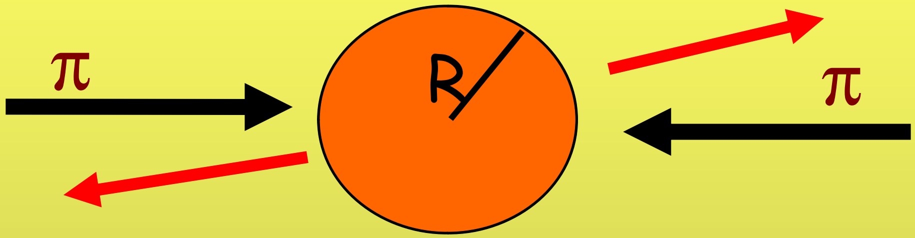
$\pi\pi$ scattering lengths measurement from K_{e4} and $K_{3\pi}$ decays at CERN SPS experiment NA48/2

Sergio Giudici
University of Pisa and INFN

On behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edimburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino , Wien

Heavy Quarks & Lepton
University of Melbourne, June 5-9 2008

$\pi\pi$ scattering lengths : why interesting ?



$$K = \frac{\sqrt{2mE}}{\hbar}$$

At low energy $KR \ll 1$ S-wave dominates total cross section
Isospin $I = 0,2$ only allowed by Bose statistics

Scattering matrix $S|\pi\pi\rangle = \exp(2i\delta)\,|\pi\pi\rangle$
may be parametrized with 2 phases: $\delta_{0,2} = a_{0,2}\;k$
related to scattering lengths $a_0\;,\,a_2$

2 clean measurements can be done
cusp-effect in $K_{3\pi}$ decay: $a_0\;,\,a_2$
phase shift in K_{e4} decays: $\delta_s - \delta_p \rightarrow a_0\;,\,a_2$

At low energy the S-wave scattering lengths are essential parameters of *Chiral Pertubation Theory* (CHPT)

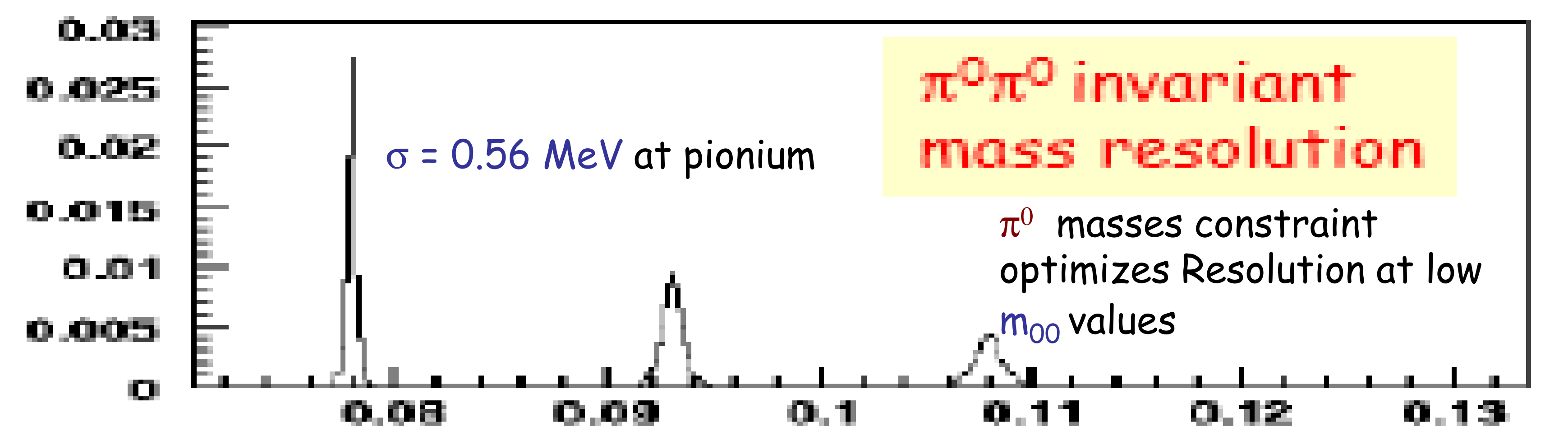
Cusp effect in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$

Initial curiosity was to observe $\pi^{+}\pi^{-}$ bound states ("pionium") annihilation in $\pi^{0}\pi^{0}$

$K^{\pm} \to \pi^{\pm} (\pi^{+}\pi^{-})_{\text{Bound = pionium}}$

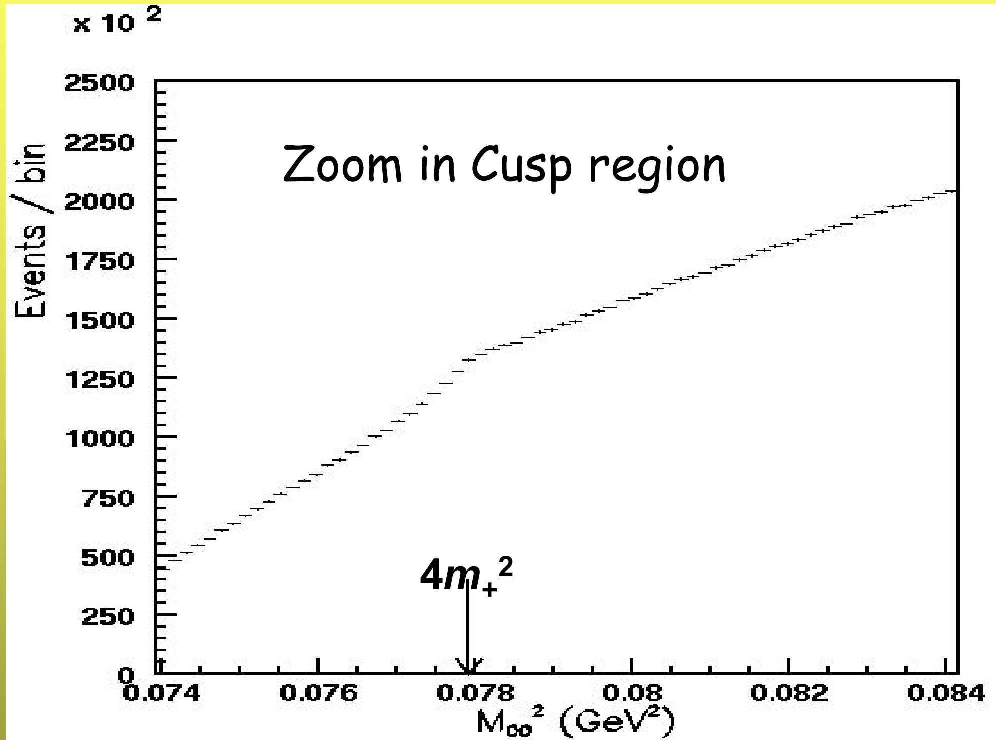
$\quad \to \pi^{0}\pi^{0}$

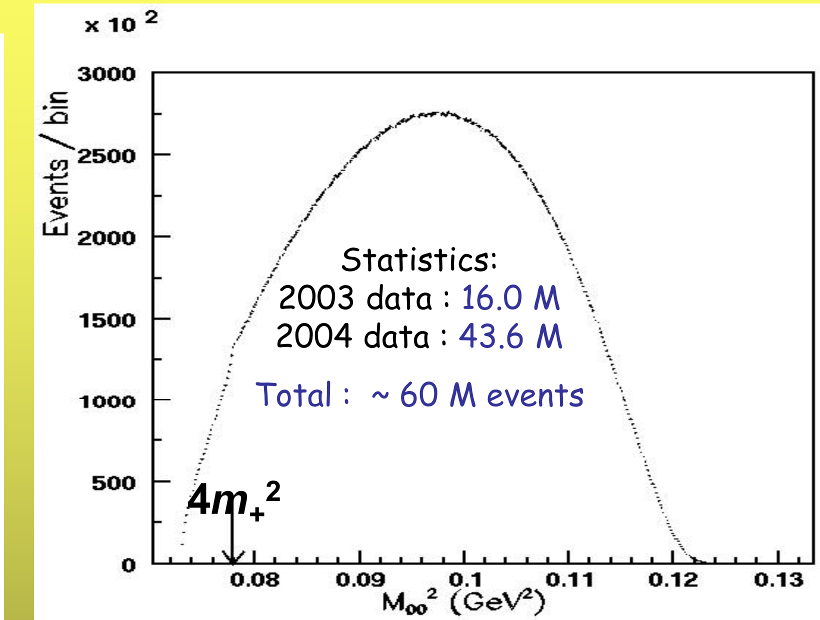
"Pionium" would produce an excess of events at the m_{00} = $2m_{+}$= 2x139.57 MeV



Good Resolution due to excellent NA48 LKR calorimeter performances and small Q-value

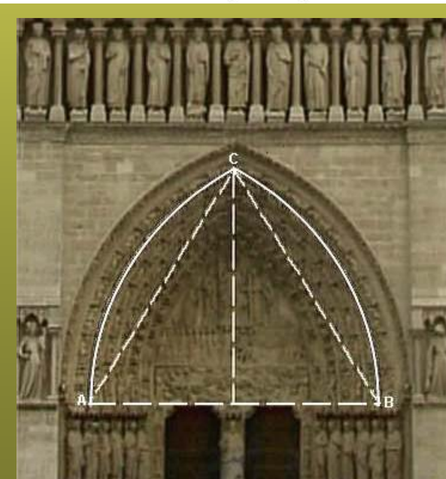
Cusp effect in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$





Clear visible discontinuity
in the first derivative = CUSP

☺ Unexpected Kaon Gothic behaviour ☺



Cusp in Notre Dame, Paris ...

Theory: ππ rescattering.

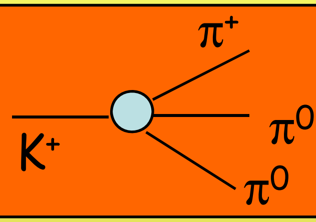
Decay Amplitude :

$M(K^\pm \to \pi^\pm\pi^0\pi^0) = M_0 + M_1 + M_2 + \ldots$

Dalitz plot variable

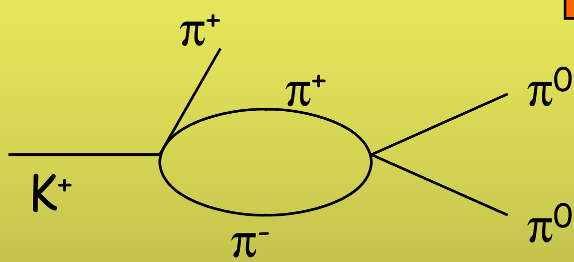
$u = 2m_K\,(m_K/3 - E_{odd})/m_\pi^2$

$v = 2m_K\,(E_1 - E_2)\,/m_\pi^2$



Direct emission

$M_0 = A_0\,(1 + g_0\,u/2 + h'u^2/2 + k'v^2/2)$



1 loop Rescattering

(N. Cabibbo , PRL 93 , 2004, 121801)

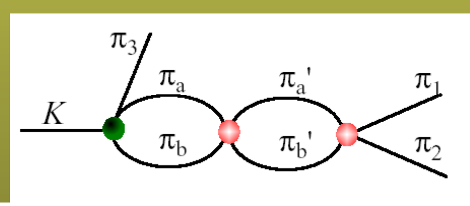
$$M_1 = -2/3\,(a_0 - a_2)\,m_+\,A_+\,\sqrt{1-\left(\frac{M_{00}}{2m_+}\right)^2}$$

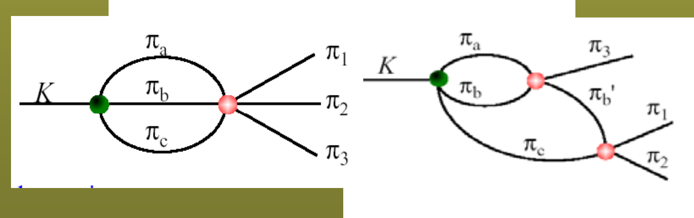
Combination of S-wave ππ scattering length

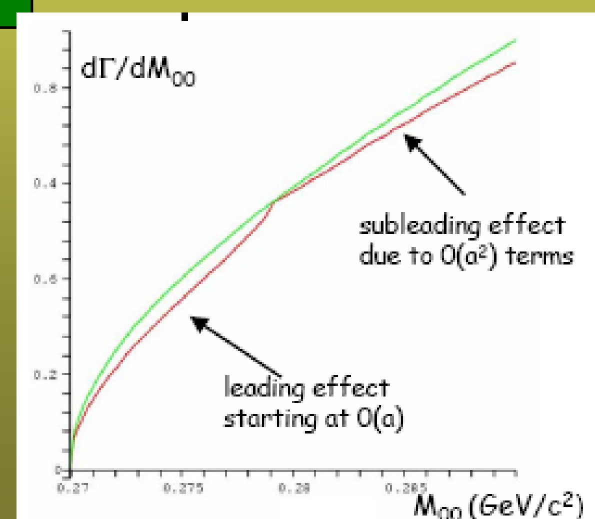
$K^\pm \to 3\pi^\pm$ Amplitude

2 loop Rescattering:

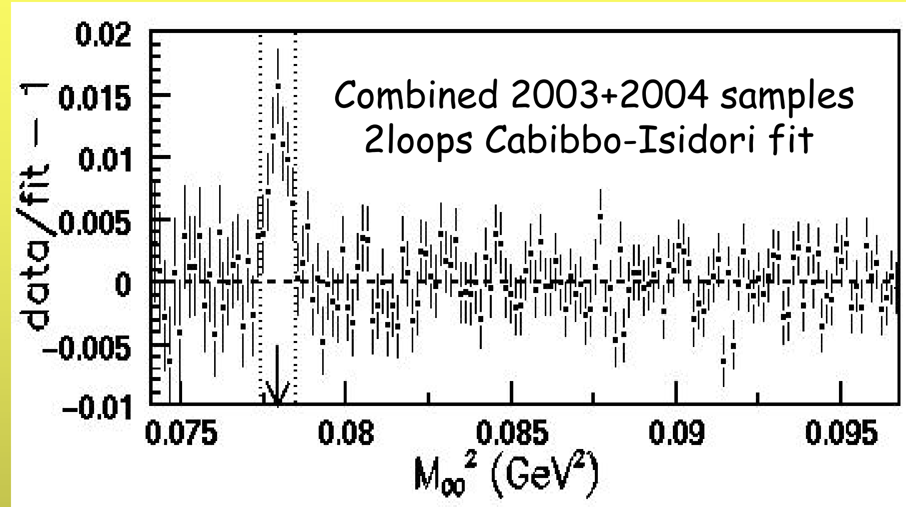
(N. Cabibbo & G. Isidori
JHEP 0503:021 , 2005)







Theory describes the data ... Fitting procedure



One dimensional fit to M_{00}^2 distribution
MINUIT minimization of χ^2
of data/MC spectra shapes

Fitting up to half spectrum
0.097 (GeV/c^2) since
Cabibbo Theory is an expansion
around $2m_+$ threshold

Fit to 5 parameters:
Norm, g, h' , $(a_0 - a_2)$ and a_2 (k' fixed)

For final result 7 bins around cusp
excluded from the fit : EM corrections
Not yet included in the model

The excess of events in this
region is interpreted as pionium
combined with E.M. corrections

$$\frac{\Gamma(K^\pm \to \pi^\pm A_{2\pi})}{\Gamma(K \to 3\pi)} = (1.82 \pm 0.21) \times 10^{-5}$$

Th. Prediction = 0.8×10^{-5}
(JTEP lett. 60, 1994, 689)

Scattering length from CUSP

$a_0 - a_2$	=	$0.261 \pm 0.006_{stat} \pm 0.003_{syst} \pm 0.0013_{ext} \pm 0.013_{th}$
a_2	=	$-0.037 \pm 0.013_{stat} \pm 0.009_{syst} \pm 0.002_{ext}$

External uncertainty: from the uncertainty on the ratio of $K^+ \to \pi^+\pi^+\pi^-$ and $K^+ \to \pi^+\pi^\circ\pi^\circ$ decay widths $A_+ / A_0 = 1.97 \pm 0.015$

Theoretical uncertainty on (a_0 – a_2) ± 5% DOMINATES !!!
(Cabibbo-Isidori Theory uncertainty from neglecting higher order diagrams and radiative corrections)

From (a_0 – a_2) and a_2 can be extracted a_0 (taken into account the statistical error correlation coefficient ≈ -0.92)

$$a_0 = 0.224 \pm 0.008_{stat} \pm 0.006_{syst} \pm 0.003_{ext} \pm 0.013_{th}$$

Uncertainties - CUSP method

Systematic effect	$(a_0-a_2) \times 10^2$	$a_2 \times 10_2$
Analysis technique	± 0.10	± 0.20
Trigger inefficiency	± negl.	± 0.50
Description of resolution	± 0.06	± 0.11
LKR non linearity	± 0.06	± 0.26
Geometric Acceptance	± 0.02	± 0.01
MC sample	± 0.03	± 0.21
Simulation of LKR shower	± 0.17	± 0.38
V – dependence on amplitude	± 0.17	± 0.38
TOTAL Systematic	± 0.28	± 0.90

Comparison: NA48 vs DIRAC

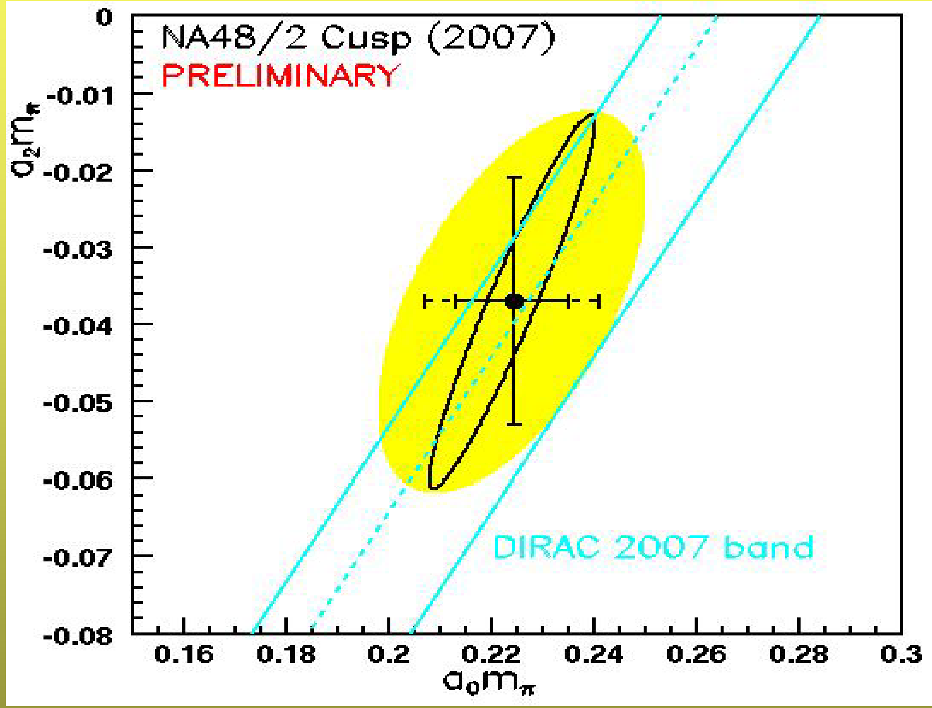
DIRAC experiment measured pionium 1S state lifetime to be

$$\tau_{1S} = 2.91 \,^{+0.49}_{-0.62} \text{ fs}$$

Corresponding to

$$|a_0 - a_2| = 0.264 \,^{+0.033}_{-0.020}$$

(PLB 619, 50, 2005)



Black Ellipse = NA48 CUSP measurement (Statistical systematic error)

Yellow area = theoretical uncertainty in Cabibbo-Isidori Model (assumed Gaussian)

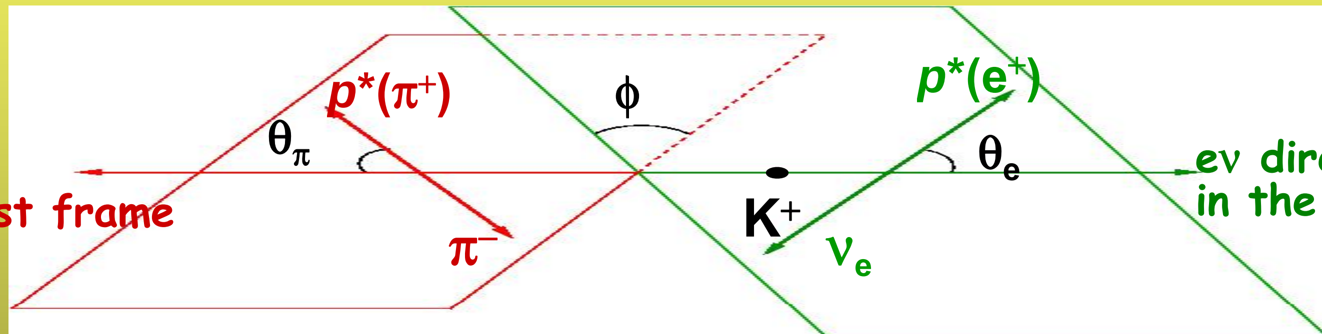
$K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$: Theory

5 kinematic variables (Cabibbo – Maksymowicz)

N.B. Kaon and electron with same sign $\Delta S = \Delta Q$ rule

$S_\pi = M^2_{\pi\pi}$, $S_e = M^2_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and Φ

ππ direction in the K⁺ rest frame



eν direction in the K⁺ rest frame

Partial wave (S,P) expansion of the Amplitude:

F,G = Axial Form Factors

$F = F_S\, e^{i\delta s} + F_P\, e^{i\delta p} \cos\theta_\pi$ + d-wave term

$G = G_P\, e^{i\delta g}$ + d-wave term

H = Vector Form Factor

$H = H_P\, e^{i\delta h}$ + d-wave term

Expansion in power of $q^2 = S_\pi/4m^2_\pi$

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e\left(S_e/4m_\pi^2\right) + \ldots$$

$$F_p = f_p + f_p' q^2 + \ldots$$

$$G_p = g_p + g_p' q^2 + \ldots$$

$$H_p = h_p + h_p' q^2 + \ldots$$

Assuming same phase for F,G,H

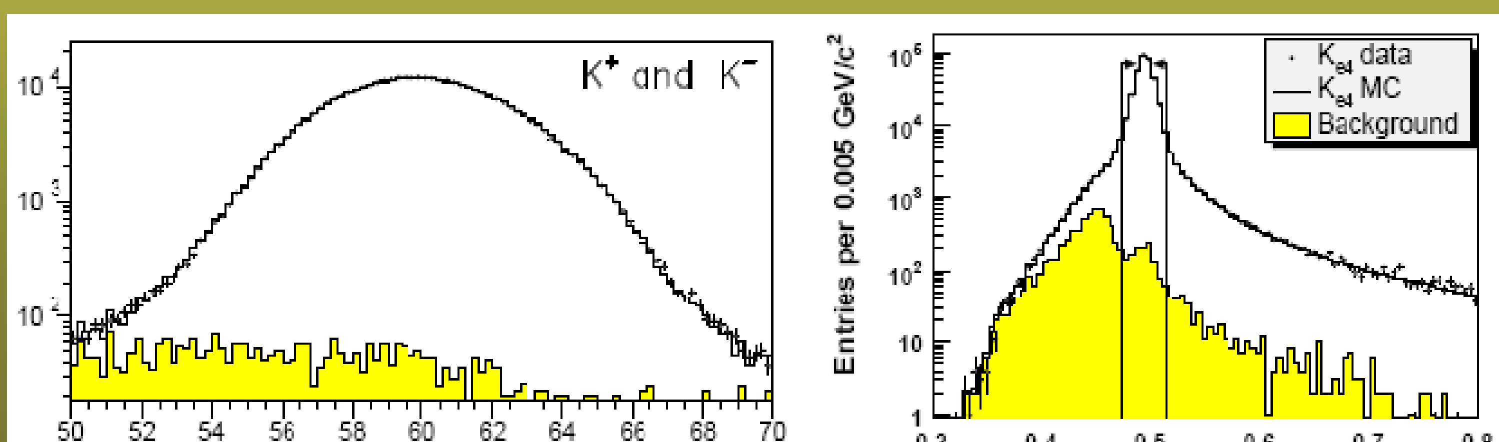
The fit parameter are : F_S F_P G_P H_P and $\delta = \delta_s - \delta_p$

$K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$: Selection and background

Topology: 3 charged track , Signal: 2π with opposite charge
1 e identified with E/p ~1 , additional Missing ν energy and pt cuts

Background main sources:
$\pi+\pi+\pi-$ decays and $\pi \rightarrow e\nu$ (dominant) or π misidentified as e
$\pi+\pi^0\pi^0$ decays and π^0 dalitz decay , γ undetected or e misidentified as π

Background estimated by Montecarlo Simulation ... But....
Wrong sign events Event $\pi+\pi+e-$ (violating $\Delta S = \Delta Q$ rule)
provide a check for MC background estimate



Fitting procedure and Statistics

- Define 10x5x5x5x12 iso-populated bins in ($\mathbf{M_{\pi\pi}}$, $M_{e\nu}$, cos θ_π , cos θ_e , ϕ)
- The form factors are extracted from the data using simulated events by minimizing a log-likehood estimator in each of the $M\pi\pi$ bins:
  - In each $M\pi\pi$ bin the form factors are assumed to be constant
  - 10 independent fits (one fit per $M_{\pi\pi}$ bin) of 4 parameters (Fp, Gp , Hp and δ) plus free normalization (related to Fs) in 4D space.
  - The correlation between the 4+1 parameters is taken into account.
  - K+ and K- fitted separately and combined.

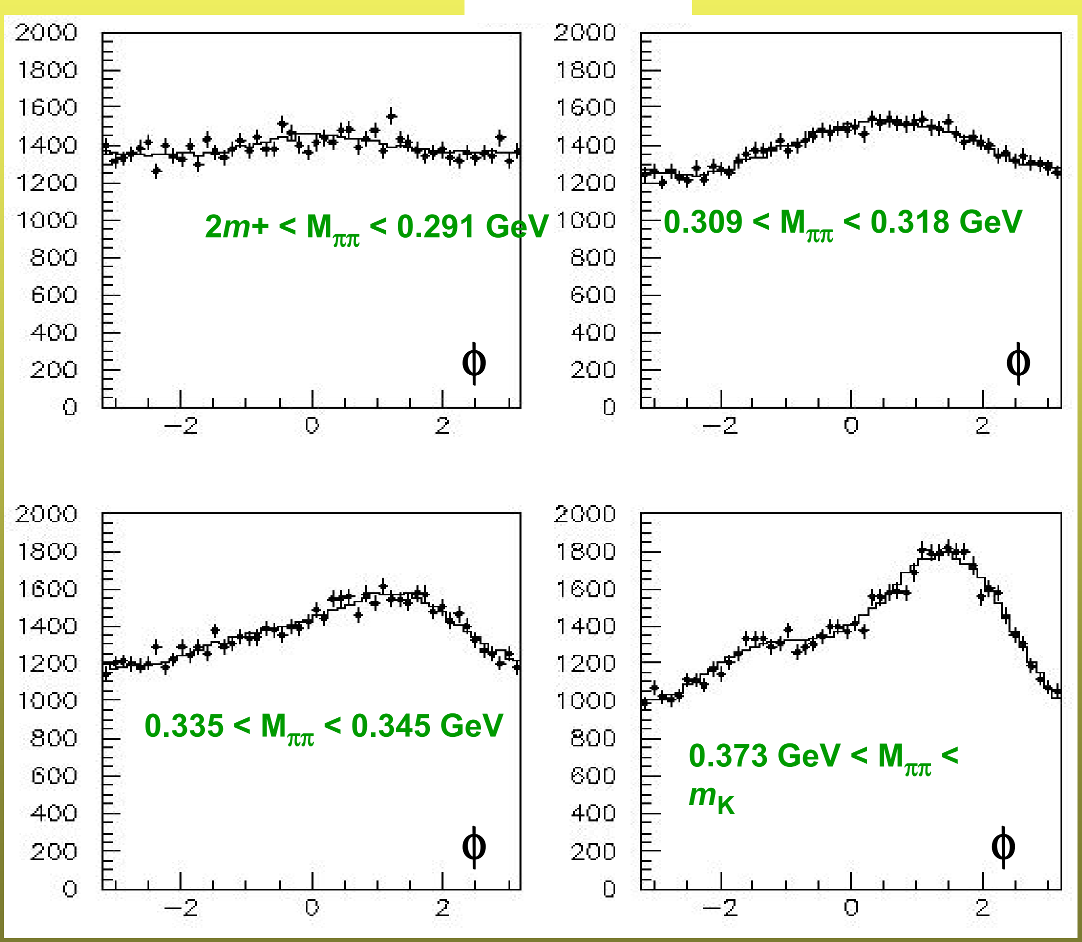
Statistics		Data (2003)	MC
K+	evts	435654	10.0 M
	Evts/bin	29	667
K-	evts	241856	5.6 M
	Evts/bin	16	373

ϕ distributions

$\delta = \delta_s - \delta_p$ of the K_{e4} decay amplitude is extracted from the measured asymmetry of the ϕ distribution as function of $M_{\pi\pi}$

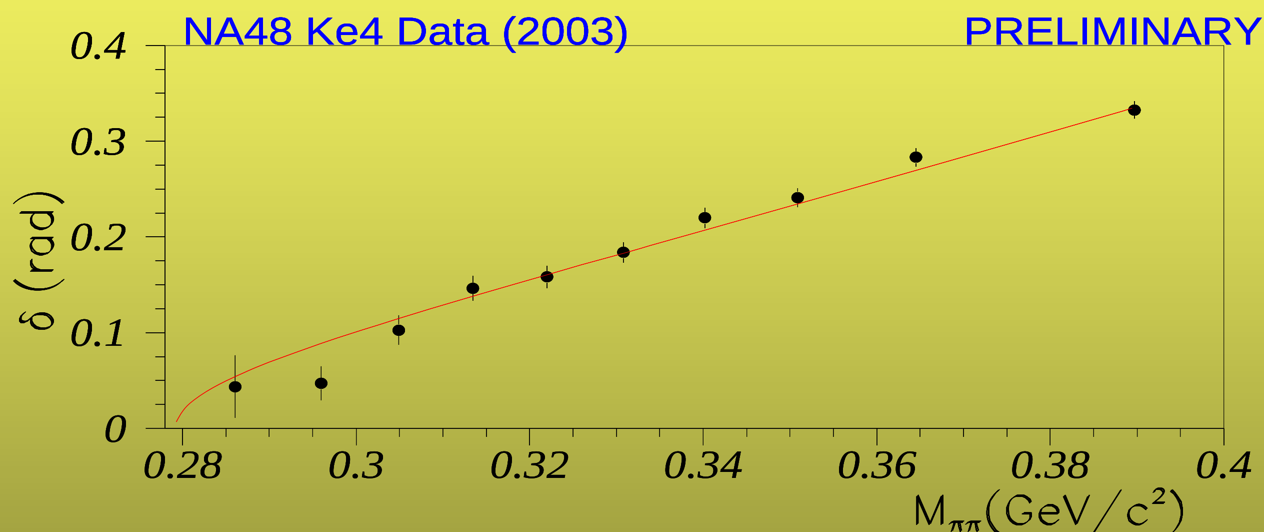
The asymmetry of the ϕ distribution increases with $M_{\pi\pi}$
$\rightarrow$ Increasing sensitivity to δ

K+ and K- have opposite ϕ asymmetry



Phase shift VS $M_{\pi\pi}$

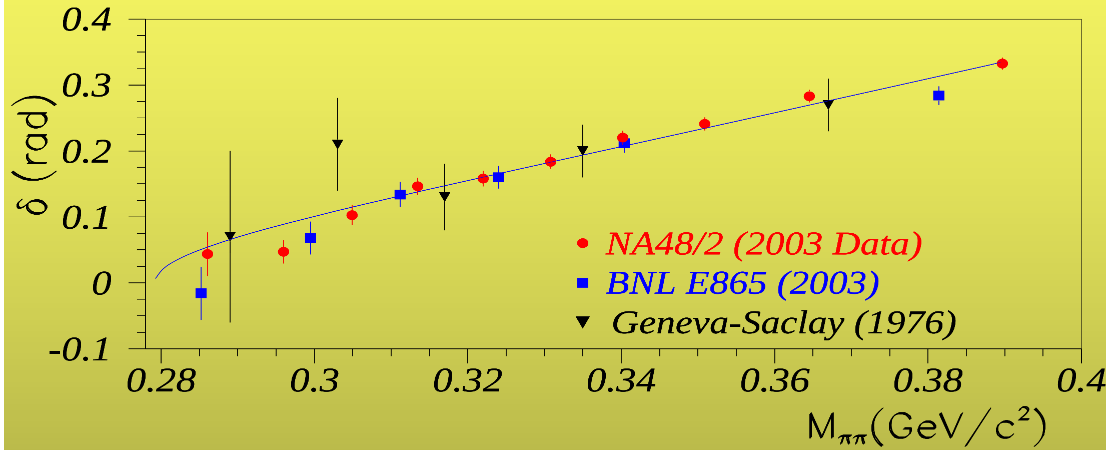
Direct measured points (NO MODEL ASSUMED SO FAR)

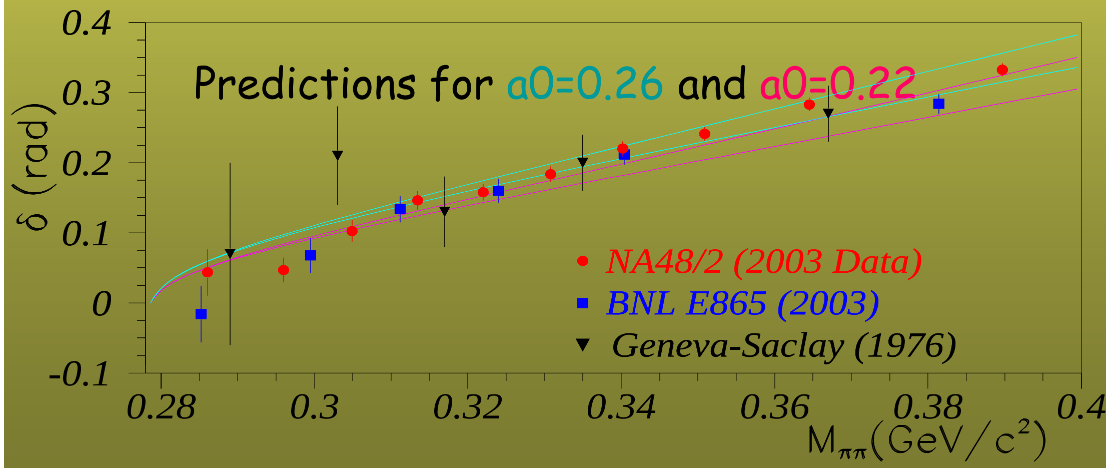


From NOW on MODEL assumptions are needed

To extract information from δ variation, some theoretical work is needed:
Numerical solution of Roy equation which relates δ and a_0 , a_2
(ACGL Phys. Rep. 352 , 2001 ; DFGS EPJ C24 , 2002)

Phase shift : Comparison

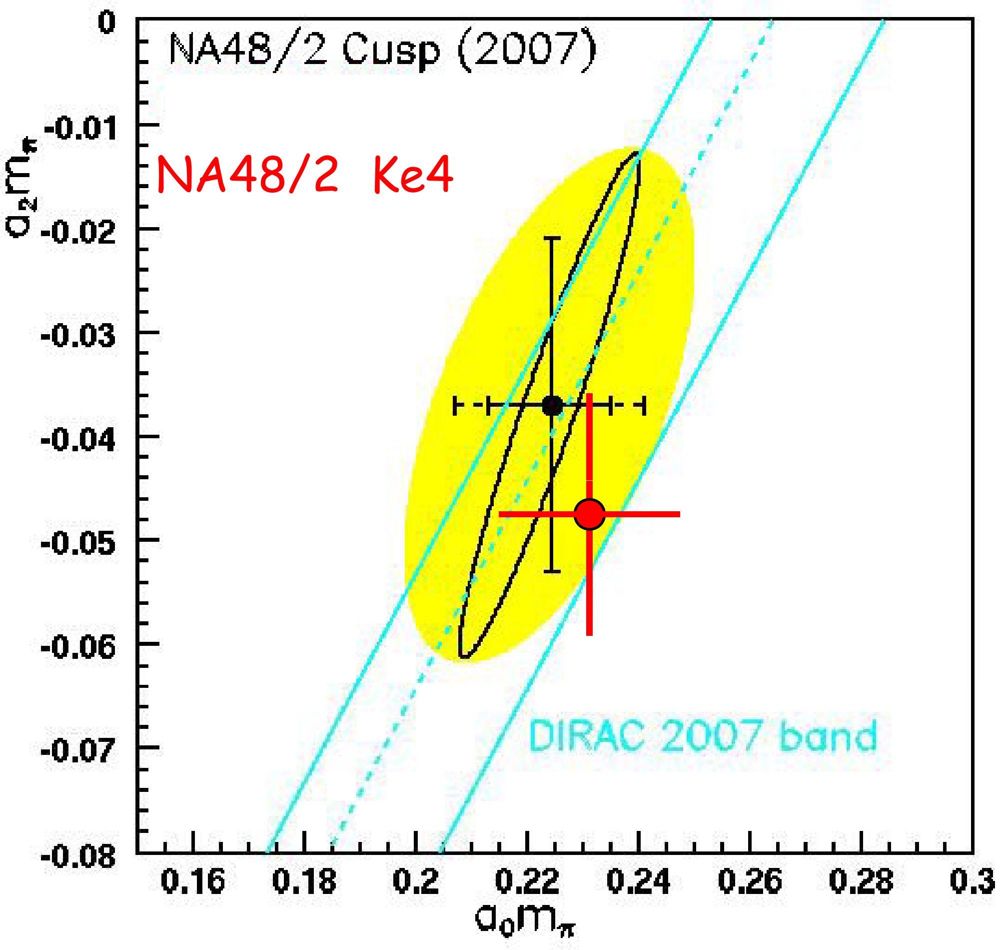




BNL E865 quotes various values ranging from a_0 = 0.203 to a_0 = 0.237

Note the last BNL point !!!

(a_0, a_2) plane K_{e4} result



Under the assumption of Isospin symmetry and using Roy Equation

$a_0 = 0.233 \pm 0.016_{\,stat} \pm 0.007_{\,syst}$

$a_2 = -\,0.0471 \pm 0.011_{\,stat} \pm 0.004_{\,syst}$

EPJC 54, 2008, 411

CHPT predictions

a0 = 0.220 ± 0.005

a2 = - 0.0444 ± 0.0010

NPB 603, 125 , 2001

Conclusions

The pion pion scattering lengths have been measured by NA48/2.

Two methods based on two different charge Kaon decay processes
Give results in good agreement.

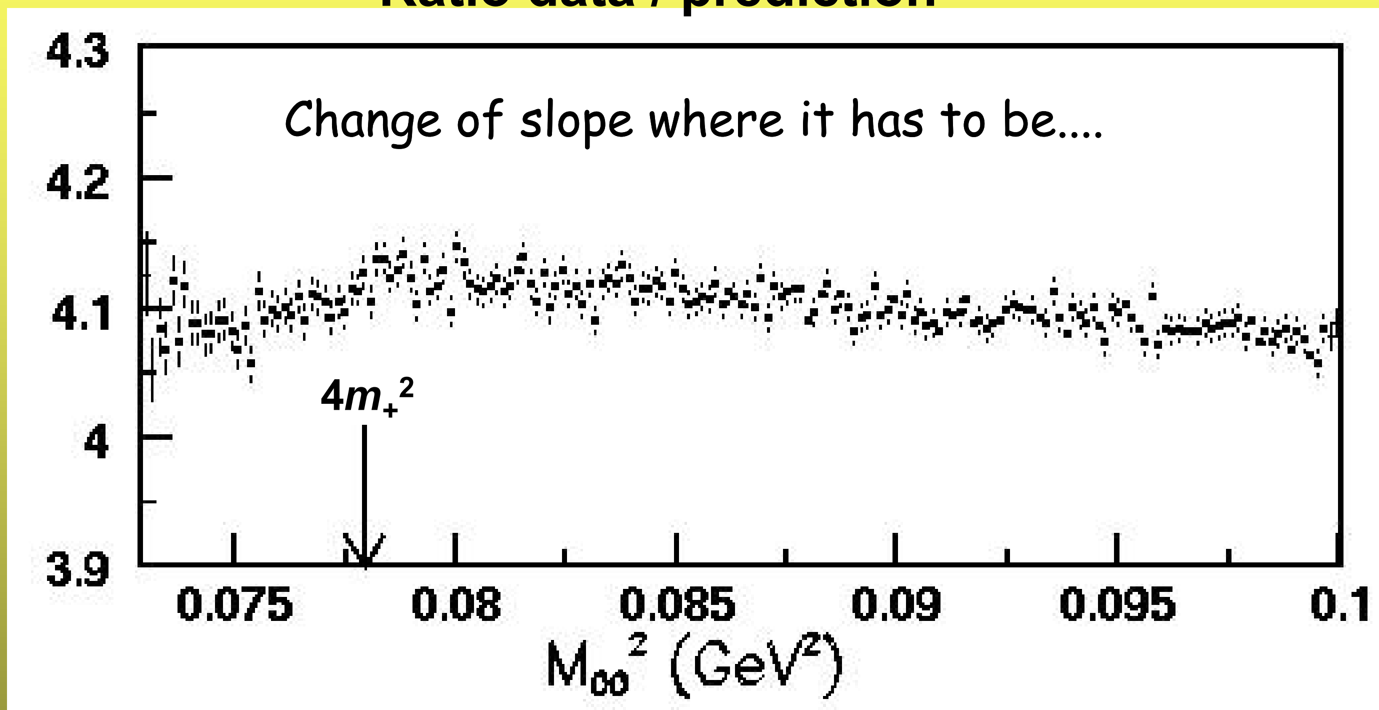
The experimental measured scattering lengths agree with
CHPT predicted values at the per cent level.

This measurement is one of the most stringent test for CHPT

... Final Invitation ...

CUSP effect in $K_L \to 3\pi^0$

Ratio data / prediction



K long sample of ~ 100M events collected in 2000
The CUSP visibility is ~ 13 smaller

CALL TO KTEV : LET THE CUSP BE SEEN IN YOUR HUGE Klong statistics

CUSP VISIBILITY

two possible $\pi^+\pi^-$ pairs

$$R(K^+) \approx \frac{2M_{++-}M_{+00}}{(M_{+00})^2} = \frac{2M_{++-}}{M_{+00}}$$

$\mathcal{M}_{++-}$: $K^+ \to \pi^+\pi^+\pi^-$ matrix element
$\mathcal{M}_{+00}$: $K^+ \to \pi^+\pi^\circ\pi^\circ$ matrix element

$$R(K_L) \approx \frac{M_{+-0}M_{000}}{(M_{000})^2} = \frac{M_{+-0}}{M_{000}}$$

$\mathcal{M}_{+-0}$: $K_L \to \pi^+\pi^-\pi^\circ$ matrix element
$\mathcal{M}_{000}$: $K_L \to \pi^\circ\pi^\circ\pi^\circ$ matrix element

Calculate matrix elements at cusp point ($M_{pp} = 2m_+$)
from measured partial width ratios and slope parameters:

$\mathcal{R}(K^+) \approx 6.1$; $\mathcal{R}(K_L) \approx 0.47$ ⟶ $\dfrac{\mathcal{R}(K+)}{\mathcal{R}(K_L)} \approx 13$

Cusp "visibility" is ~ 13 times higher in $K^+ \to \pi^+\pi^\circ\pi^\circ$ decays than in $K_L \to \pi^\circ\pi^\circ\pi^\circ$ decays