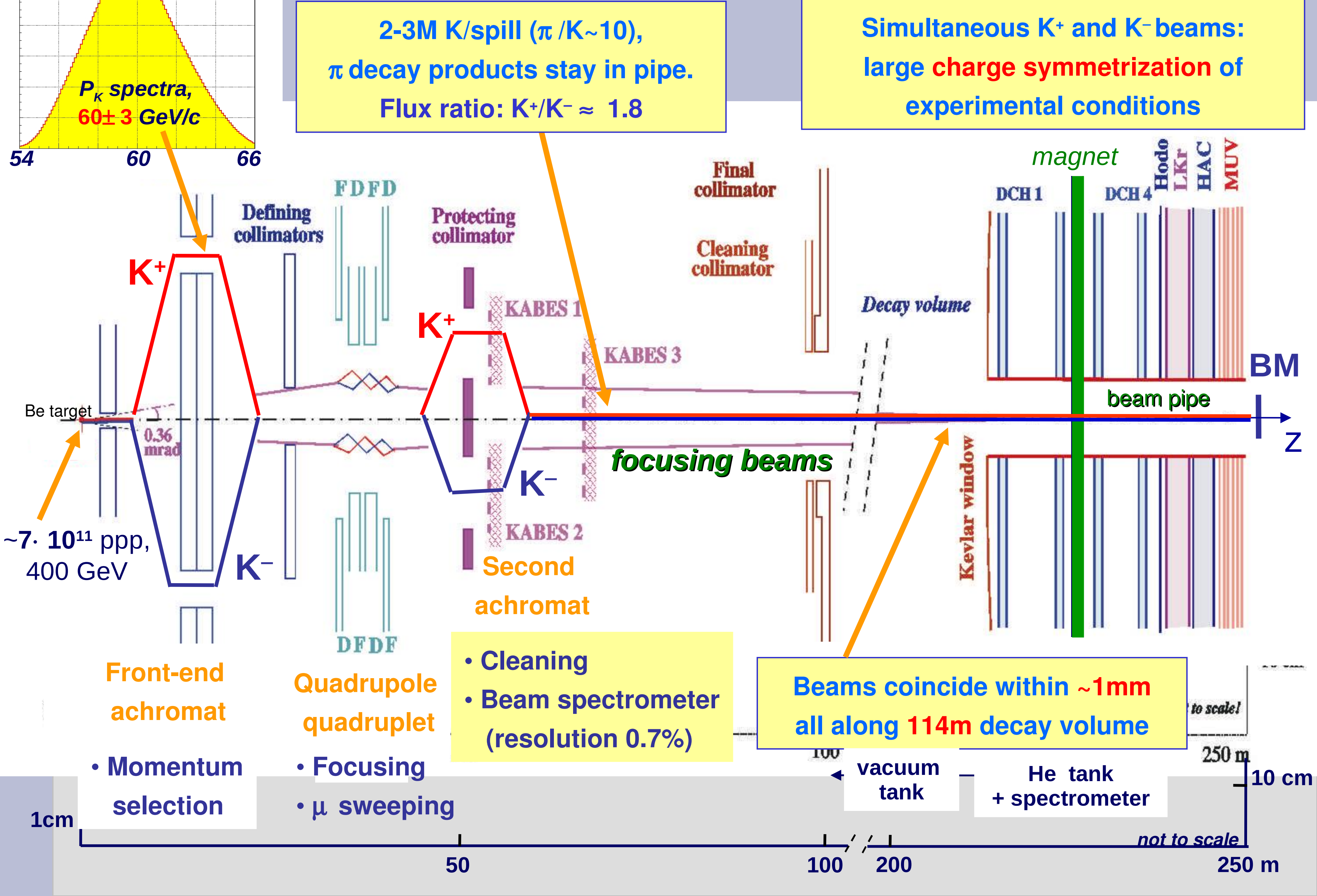
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Overview

- NA48/2 experiment at CERN: setup, data

- Study of $\mathbf{K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}}$ Dalitz plot distribution: preliminary final measurement of $\pi\pi$ scattering lengths ($a_0 - a_2$, a_2) and the Dalitz plot slopes

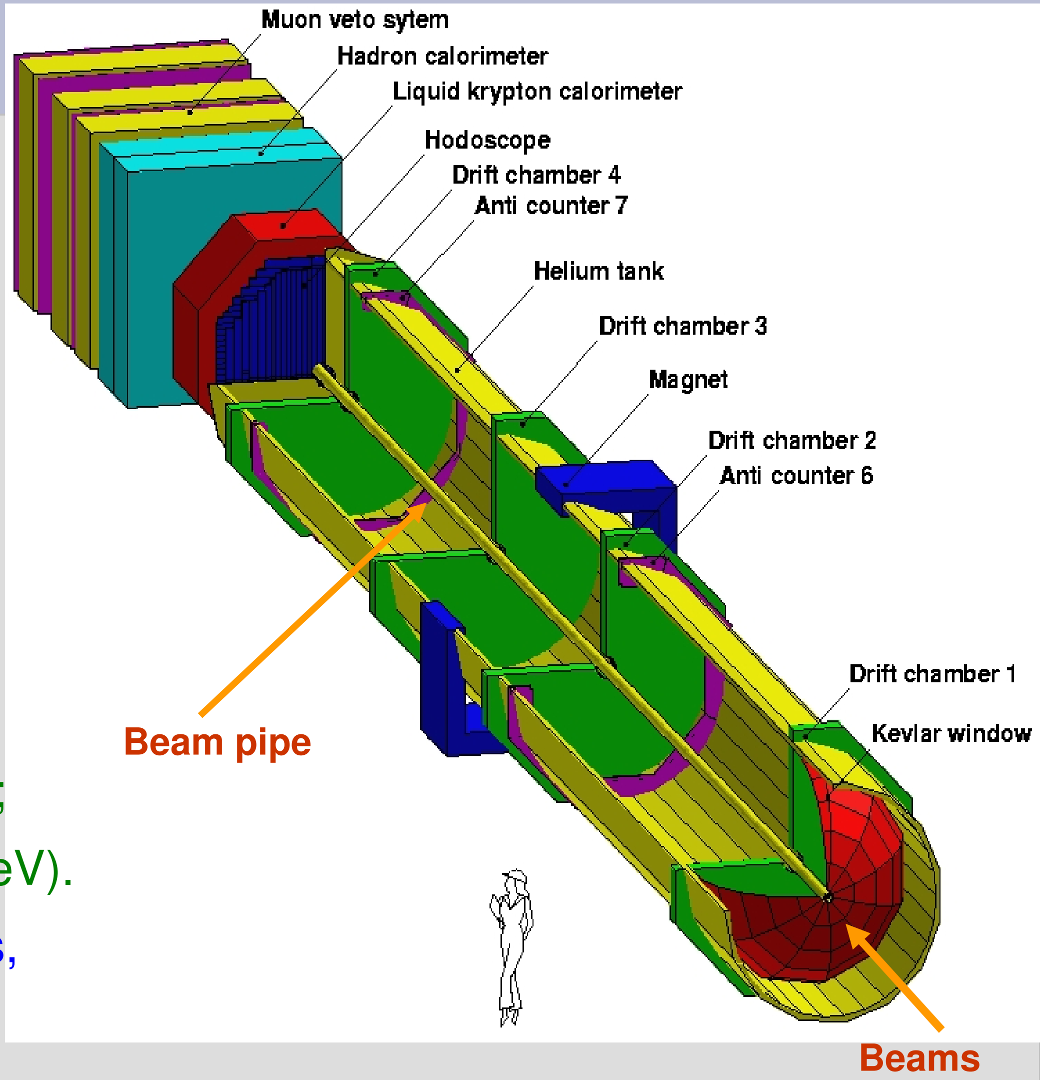
NA48/2 beam line



The NA48 detector

Main detector components:

- Magnetic spectrometer (4 DCHs):
  4 views/DCH: redundancy ⇒efficiency;
  used in trigger logic;
  $\Delta p/p = 1.0\% + 0.044\%*p$ [GeV/c].
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious;
  $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$ [GeV];
  $\sigma_x=\sigma_y=0.42/E^{1/2} + 0.6$mm (1.5mm@10GeV).
- Hadron calorimeter, muon veto counters,
  photon vetoes.



NA48/2 data:

2003 run: ~ 50 days

2004 run: ~ 60 days

A view of the NA48/2 beam line

Total statistics in 2 years:

$K^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$: ~$4 \cdot 10^{9}$

$K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$: ~$1 \cdot 10^{8}$

Rare $K^{\pm}$ decays:

BR's down to 10^{-9}

can be measured

>**200 TB of data recorded**

Pion scattering lengths

The important free parameter of ChPT is the quark condensate <qq>, it determines the relative size of mass and momentum terms in the power expansion.

a_0 and a_2 are S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2, correspondingly. They enter into all $\pi\pi$ scattering amplitudes.

The relation between <qq> and the scattering lengths a_0 and a_2 is known from this theory with a high precision, so the experimental measurement of a_0 and a_2 provides an important constrains for ChPT Lagrangian parameters.

Pion scattering lengths can be measured in the

study of the cusp-effect in $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ decays

[2003 result: PLB 633 (2006) 173, Cabibbo-Isidori theoretical framework]

$K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ selection

For each photon pair (i,k) a decay vertex reconstructed along beam axis under the assumption of $\pi^0\rightarrow\gamma\gamma$ decay

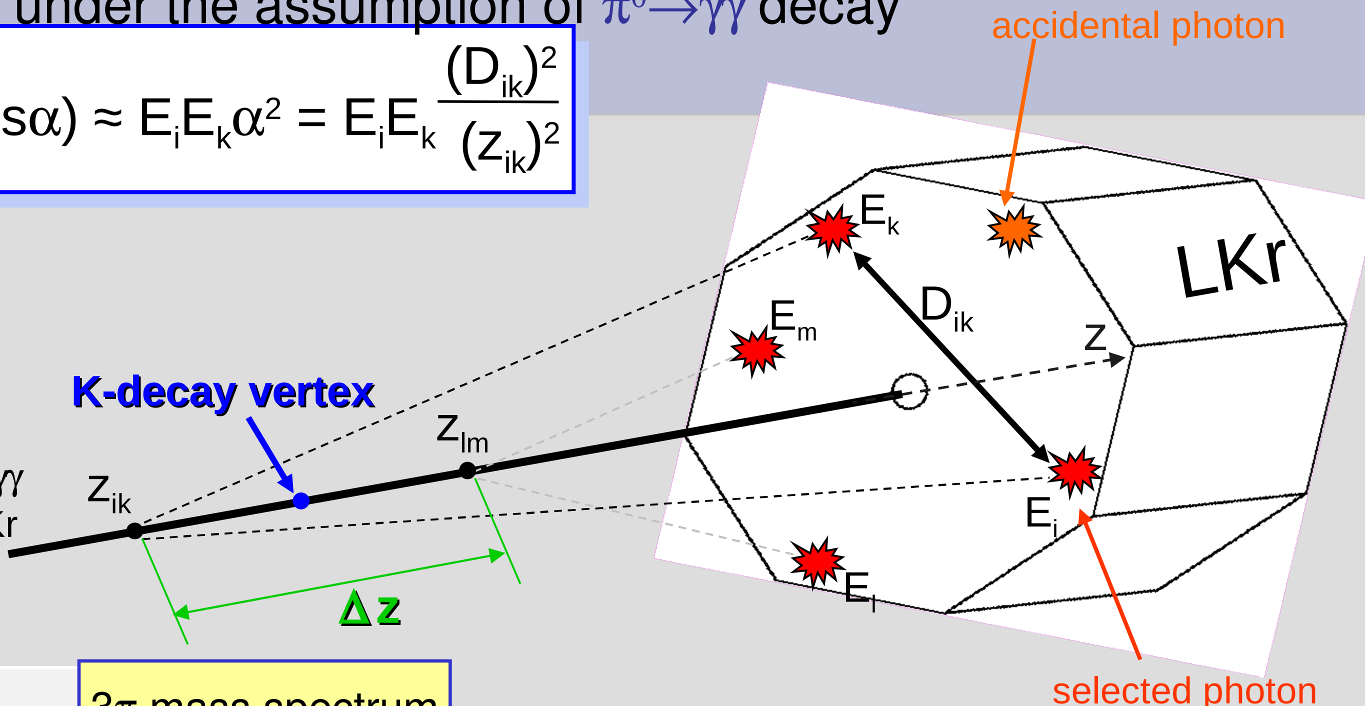
$$m_0^2 = 2E_iE_k(1-\cos\alpha) \approx E_iE_k\alpha^2 = E_iE_k\frac{(D_{ik})^2}{(z_{ik})^2}$$

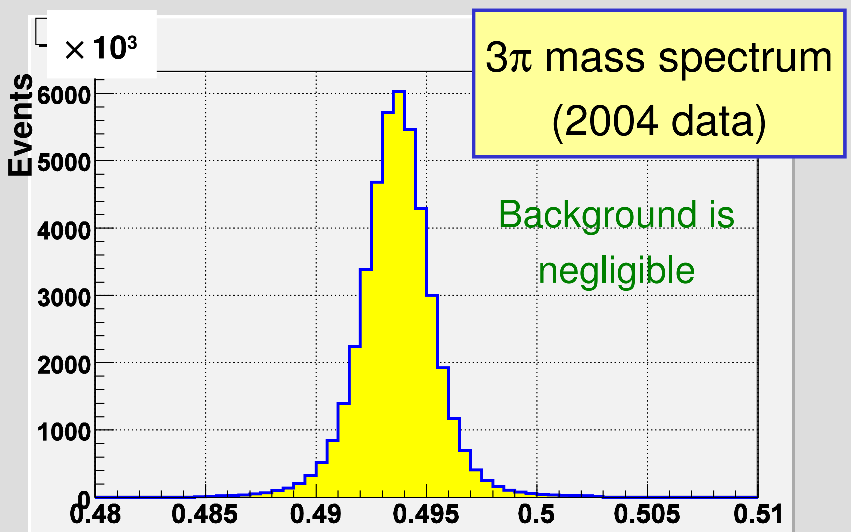
m_0^2 – mass of π^0

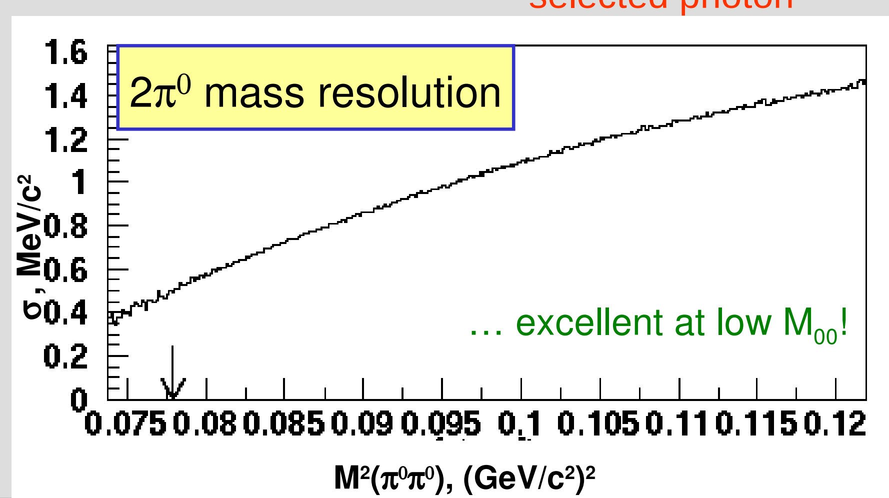
E_i, E_k – energy of γ_i, γ_k

D_{ik} – distance between γ_i and γ_k on LKr

z_{ik} – distance from $\pi^0\rightarrow\gamma\gamma$ decay vertex to LKr





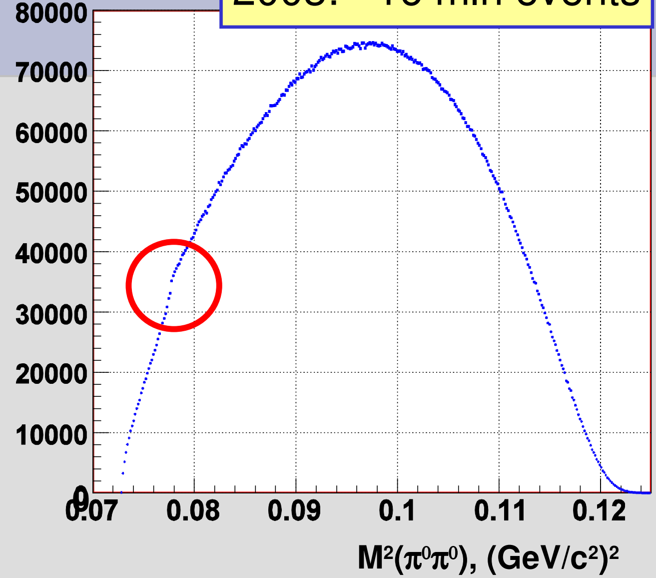


Observation of the cusp

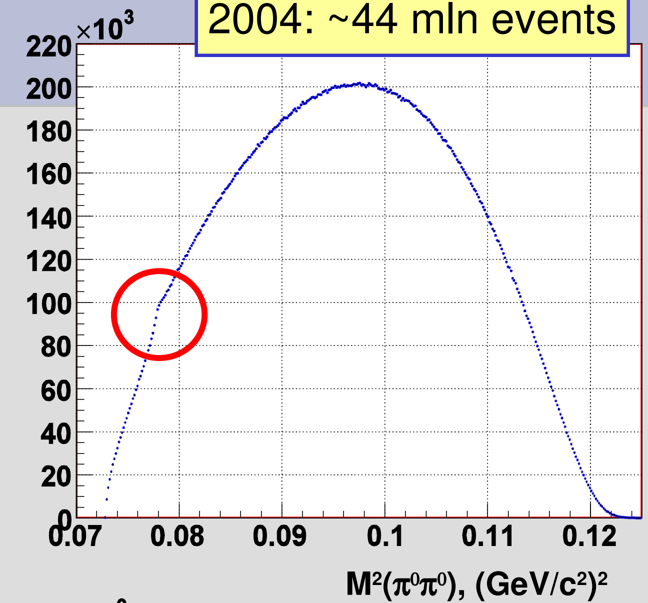
First observation of the cusp was made with the 2003 data

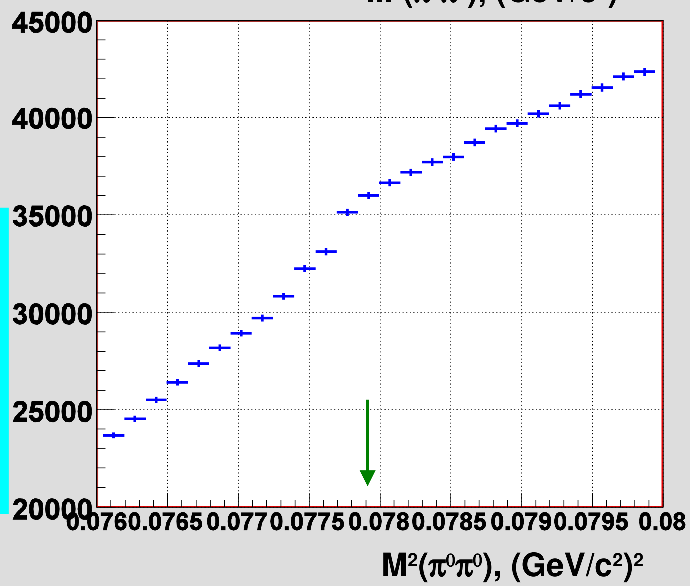
Inclusion of 2004 data: statistics increased by a factor of 3.7

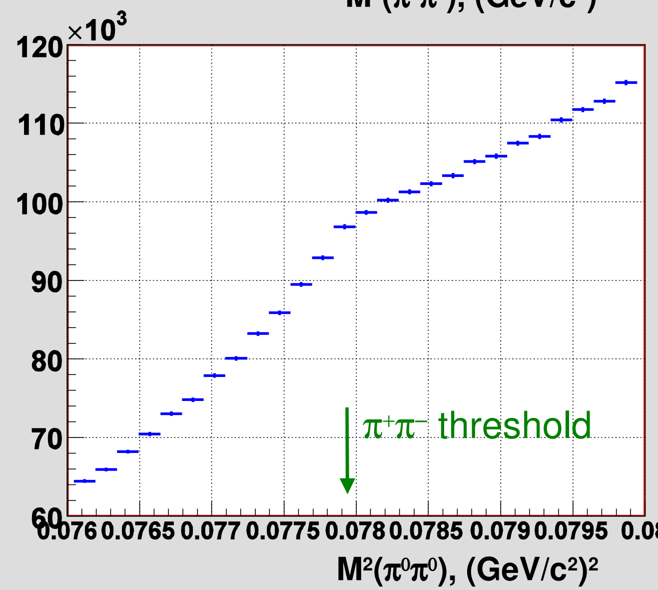
2003: ~16 mln events



2004: ~44 mln events







Now - uniform selection conditions and MC/Data statistics, so we sum the 2003 and 2004 data for the joint analysis

Theory: final state rescattering

N. Cabibbo, PRL 93 (2004) 121801

$$M(K^\pm \to \pi^\pm\pi^0\pi^0) = M_0 + M_1$$

Direct emission (k`,h` << g):

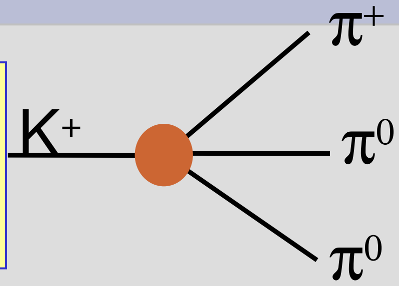
$$M_0 = A_0(1+g_0u/2+h'_0u^2/2+k'_0v^2/2)$$

$$M_+ = A_+(1+g_+u/2+h'_+u^2/2+k'_+v^2/2)$$

Kaon rest frame:

$u = 2m_K\cdot(m_K/3-E_{odd})/m_\pi^2$

$v = 2m_K\cdot(E_1-E_2)/m_\pi^2$



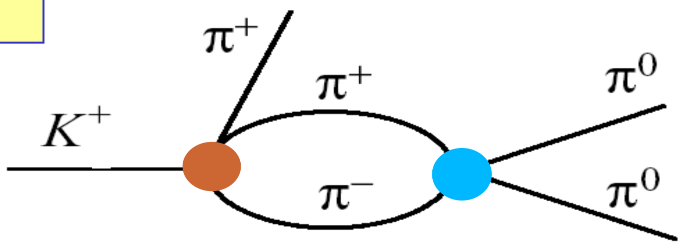
Rescattering amplitude:

$$M_1 = -2/3(a_0-a_2)m_+M_+\sqrt{1-\left(\frac{M_{00}}{2m_+}\right)^2}$$

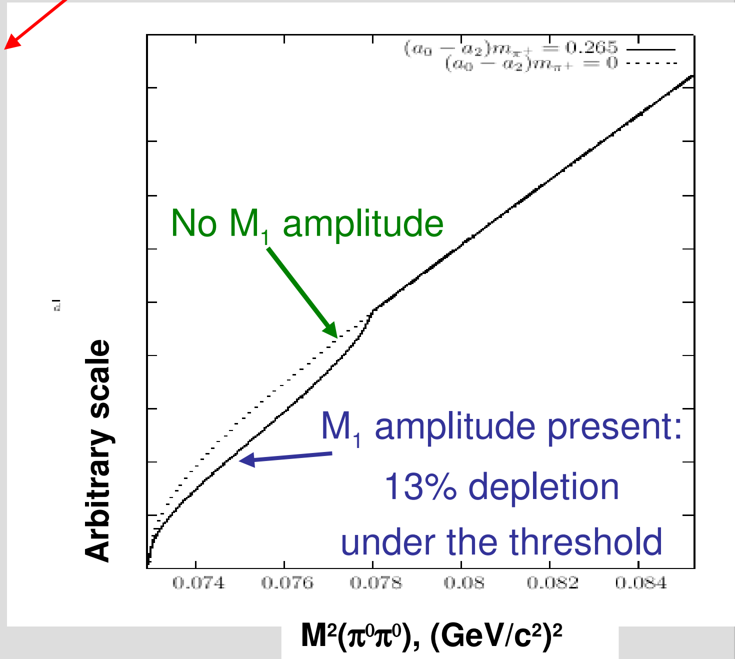
Combination of S-wave ππ scattering lenghts

(isospin symmetry assumed here)

$K^\pm \to 3\pi^\pm$ amplitude at threshold



Negative interference under threshold

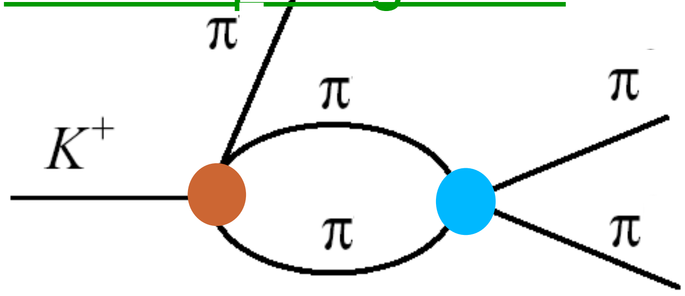


Theory: two-loop diagrams

N. Cabibbo and G. Isidori (CI), JHEP 503 (2005) 21

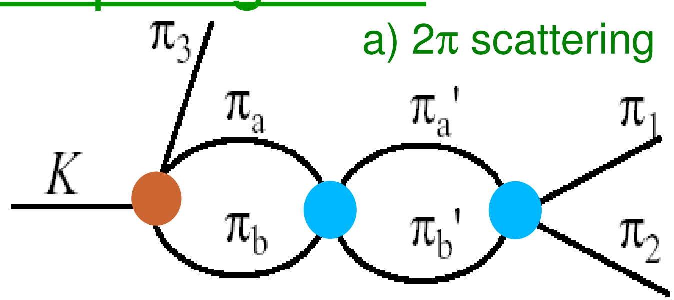
- Five S-wave scattering lengths (a_x, a_{++}, a_{+-}, a_{+0}, a_{00}) expressed as linear combinations of a_0 and a_2
- Isospin symmetry breaking accounted for following J. Gasser. For example, $a_x = (1+\varepsilon/3)(a_0-a_2)/3$, where $\varepsilon=(m_+^2-m_0^2)/m_+^2=0.065$ is isospin breaking parameter
- Radiative corrections missing; (a_0-a_2) precision ~5%
- ***V-dependent terms ~ $(k`/2)V^2$ introduced recently both into "unperturbed" $K^\pm\to\pi^\pm\pi^0\pi^0$ and $K^\pm\to\pi^\pm\pi^+\pi^-$ amplitudes.***

One-loop diagrams:

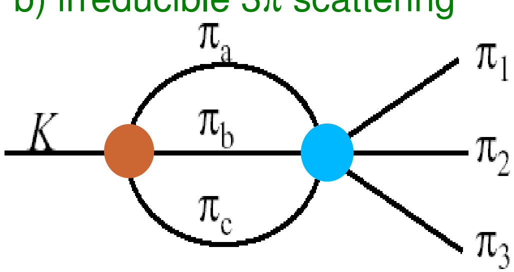


Two-loop diagrams:

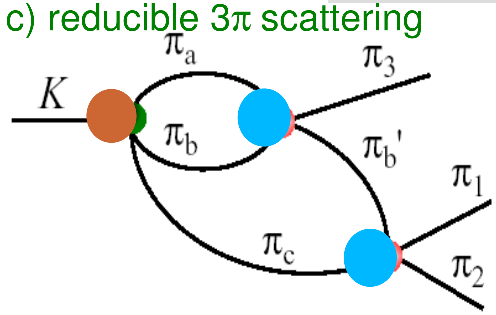
a) 2π scattering

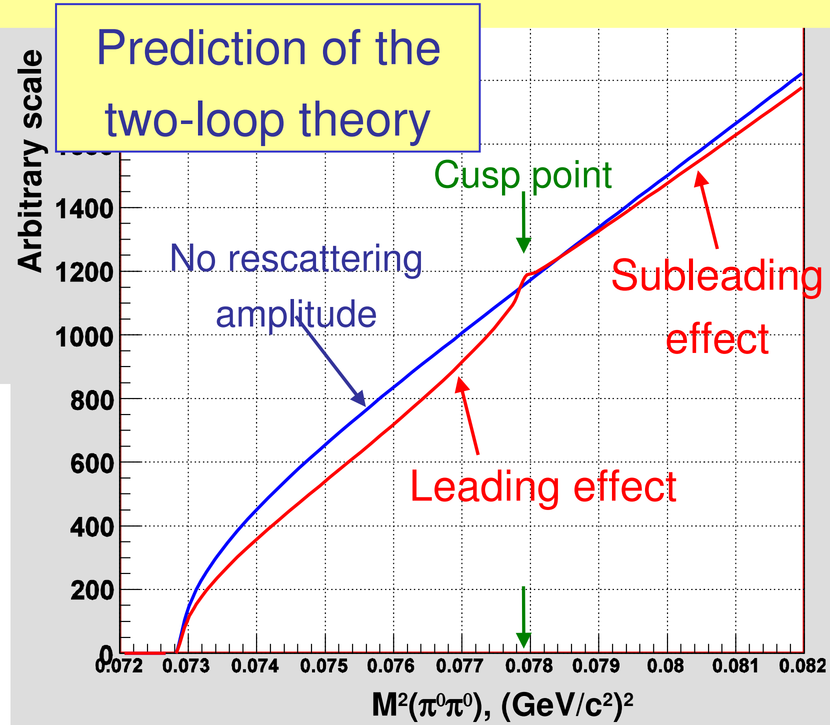


b) irreducible 3π scattering



c) reducible 3π scattering





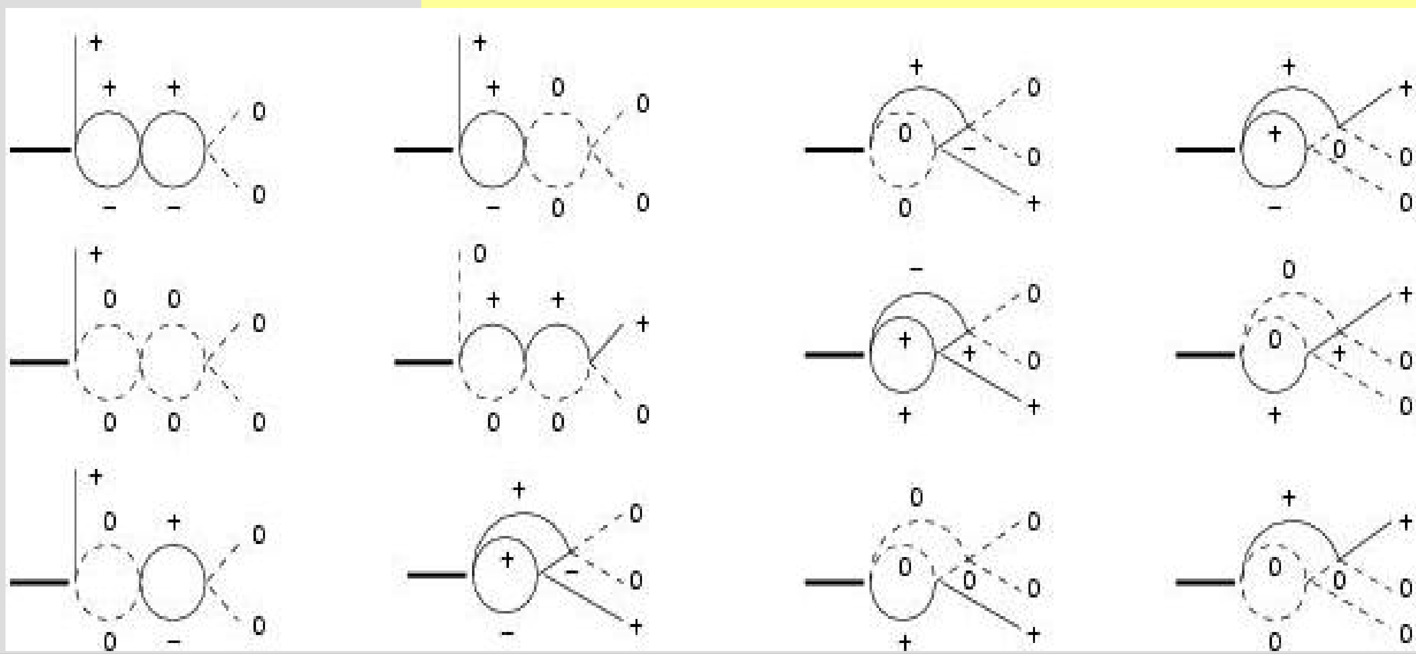
Theory: effective fields

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky (CGKR)

Phys.Lett. B638 (2006) 187-194

- Non-relativistic Lagrangian for effective fields; expanding in another small parameters.
- Valid in the whole decay region.
- Another (in comparison with CI) part of amplitude is absorbed in the polinomial terms (so another correlations).
- At two loops, algebraically different formulae for amplitude
- FORTRAN code written by authors

NA48 small technical intervention: polinomial parts of amlitudes are expressed in terms of (U,V)-slopes g,h,k (numerically different from CI ones), and faster integration of the amplitude over V.



Fitting procedure

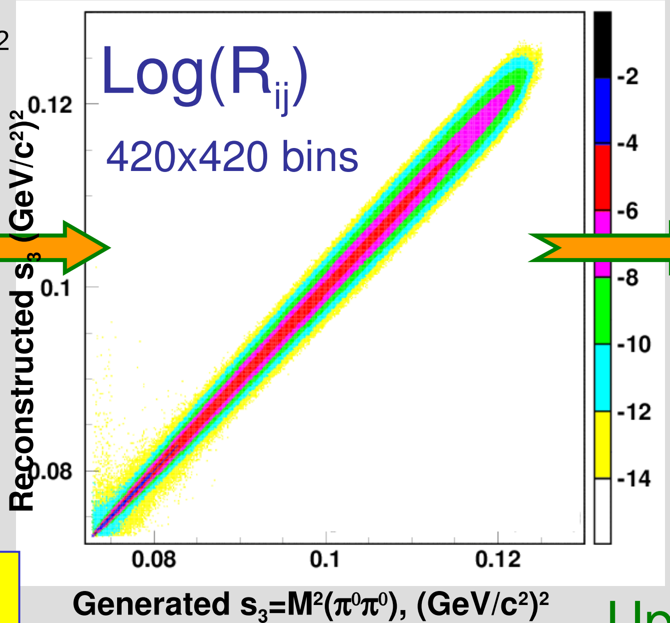
1-dimensional fit of the M_{00} projection

Detector response matrix R_{ij} obtained with a GEANT-based Monte-Carlo simulation

Bin width 0.00015 $(GeV/c^2)^2$

Generated distribution

$G(M_{00}) = G(g_0,h',a_0,a_2,M_{00})$



Reconstructed distribution:

$F_j^{MC} = \Sigma R_{ij} G_i$

MINUIT minimization of χ^2 of data/MC spectra shapes

Up to 5 free parameters

$$\chi^2(g,h',m_+(a_0-a_2),m_+a_2,N)=\sum_{s_3\ \text{bins}} \frac{(F_{DATA}-NF_{MC})^2}{\delta F_{DATA}^2+N^2\delta F_{MC}^2}$$

If one fix a_2 or link it to a_0 by some relation, result for (a_0-a_2) is more precise

Constants, used in the fit (recent development)

There are external experimental constants used in the amplitude calculation

The largest sensitivity is to the ratio of amplitude norms $R=A_+/A_0$, fixed from branching ratios and amplitude shapes: $R = 1.975 \pm 0.015$

- We fix the constant k'_0 of the V-dependent term $k'_0 v^2/2$ from our measurement by two methods: fit of 2d Dalitz plot 1) using 4d acceptance matrix and 2) by reweighting of all MC events in the selected sample. Result the measurement:
- $k'_0 = 0.00950 \pm 0.00017(stat) \pm 0.00048(syst) = 0.0095 \pm 0.0005$

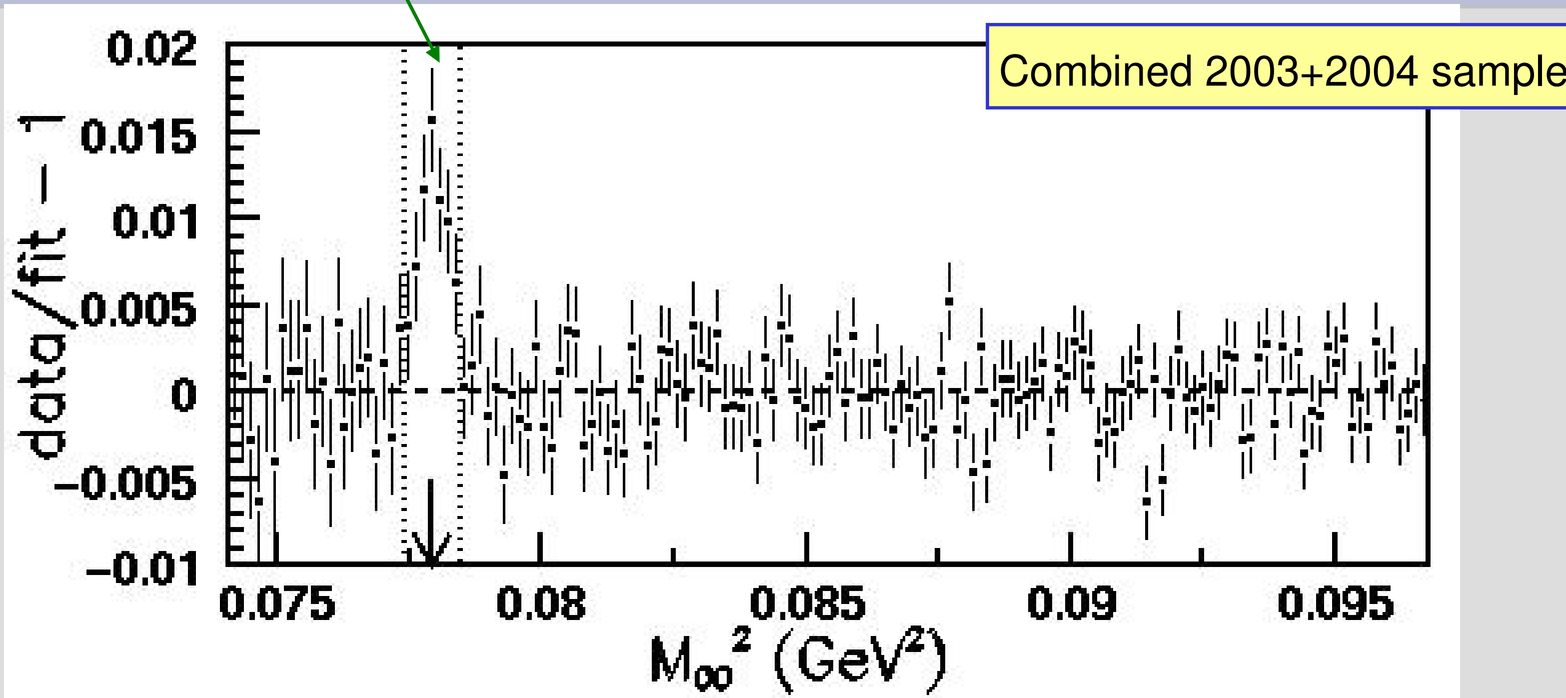
- For the CI fit we take the recently measured by NA48/2 parameters of the $K^\pm \to 3\pi^\pm$ amplitude $M_+ = A_+(1+g_+u/2+h'_+u^2/2+k'_+v^2/2)$:
- $g_+ = -0.21117(15)$; $h'_+ = 0.00671(26)$; $k'_+ = -0.00477(8)$

- For CGKR the similar parameters are not numerically the same due to the presence of additional rescattering terms in the in M_+ amplitude. To fix them: simultaneous fit of the NA48/2 $K^\pm \to 3\pi^\pm$ and $K^\pm \to \pi^\pm\pi^0\pi^0$ Dalitz plots by the CGKR code; result is:
- : $g_+ = -0.1837$; $h'_+ = 0.00043$; $k'_+ = -0.0059$

Pionium signature

Points excluded from the fit due to absence of EM corrections in the model

7 data bins skipped around the $M(\pi^+\pi^-)$ threshold



Excess of events in the excluded interval (CI fit),

if interpreted as due to pionium decaying as $A_{2\pi} \to \pi^0\pi^0$,

gives $R = \Gamma(K^\pm \to \pi^+ A_{2\pi}) / \Gamma(K^\pm \to \pi^\pm\pi^+\pi^-) = (1.8 \pm 0.3) \times 10^{-5}$.

⇨ Prediction [Z.K. Silagadze, JETP Lett. 60 (1994) 689]: $R = 0.8 \times 10^{-5}$.

Electromagnetic corrections to final state interactions in $K \to 3\pi$ decays

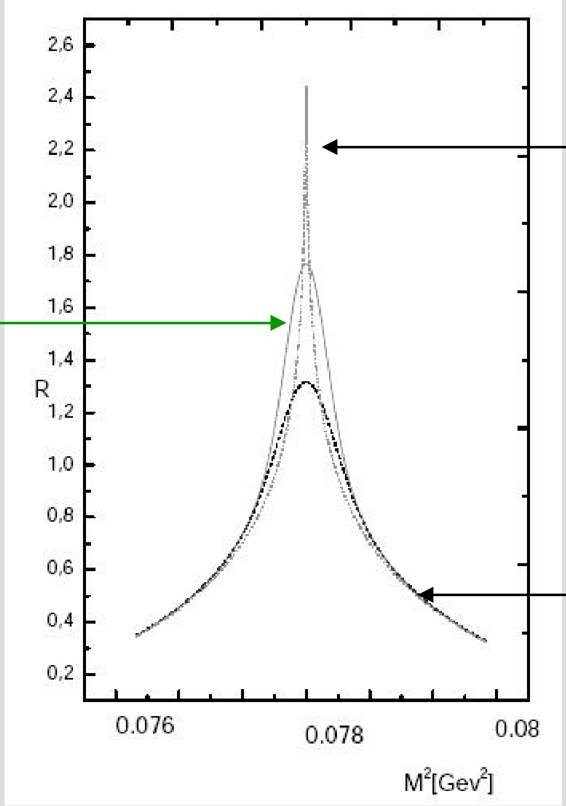
(Gevorkian, Tarasov, Voskresenskaya, hep-ph / 0612129)

Two contributions from $K^\pm \to \pi^\pm\pi^+\pi^-$ decay to the $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ cusp region:

- **Pionium formation : $\pi^+\pi^-$ atom $\to \pi^\circ\pi^\circ$ (negligible width)**
- **Additional $\pi^+\pi^-$ unbound states with resonance structure $\to \pi^\circ\pi^\circ$**

Our observable is an additional joint narrow (with respect to our resolution) contribution into the decay width at the very cusp point.

$\pi^+\pi^-$ atoms plus
$\pi^+\pi^-$ resonant structure
with experimental resolution



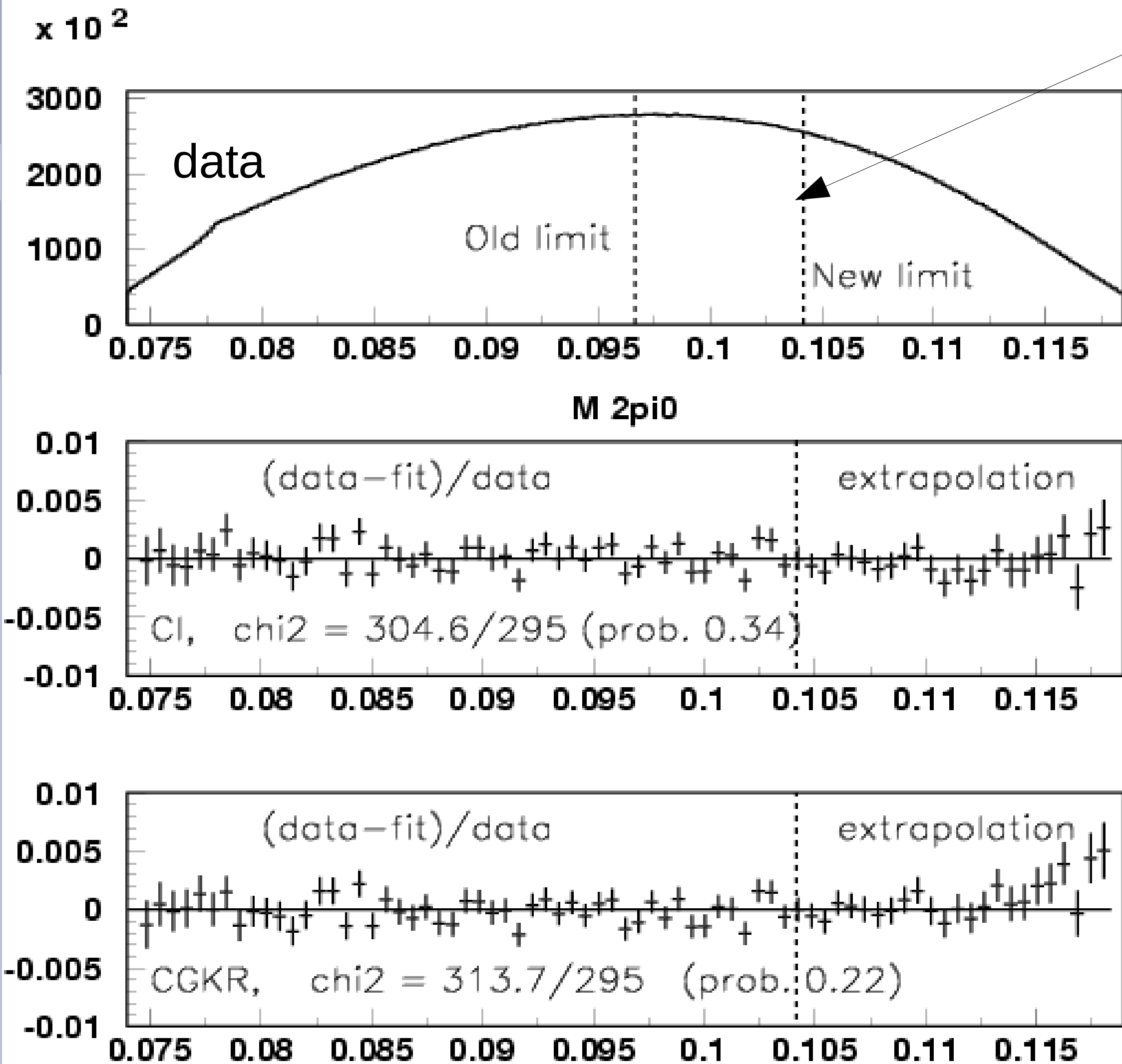
$\pi^+\pi^-$ resonant structure
(no experimental resolution)

$\pi^+\pi^-$ resonant structure
with experimental resolution

This combined narrow contribution N with respect to $\Gamma(K^\pm \to \pi^\pm\pi^\circ\pi^\circ)$:

$N / \Gamma(K^\pm \to \pi^\pm\pi^\circ\pi^\circ) =$
$(5.6 \pm 1.0) \times 10^{-5}$

Fit region and results validity area



New maximum is choosen to reach a minimum of total error (systematical one grows with maximum s_3) -
226-th bin instead of old 176-th

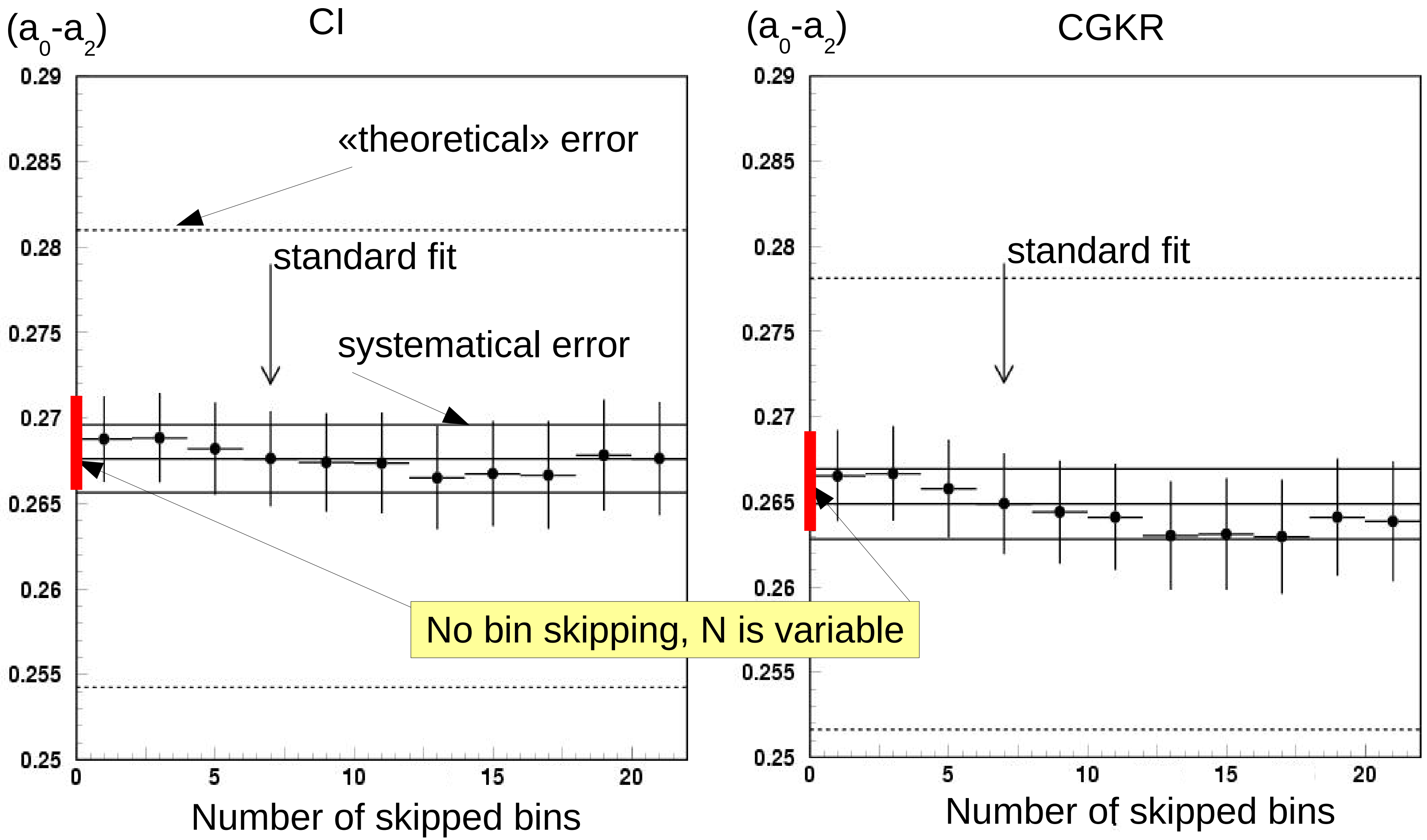
The fit results seem to be valid in the full area for both CI and CGKR parametrisations. On the plots the parameters are fixed from the fit region and extrapolated further.

Bins are combined by 4 for more clear picture, narrow contribution at the cusp point is fixed from the full fit

Stability vs skipped area around the cusp

With a chiral symmetry constraint (linked a_2 and a_0)

[Colangelo et al., PRL 86 (2001) 5008]:



(preliminary)

Uncertainties & results (1)

Using a chiral symmetry constraint [Colangelo et al., PRL 86 (2001) 5008]:

$a_2 = (-0.0444 +/- 0.0008) + 0.236(a_0-0.22) - 0.61(a_0-0.22)^2 - 9.9(a_0-0.22)^3$

CI :$(a_0-a_2)m_+ = 0.268 \pm 0.003_{stat.} \pm 0.002_{syst.} \pm 0.001_{ext.}$

CGKR: $(a_0-a_2)m_+ = 0.266 \pm 0.003_{stat.} \pm 0.002_{syst.} \pm 0.001_{ext.}$

➡ External uncertainty: mainly due to $R = (A_{++-}/A_{+00})|_{threshold} = 1.975 \pm 0.015$;

Theory precision uncertainty for CI case: $\delta(a_0-a_2)m_+ = 0.013$.

Table - in units of 10^{-4}

Source	CI	CGKR
Acceptance	4	4
Trigger efficiency	9	11
LKr resolution	2	2
LKr nonlinearity	10	11
LKr shower size	3	3
Kaon P spectrum	0	0
MC(T)	4	4
k' error	0	0
ChPT link error	5	3
Total systematics	*16*	*17*

(preliminary)

Uncertainties & results (2)

If one want to measure also a2, it become a free parameter of the fit

CI case:

$$(a_0 - a_2)m_+ = 0.266 \pm 0.005_{stat.} \pm 0.002_{syst.} \pm 0.001_{ext.}$$
$$a_2 m_+ = -0.039 \pm 0.009_{stat.} \pm 0.006_{syst.} \pm 0.002_{ext.}$$

Theory precision uncertainty: 5%.

CGKR case:

$$(a_0 - a_2)m_+ = 0.273 \pm 0.005_{stat.} \pm 0.002_{syst.} \pm 0.001_{ext.}$$
$$a_2 m_+ = -0.065 \pm 0.015_{stat.} \pm 0.010_{syst.} \pm 0.002_{ext.}$$

Theory precision uncertainty is not given.

- In CGKR case correlations between a_2 and polinomial terms of amplitude are larger.
- In CI polinomial parameters are very close to PDG ones, as scattering lengths look to be better disentangled with the slopes g and h'.

(preliminary)

Systematical errors in case of free a_2

In units of 10^{-4}

	CI	CI	CGKR	CGKR
Source \value	a0-a2	a2	a0-a2	a2
Acceptance	3	15	3	26
Trigger efficiency	9	32	11	53
LKr resolution	6	9	7	15
LKr nonlinearity	11	44	10	73
LKr shower size	3	9	2	10
Kaon P spectrum	5	8	5	14
MC(T)	4	2	4	4
k' error	2	4	2	7
Total systematics	***17***	***59***	***18***	***97***

(preliminary)

Results: Dalitz plot slopes

$$M(K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}) = M_0 + M_1$$

Unperturbed amplitude is $M_0 \sim (1+g_0u/2+h'u^2/2+k'v^2/2)$

NB: not the same parameterization as the PDG one:

$|M_0|^2_{(PDG)} \sim (1+gu+hu^2+kv^2)$ [$g_0 \approx g$, $h' \approx h - g^2/4$, $k' \approx k$]

- Technique:
  1. k' is extracted from 2-dimensional CI fits
  2. (a_0-a_2,g_0,h'); ChPT $a_2(a_0)$; fixed k' (k uncertainty is in systematics)

$$k' = -0.0095 \pm 0.0002_{stat.} \pm 0.0005_{syst.}$$

CI parameters:	CGKR parameters:
$g = 0.652 \pm 0.001_{stat.} \pm 0.003_{syst.}$	$g = 0.621 \pm 0.001_{stat.} \pm 0.003_{syst.}$
$h' = -0.039 \pm 0.001_{stat.} \pm 0.003_{syst.}$	$h' = -0.049 \pm 0.001_{stat.} \pm 0.003_{syst.}$

For the free a_2 the errors of g,h' slopes are larger.

(preliminary)

Sensitivity to V-dependence of amplitudes

Change in the case $k'_0=0$

Fit \value	δg_0	$\delta h'$	$\delta(a_0-a_2)$	δa_2
CI, link $a_2(a_0)$	-13	-13	1	0
CGKR, link $a_2(a_0)$	-13	-13	1	0
CI, a_2 free	-15	-11	-3	9
CGKR, a_2 free	-9	-8	-3	14

Here - in units of 10^{-3}

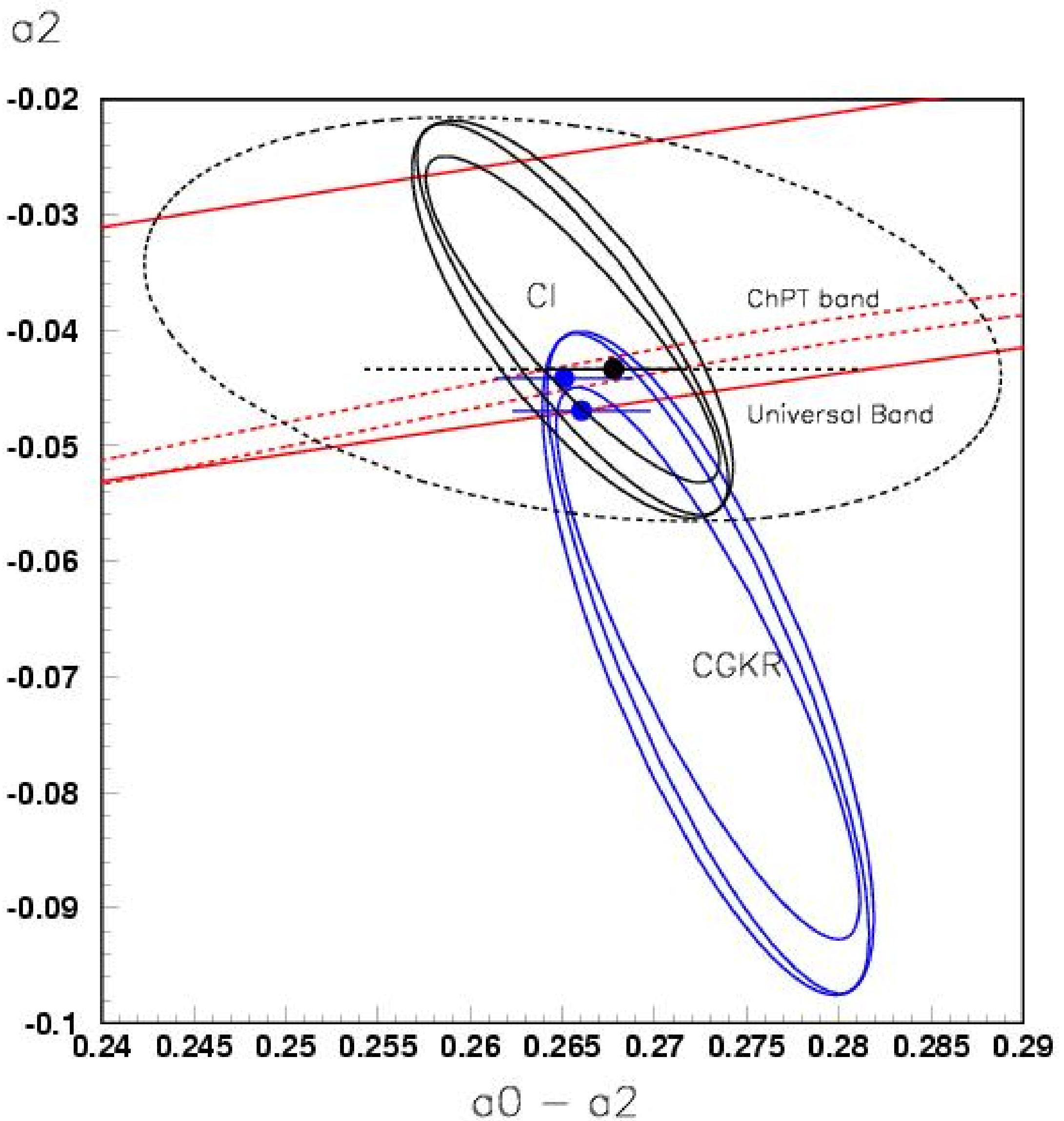
Change in the case $k'_+=0$ (and 2004 g_+,h_+ for CI)

Fit \value	δg_0	$\delta h'$	$\delta(a_0-a_2)$	δa_2
CI, link $a_2(a_0)$	1	0	-2	-1
CGKR, link $a_2(a_0)$	1	1	-3	-1
CI, a_2 free	1	0	-1	-2
CGKR, a_2 free	0	-1	-2	-5

To be taken into account (additively) in comparisons with the earlier 1D measurements, where both k' are assumed to be zero

(preliminary)

NA48/2 Cusp fit results



Red lines — theoretical limits;

Black (68.27% probability) ellipses — CI fit results with:

- -statistical only error;
- -statistical and systematical;
- -stat., syst. and external;
- -all including theoretical error;

Black circle — Ci fit with linked lengths (by Ch. constraint)

Blue ellipses — CGKR fit results with:

- -statistical only error;
- -statistical and systematical;
- -stat., syst. and external;

Blue circles — CGKR fits with a linked lengths (by Ch. Constraint and by the lower limit of Universal Band)

Conclusions

- Analysis of $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ decay Dalitz plot taking into account the cusp effect is in its final phase.
- Full statistics (2003+2004, 60 mln decays), a uniform and cross-checked analysis.
- Fits and systematics are done both for **Cabibbo-Isidori** and **Colangelo-Gasser-Kubis-Rusetsky** formulaes with a reasonable accordance, especially for the linked scattering lengthes.
- Full description of 2-dimensional Dalitz plot exists in two forms with two sets of parameters.