

## Recent results from NA48/2 on $K_{e4}$ decays and interpretation in term of $\pi\pi$ scattering lengths

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Preliminary results from a new measurement of the  $K_{e4}$  decay  $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$  by the Na48/2 collaboration at the CERN SPS are reported. An unprecedented sample of more than 670000  $K_{e4}$  decays in both charged modes have been collected in 2003. The form factors of the hadronic current (F, G, H) and phase shift ( $\delta_0^0 - \delta_1^1$ ) of the  $\pi\pi$  scattering have been measured using a model independent method and their variation with the  $\pi\pi$  mass has been investigated. First evidence for a non zero  $f_p$  term is reported. Thanks to a sizeable acceptance at large  $\pi\pi$  mass, a low background and a very good resolution, an improved accuracy ( $\pm 0.006_{\text{stat}} \pm 0.005_{\text{syst}}$ ) can be reached when extracting the  $\pi\pi$  scattering length  $a_0^0$ . Using more elaborated theory inputs and another NA48/2 result from  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$  decays, a consistent picture can be drawn for the scattering lengths  $a_0^0$  and  $a_0^2$ , in good agreement with Chiral Perturbation Theory predictions.

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## Introduction

Charged  $K_{e4}$  data are of particular interest as they give access to the  $\pi\pi$  phase shift  $\delta = \delta_0^0 - \delta_1^1$  in absence of any other hadron. The measured variation of the phase shift with the invariant mass  $M_{\pi\pi}$  near threshold can be related to  $a_0^0$  and  $a_0^2$  (the  $\pi\pi$  s-wave scattering lengths for Isospin states 0 and 2) using dispersion relations and data at intermediate energies [1, 2, 3]. Predictions on the behavior of the hadronic form factors have also been developed in the framework of Chiral Perturbation Theory (ChPT) [4] and can be compared to precise experimental measurements. In the past years, only two experiments were able to collect large samples of  $K_{e4}$  decays [5, 6] and study their properties. The NA48/2 experiment was primarily dedicated to search for CP violation in charged kaon decays into  $3\pi$  [7] but the high intensity charged kaon beams allow to record at the same time large samples of rare decay modes like  $K_{e4}$  with branching fraction  $\sim 4 \cdot 10^{-5}$ .

This contribution presents preliminary results from the analysis of more than 670000  $K_{e4}$  decays recorded in 2003. The form factors and  $\pi\pi$  phase shift which characterize the decay are measured in a model independent method and compared with previous measurements. The result in terms of s-wave  $\pi\pi$  scattering lengths is compared with a complementary measurement performed by NA48/2 in the  $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$  mode and presented at this Conference [8].

## 1. Experimental setup

Two simultaneous  $K^\pm$  beams were produced by 400 GeV protons from the CERN/SPS, impinging on a beryllium target. The beams were then deflected in a front-end achromat to select momenta in the range  $(60 \pm 3)$  GeV/c and focused  $\sim 200$  m downstream at the first spectrometer chamber. A schematic view of the beam line can be found in [7]. The NA48 detector and its performances are described elsewhere [9]. The main components used in the  $K_{e4}$  analysis are:

- a magnetic spectrometer consisting of a dipole magnet surrounded by two sets of drift chambers achieving a momentum resolution  $\sigma(p)/p = (1.02 \oplus 0.044 p)\%$  ( $p$  in GeV/c).
- a 27 radiation length liquid krypton calorimeter used to measure electromagnetic deposits and identify electrons through their  $E/p$  ratio. The transverse segmentation into 13248 projective cells gives an energy resolution  $\sigma(E)/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$  ( $E$  in GeV) and a space resolution for isolated showers  $\sigma_x = \sigma_y = (0.42/\sqrt{E} \oplus 0.06)$  cm.

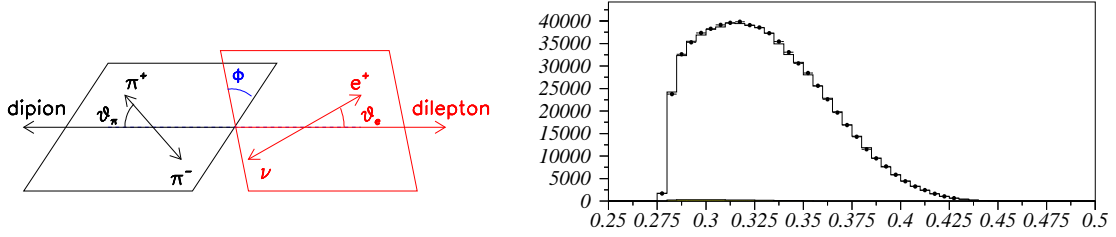
## 2. The $K_{e4}$ decay analysis

The 2003 data were selected for three well reconstructed charged tracks topologies, requiring two opposite sign pions and one electron carrying the same charge as the total charge, identified from their  $E/p$  ratio. The reconstructed 3-track invariant mass (assigning a pion mass to each track) and  $p_t$  relative to the beam axis had to be outside an ellipse centered on the kaon mass and zero  $p_t$ , with semi-axes  $\pm 20$  MeV/c<sup>2</sup> and  $\pm 35$  MeV/c, allowing missing energy and  $p_t$  for the neutrino. The background sources are  $K^\pm \rightarrow \pi^+\pi^-\pi^\pm$  decays with subsequent  $\pi \rightarrow e\nu$  decay or a pion misidentified as an electron and  $K^\pm \rightarrow \pi^\pm\pi^0(\pi^0)$  decays with subsequent Dalitz decay of a  $\pi^0$  with an electron misidentified as a pion and photon(s) undetected. The relative level of background to signal is  $\sim 0.5\%$  and has been cross-checked using Monte Carlo simulated events. The  $K_{e4}$

decay is fully described by the five kinematic Cabibbo-Maksymowicz variables: two invariant masses  $M_{\pi\pi}$  and  $M_{e\nu}$  and three angles  $\theta_\pi$ ,  $\theta_e$  and  $\Phi$  as shown in Figure 1. Three axial ( $F, G, R$ ) and one vector ( $H$ ) form factors contribute to the transition amplitude and can be developed in a partial wave expansion of  $s, p, d$  waves.

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi + \dots, \quad G = G_p e^{i\delta_s} + \dots, \quad H = H_p e^{i\delta_h} + \dots$$

The form factor  $R$  is suppressed by a factor  $m_e^2/S_e$  and cannot be measured in  $K_{e4}$  decays. Neglecting  $d$  wave terms and assuming the same phase for  $F_p, G_p, H_p$ , only one phase ( $\delta(q^2) = \delta_s - \delta_p$ ) and four form factors are left, which are expanded further [4] in powers of  $q^2 = (M_{\pi\pi}^2/4m_\pi^2) - 1$ :  $F_s = f_s(1. + f'_s/f_s q^2 + f''_s/f_s q^4 + \dots)$ ,  $F_p = (f_p + \dots)$ ,  $G_p = (g_p + g'_p q^2 + \dots)$ ,  $H_p = (h_p + \dots)$



**Figure 1:** Left: Topology of the charged  $K_{e4}$  decay. Right: Distribution of the reconstructed  $\pi\pi$  mass ( $\text{GeV}/c^2$ ). Data are shown as symbols with error bars, simulation after fit as open histogram and background (hardly visible) from wrong sign events as shaded area.

From the data sample, a total of 15000 equi-populated bins are defined in the five-parameter space (ten along  $M_{\pi\pi}$ , five along  $M_{e\nu}$ , five along  $\cos\theta_\pi$ , five along  $\cos\theta_e$  and twelve along  $\Phi$ ). Ten independent four-parameter fits are performed, one for each bin in  $M_{\pi\pi}$ . The set of form factors and the phase shift are used to minimize the differences summed over all bins between data events and predicted events from a detailed simulation. A particular attention was given to the acceptance and resolution in the five-variable space. Radiative corrections, including Coulomb attraction between the two pions, were implemented using a dedicated generator code (PHOTOS).

### 3. Results and interpretation

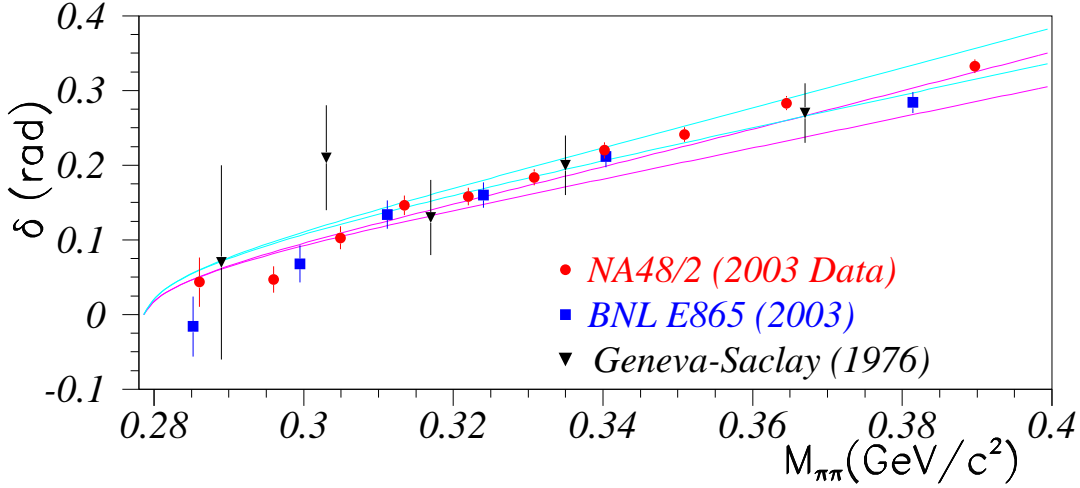
Two series of fits are performed separately for the  $K^+$  and  $K^-$  samples, using the same  $M_{\pi\pi}$  bins definition. The results are checked for consistency and then combined in each bin according to their statistical weight under the assumption of CP conservation (the  $\phi$  distribution of  $K^+$  decays is opposite to the  $\phi$  distribution of  $K^-$  decays with the same  $|H_p|$  value). The Data/MC normalizations are rescaled to have a mean value equal to unity. Last, values of  $F_p/F_s, G_p/F_s, H_p/F_s$  are deconvoluted of the observed  $F_s$  variation in each bin and plotted against  $M_{\pi\pi}$  to investigate a possible further dependence. A residual variation of  $F_s^2$  with  $S_e (= M_{e\nu}^2)$  suggests to measure the variation of the normalizations in the plane ( $q^2, S_e/4m_\pi^2$ ). Polynomial in the dimensionless variable  $q^2$  (and  $S_e/4m_\pi^2$ ) are used to fit the form factor variations and the Universal Band center line constraint is used to deduce a value of the scattering length  $a_0^0$ .

Systematic uncertainty studies include comparison of two independent analyses, trigger efficiency, acceptance control, background contamination, electron mis-identification, implementation

of radiative corrections and neglected  $S_e$  dependence of the form factors in the simulation. The numerical results are given below, including also a 2-parameter fit where both  $a_0^0$  and  $a_0^2$  are free :

$f'_s/f_s = 0.172 \pm 0.009_{\text{stat}} \pm 0.006_{\text{syst}}$	$f''_s/f_s = -0.090 \pm 0.009_{\text{stat}} \pm 0.007_{\text{syst}}$
$f'_e/f_s = 0.081 \pm 0.008_{\text{stat}} \pm 0.008_{\text{syst}}$	$f_p/f_s = -0.048 \pm 0.004_{\text{stat}} \pm 0.004_{\text{syst}}$
$g_p/f_s = 0.873 \pm 0.013_{\text{stat}} \pm 0.012_{\text{syst}}$	$g'_p/f_s = 0.081 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}}$
$h_p/f_s = -0.411 \pm 0.019_{\text{stat}} \pm 0.007_{\text{syst}}$	$a_0^0(1p) = 0.256 \pm 0.006_{\text{stat}} \pm 0.005_{\text{syst}}$
$a_0^0(2p) = 0.233 \pm 0.016_{\text{stat}} \pm 0.007_{\text{syst}}$	$a_0^2(2p) = -0.047 \pm 0.011_{\text{stat}} \pm 0.004_{\text{syst}}$

A comparison of NA48/2 phase shift measurements with those of previous experiments and the Universal Band predictions for two values of  $a_0^0$  is shown in Figure 2: the data are in good agreement (apart from the highest energy point of E865) and favour rather large values of  $a_0^0$ .

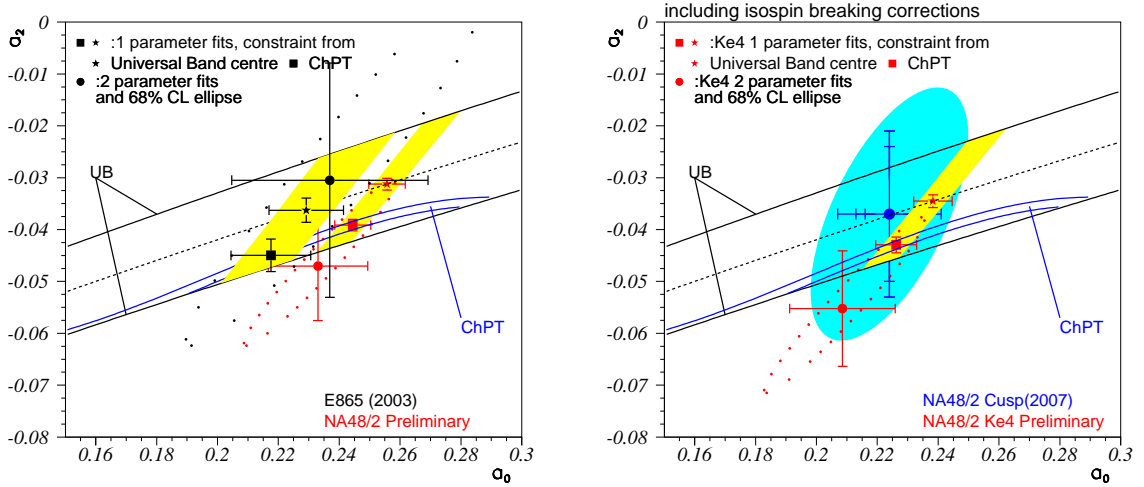


**Figure 2:** Phase shifts measurement from the three experiments. The top band corresponds to the predictions of the Roy equations for  $a_0^0 = 0.26$ , the lower band for  $a_0^0 = 0.22$ .

To investigate the compatibility of the two high statistic experiments, the results of the 1 and 2-parameter fits are plotted in the plane  $(a_0^0, a_0^2)$  where they show marginal consistency (Figure 3). This is mainly due to the highest energy point of E865 which received a specific treatment in the analysis [6]. Further theoretical developments [10] suggest that isospin symmetry breaking effects, neglected so far, should be considered when extracting  $\pi\pi$  scattering lengths from phase measurements. Under this assumption, measurements of the same scattering lengths in the  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$  decays [8] show very consistent results, in good agreement with the predictions from Chiral Perturbation Theory [2].

## Conclusion

The axial and vector form factors of the  $K_{e4}$  decay have been measured with an unprecedented relative precision of few percents and evidence for a non zero  $f_p$  term of  $\sim 5\%$  has been established.



**Figure 3:** Left: comparison of E865 and NA48/2  $K_{e4}$  results in the plane  $(a_0^0, a_0^2)$ ; the results are marginally compatible. Right: comparison of NA48/2  $K_{e4}$  and cusp results, including isospin symmetry breaking corrections. The blue ellipse of the cusp result is still affected by a large theoretical uncertainty.

The form factor dependence on the invariant masses has been measured with a relative precision  $\sim 15\%$ . The phase shift  $\delta$  has been extracted in a model independent way which allows comparison and combination with previous measurements. The extraction of the  $\pi\pi$  scattering lengths  $a_0^0$  and  $a_0^2$  is subjected to theoretical external inputs which uncertainties are of the same order as the NA48/2 experimental precision and could even bias the extracted values. This opens the way to new interesting developments, thanks to a very positive collaboration with theorists. Preliminary calculations of isospin symmetry breaking corrections to the phase predictions already suggest a good agreement with the other NA48/2 measurement and the prediction of Chiral Perturbation Theory.

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