

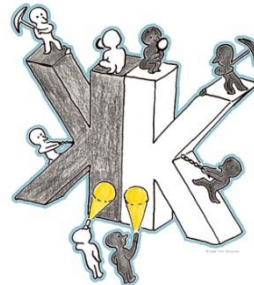


Precision measurement of $\pi\pi$ scattering lengths in $K\pi 4$ decays by NA48/2



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KAON 09, EPOCHAL Tsukuba, 9-12 June 2009



On behalf of the NA48/2 collaboration:

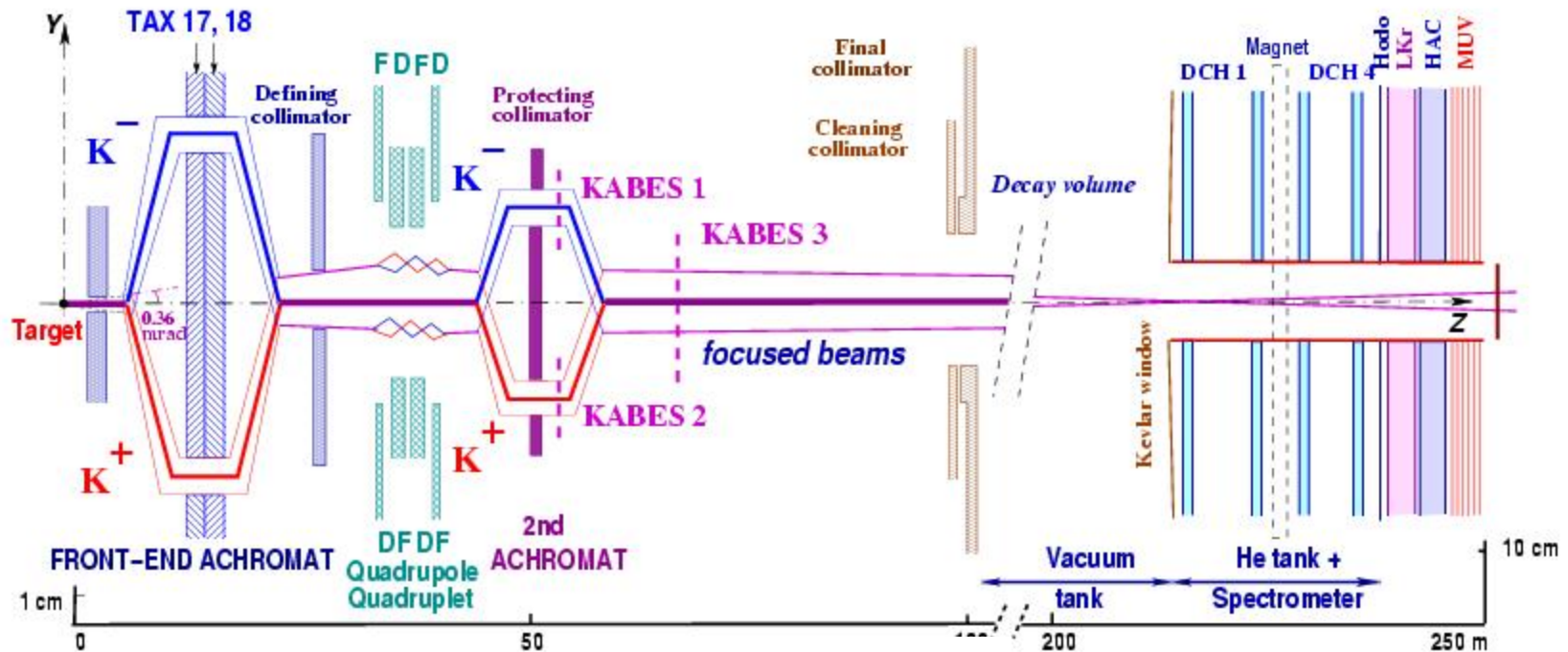
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze,
Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

outline

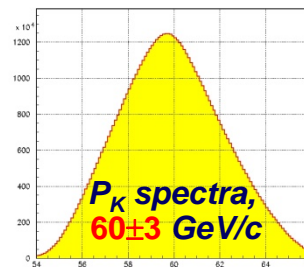
- Brief introduction to NA48/2
- QCD tests from study of Kaon decays @ NA48/2
- Ke4 decays ($K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$):
Form Factors and $\pi\pi$ scattering lengths
- Combining cusp and Ke4 results
- Summary

The NA48/2 experiment at the CERN-SPS : primarily designed for CP violating charge asymmetries studies in $K_3\pi$ decays

2003 run: ~ 50 days + 2004 run: ~ 60 days



Simultaneous K^+ and K^- beams:
large charge symmetrization of
experimental conditions



Beams coincide within ~1mm
all along the 114m decay volume
flux ratio $K^+/K^- \sim 1.8$

The NA48/2 experiment: detector and performances

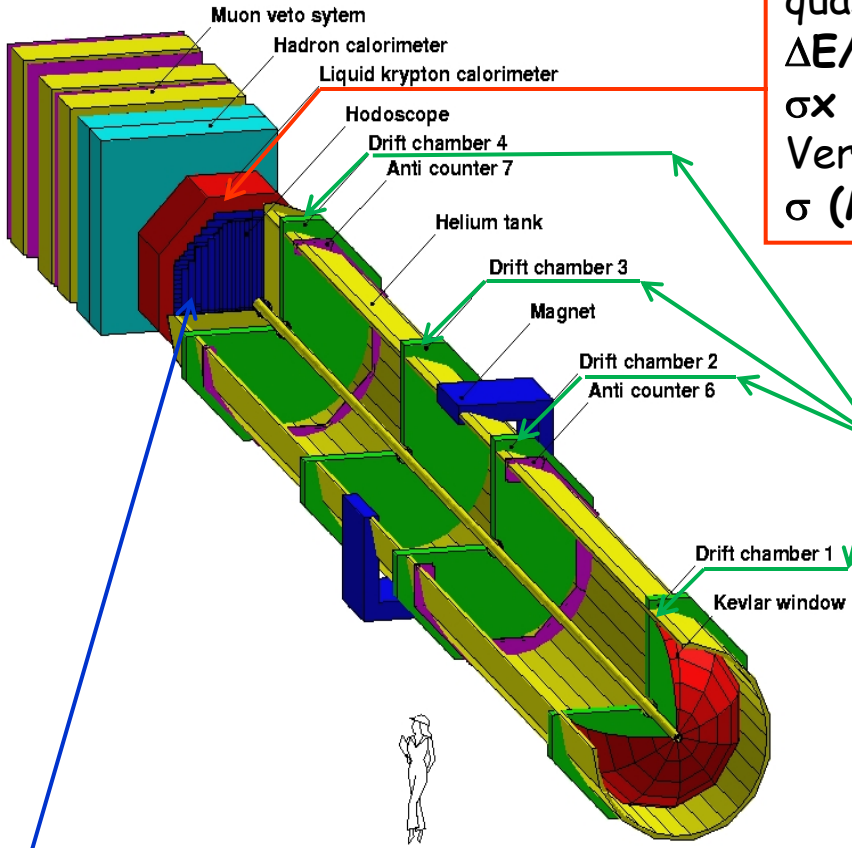
LKr electromagnetic calorimeter :
 quasi-homogenous and high granularity
 $\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)
 $\sigma_x = \sigma_y \sim 1.5 \text{ mm}$ for E=10 GeV
 Very good resolution for neutrals ($\pi^0 \rightarrow \gamma\gamma$)
 $\sigma (M_{\pi\pi^0\pi^0}) = 1.4 \text{ MeV}/c^2$

Magnetic spectrometer :
 4 high-resolution DCH's + dipole magnet
 $\Delta p/p = (1.0 \oplus 0.044 p)\%$ (p in GeV/c)
 Very good resolution for charged invariant masses: $\sigma (M_{3\pi^\pm}) = 1.7 \text{ MeV}/c^2$

+



E/p ratio used for e / π discrimination



Hodoscope for charged fast trigger
 $\sigma_t = 150 \text{ ps}$

Kaon decays : what can be learned on QCD @ Low Energy ?

Hadronic decay modes into 3 pions: (D.Madigojine 's talk)

- large Br's : $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ (1.7 %) and $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$ (5.6 %),

60 Millions events now analyzed (PRL B633 (2006) partial sample)

- three pions : $\pi^0 \pi^0$ system + nearby hadron
- accessible $M_{\pi\pi}$ range from $\pi^0 \pi^0$ threshold to $(M_K - M_\pi)$

Semileptonic decay mode Ke4: (this talk)

- small Br's : $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ ($4.1 \cdot 10^{-5}$),

1.1 Million events now analyzed (EPJC 54 (2008) partial sample)

- only two $\pi^+ \pi^-$ pions, very clean environment
- accessible $M_{\pi\pi}$ range from $\pi^+ \pi^-$ threshold to $(M_K - M_e) \cong M_K$

Two **different** but **complementary** approaches to $\pi\pi$ scattering near threshold to extract s-wave scattering lengths (a_0, a_2) for Isospin $I = 0$ and $I = 2$

in press

preliminary

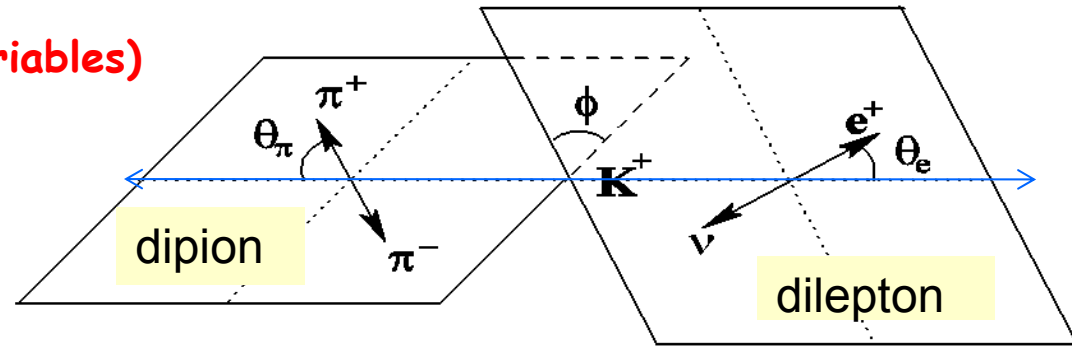
Ke4 decays : formalism

Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

$S_\pi (M^2_{\pi\pi}), S_e (M^2_{e\nu}),$

$\cos\theta_\pi, \cos\theta_e$ and ϕ .



Partial Wave expansion of the amplitude

into s and p waves (Pais-Treiman 1968)

+ Watson theorem (T-invariance) for δ_l^I

$$\delta_0^0 \equiv \delta_s \text{ and } \delta_1^1 \equiv \delta_p$$

F, G = 2 Axial Form Factors

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi$$

$$G = G_p e^{i\delta_g}$$

H = 1 Vector Form Factor

$$H = H_p e^{i\delta_h}$$

Map the distributions of the Ca.Ma. variables in the **five-dimensional space** with 4 Form factors and one phase shift, assuming identical phases for the p-wave Form Factors F_p, G_p, H_p :

The fit parameters are :

$$F_s, F_p, G_p, H_p \text{ and } \delta = \delta_s - \delta_p$$

(F_s, F_p, G_p, H_p are real)

F, G, H are complex and dimensionless

Ke4 decays: event selection and background rejection

Signal ($\pi^+\pi^- e^\pm \nu$) topology:

- 3 charged tracks and a good vertex
- two opposite sign pions,
- 1 electron ($E(LKr) / p \sim 1$),
- some missing energy and p_T (ν)
- reconstruct PK (missing ν hypothesis)

Background main sources :

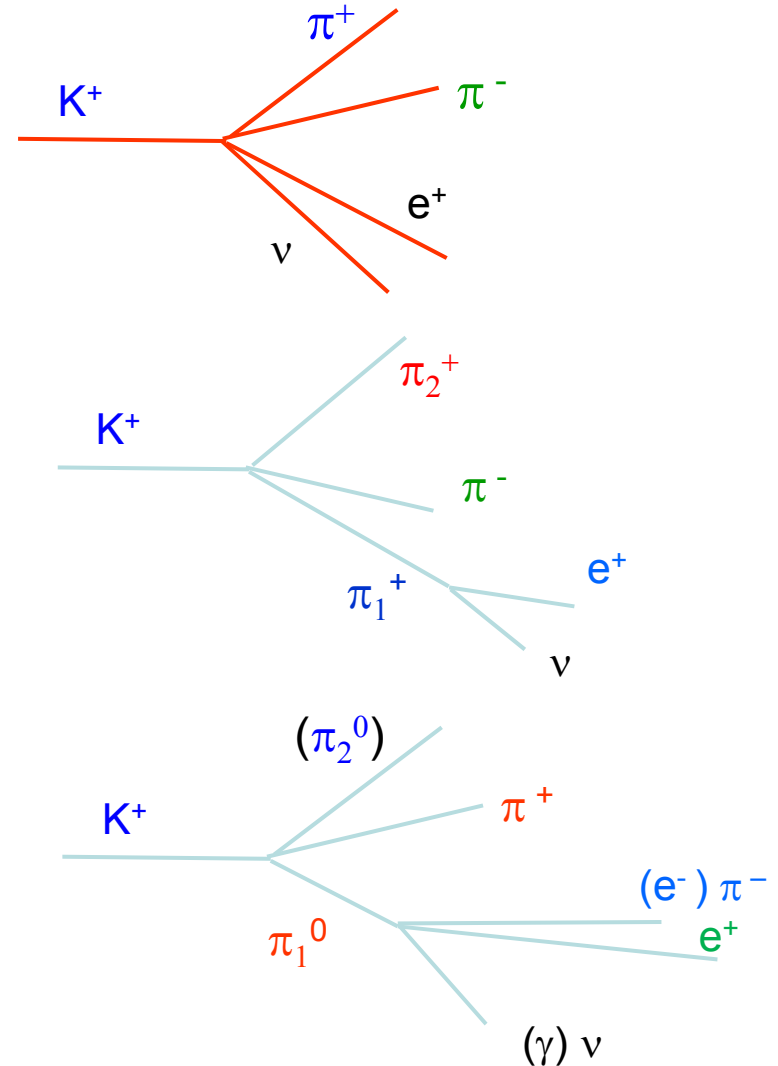
$K^+ \rightarrow \pi^- \pi_2^+ \pi_1^+$ (dominant)

↳ $e^- \nu$ or mis-ident e^-

$K^+ \rightarrow \pi^0 (\pi^0) \pi^+$

↳ $(e^+e^- \gamma)$

↳ mis-ident π^+ and γ (s) undetected



Ke4 decays: background rejection

Control sample from data
(assuming $\Delta S = \Delta Q$ valid)

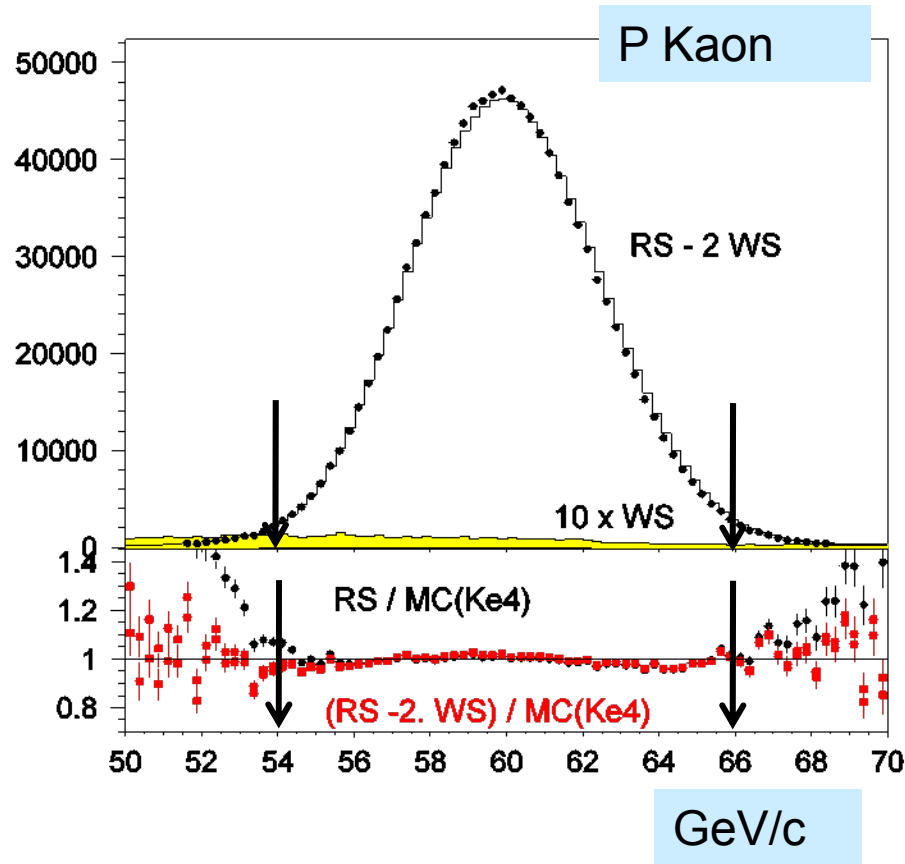
$K^\pm \rightarrow \pi^\pm \pi^\pm e^\mp \nu$ "Wrong Sign" events

- total charge (± 1) as "Right Sign" events
- electron charge opposite to total charge
- same sign pions

Ratio (RS/WS) events:

2/1 if coming from $K3\pi$ (dominant)

1/1 if coming from $K2\pi (\pi^0)$



Total background level can be kept at $\sim 2 \times 0.3\%$ relative level estimated from WS events rate and checked from MC simulation

Ke4 decays : fitting procedure

Total (2003+2004) 1.13 Million Ke4 decays

Using **iso-populated boxes** in the 5-dimension space of the Ca.Ma. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_{\pi}$, $\cos\theta_e$ and ϕ) one defines a grid of

10x5x5x5x12=15000 variable size boxes.

In each $M_{\pi\pi}$ "slice" (1500 boxes), a set of 4 fit parameters is found which minimizes the difference between the data and predicted populations

The normalisation F_s^2 is obtained in each bin/slice by the ratio $x_{\text{slice}} = \sum_{j \text{ in slice}} N_j / \sum_{j \text{ in slice}} MC_j$

K⁺ sample (726 400 events) 48 events/box

K⁻ sample (404 400 events) 27 events/box

Data sample

K⁺ MC (17.4 Million events) 1160 events/box

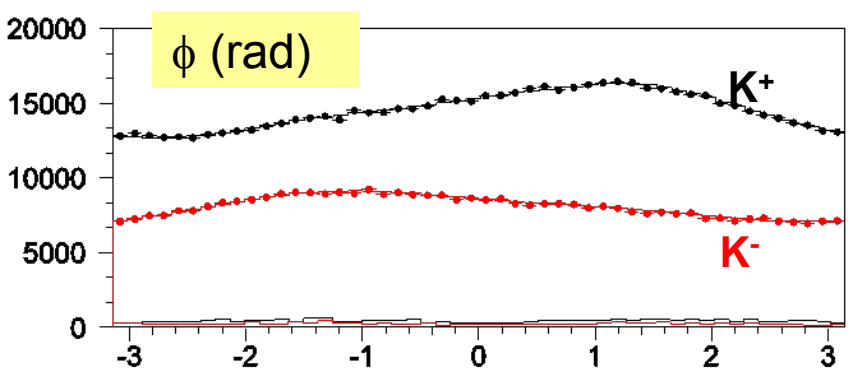
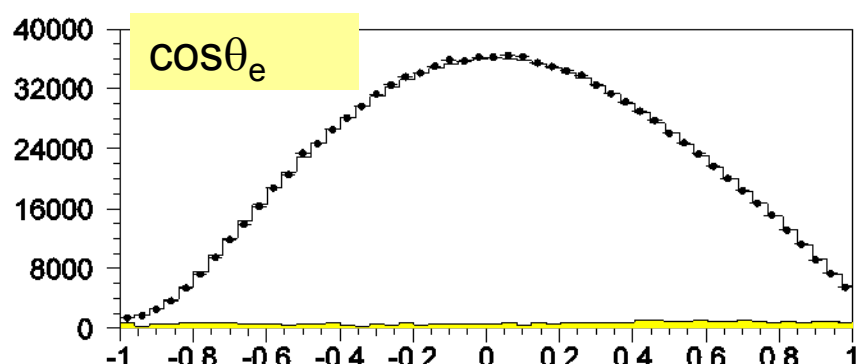
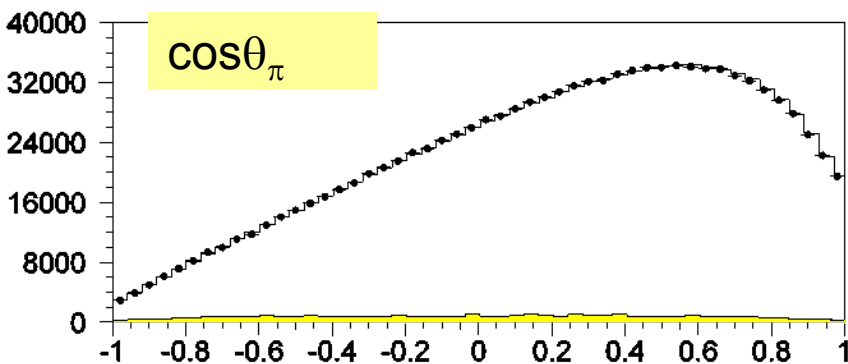
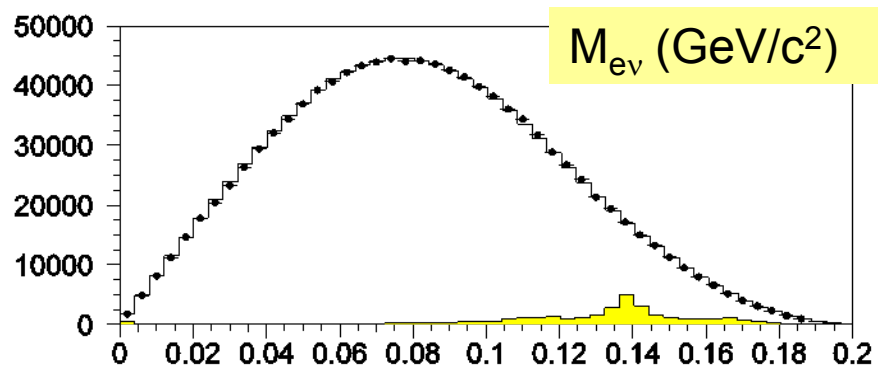
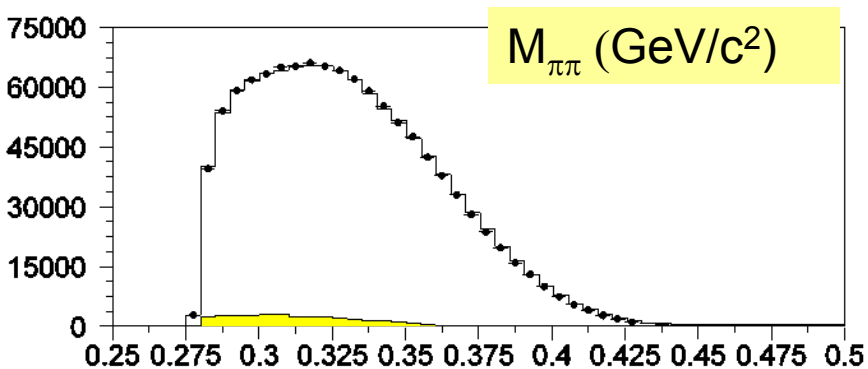
K⁻ MC (9.7 Million events) 650 events/box

MC sample

K⁺ and K⁻ samples fitted separately in 10 independent $M_{\pi\pi}$ bins/slices, then combined in each slice according to their statistical error.

No assumption is made on the shape of the variation of the phase δ (and FF) from one $M_{\pi\pi}$ slice to the next (i.e. "model independent" analysis)

Ke4 decays : Data/MC comparison after fit



	= Data
	= Simulation after fit
	= WS Background (x 10 to be visible)

CP symmetry :
 $(K^+) \phi$ distribution is opposite of $(K^-) \phi$ distribution

Ke4 Form Factors : fit results

Series expansion with q^2 ($q^2 = S_\pi / 4m_\pi^2 - 1$) and $S_e / 4m_\pi^2$ used to describe the FF variations, **in the isospin symmetry limit (Amoros Bijens 1999)**

$$F_s^2 = f_s^2(1 + f_s'/f_s q^2 + f_s''/f_s q^4 + f_e'/f_s S_e/4m_\pi^2)^2$$

Correlation	f_s''/f_s	f_e'/f_s
f_s'/f_s	-0.95	0.08
f_s''/f_s		0.02

$$G_p/f_s = g_p/f_s + g_p'/f_s q^2$$

Correlation -0.91

systematics

- mostly from background + acceptance control
- ~ same size as statistical error or smaller

Preliminary (2003+2004)

	value	stat	syst
f_s'/f_s	0.152	± 0.007	± 0.005
f_s''/f_s	-0.073	± 0.007	± 0.006
f_e'/f_s	0.068	± 0.006	± 0.007
f_p/f_s	-0.048	± 0.003	± 0.004
constant			
g_p/f_s	0.868	± 0.010	± 0.010
g_p'/f_s	0.089	± 0.017	± 0.013
h_p/f_s	-0.398	± 0.015	± 0.008
constant			



first evidence by NA48

Ke4 decays: from phase shifts to scattering lengths (a_0, a_2)

$\pi\pi$ phases at threshold can be predicted from data above 0.8 GeV using **Roy equations** (unitarity, analyticity and crossing symmetries) and 2 subtraction constants **a_0 and a_2**

Numerical solutions have been developed (ACGL, DFGS) valid in the **Isospin symmetry limit (Universal Band in the $[a_2, a_0]$ plane)**, but broken in the experimental world.

factorization of electromagnetic and mass effects :

Gamow factor x PHOTOS generator

x

Isospin corrections

Radiative effects (except mass effects) included in the simulation,

Gamow factor : "classical" Coulomb attraction between the 2 charged pions

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

(-> effect on event selection + possible bias on reconstructed quantities)

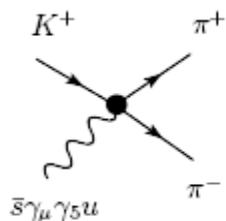
Mass effects:

- recently computed as a correction to the measurements
 - even larger than current experimental precision !
- (CGR **EPJ C59 (2009) 777**,
DK preliminary **June 2008** in progress)

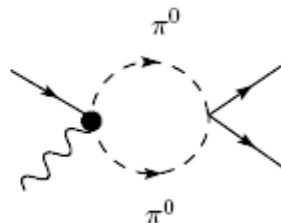
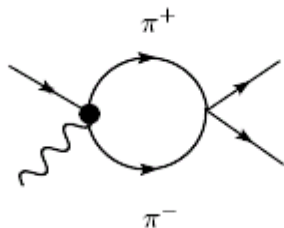
Ke4 charged decays : isospin corrections to δ

CGR EPJ C59 (2009) 777 formulation developed in close contact with NA48

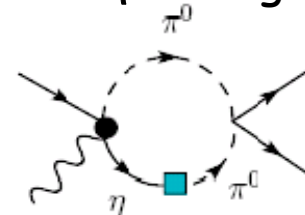
tree



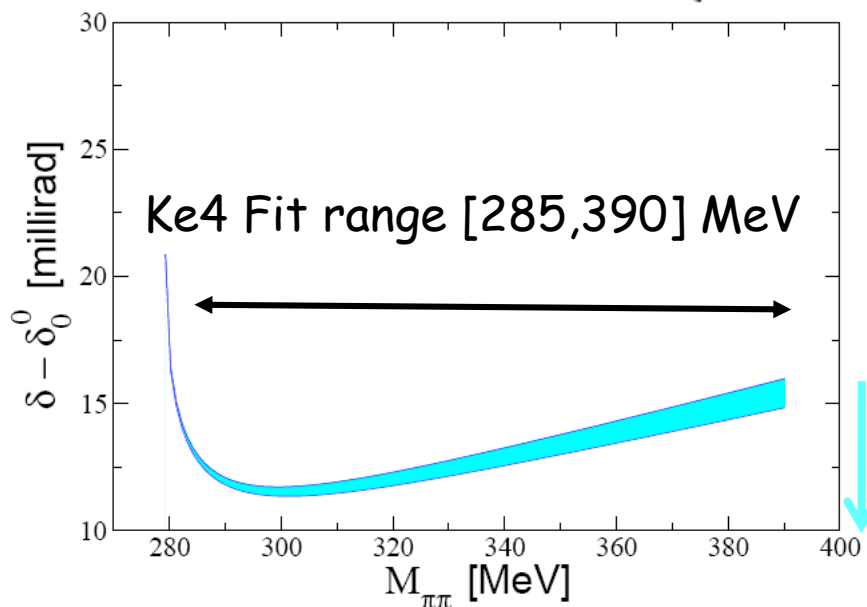
one loop



π^0 - η mixing



$$\delta_0^0 \rightarrow \delta = \frac{1}{32\pi F^2} \left\{ (4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2) \left(1 + \frac{3}{2R}\right) \sigma_0 \right\}$$



$$\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2,$$

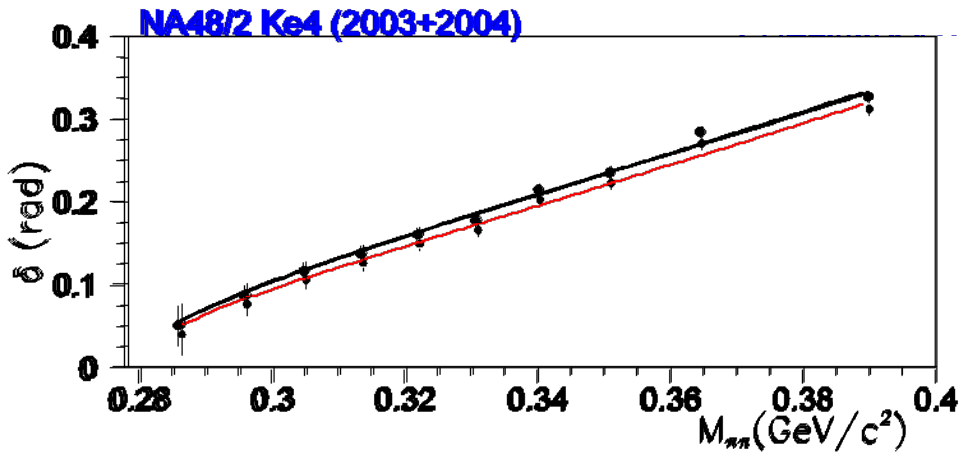
$$\sigma = \sqrt{1 - \frac{4M_\pi^2}{s}},$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

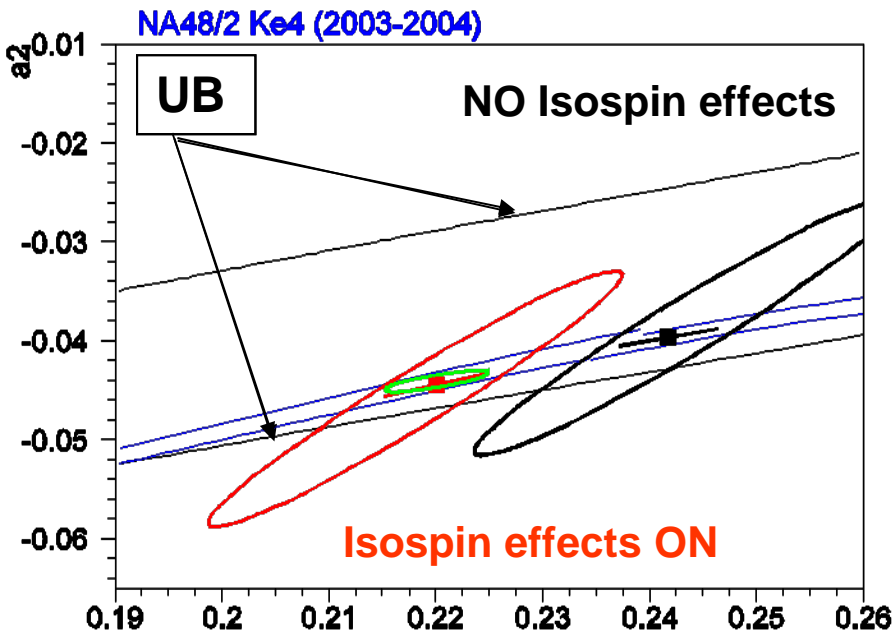
Correction is ~ 10 - 15 mrad

Exp. stat precision (δ) is ~ 7 - 8 mrad

Ke4 decays: from phase shifts to scattering lengths (a_0, a_2)



a tiny effect from theory... a big change in now precise experimental measurement !



This induces a large **change** on (a_0, a_2) values

from a 2p fit	from a 1p fit
$\Delta a_0 = -0.025$, $\Delta a_2 = -0.007$	$\Delta a_0 = -0.022$
error stat syst	stat syst
$\sigma(a_0): \pm 0.0128 \pm 0.0050$	$\pm 0.005 \pm 0.002$
$\sigma(a_2): \pm 0.0084 \pm 0.0034$	

Ellipses are 68% CL contours in 2p fits

Ke4 decays: comparison with theoretical predictions

Preliminary
(2003+2004)

THEORY prediction

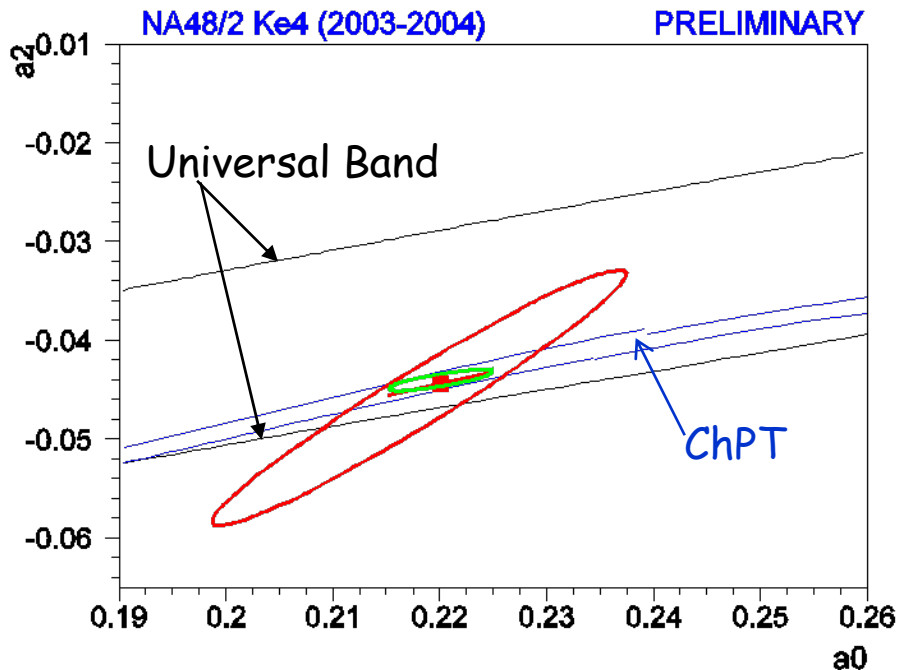
Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001),PRL86(2001)):

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0008$$

Experimental measurement

a_0 ChPT 1p fit	0.2206 ± 0.0049 stat ± 0.0018 syst ± 0.0064 theo *
a_0 free	0.2220 ± 0.0128 stat ± 0.0050 syst ± 0.0037 theo*
a_2 free 2p fit	-0.0432 ± 0.0086 stat ± 0.0034 syst ± 0.0028 theo*
Correlation 96.7%	



*Theory error evaluated from control of the isospin corrections & inputs to Roy equation numerical solutions (CGR EPJ C59 (2009)777)

Comparison of Ke4 phase shift experimental measurements

Apply Isospin corrections (10-15 mrad) to all published points :

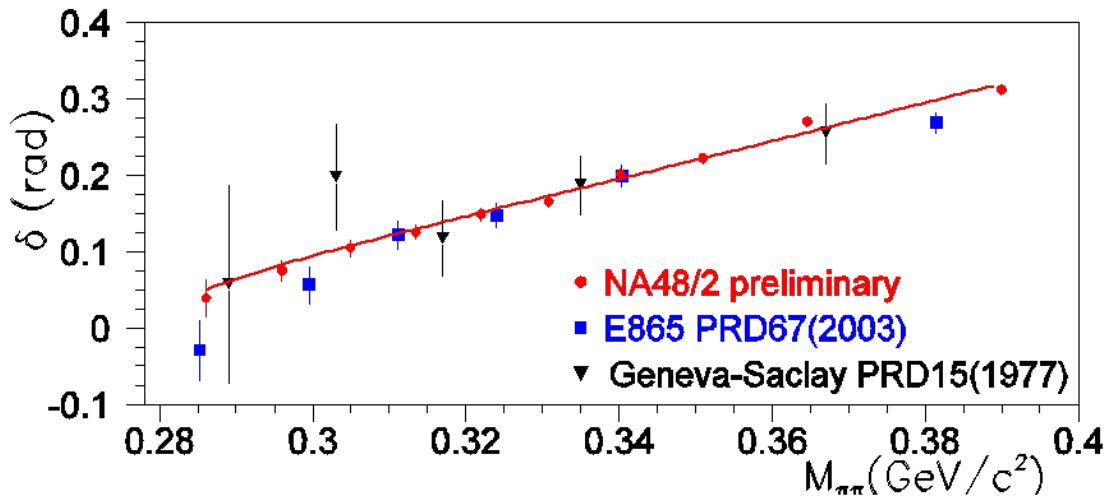
S118 (Geneva-Saclay): typical error 40-50 mrad

E865: typical error 15-20 mrad

Correction small wrt experimental error but coherent shift downwards for all data points

NA48/2 typical error 7-8 mrad

improved precision due to both
 -larger statistics $\sim 3 \times$ E865
 -larger acceptance at high $\pi\pi$ mass



- All Phase points corrected for isospin mass effects
- Independent experiments
- Errors = stat + syst

Line from a 2p fit to NA48 data alone

Fit to all data points (21 points) : dominated by NA48/2 measurements

2p fit:

$$a_0 = 0.2199 \pm 0.0125_{\text{exp}} \pm 0.0037_{\text{theo}}$$

$$a_2 = -0.0430 \pm 0.0083_{\text{exp}} \pm 0.0028_{\text{theo}}$$

1p fit :

$$a_0 = 0.2168 \pm 0.0048_{\text{exp}} \pm 0.0064_{\text{theo}}$$

(theory error common to all expts)

Cusp and Ke4 scattering lengths results

- Two **statistically** independent measurements by NA48/2
 - 60 Millions $K3\pi$ decays \Leftrightarrow 1.13 Million Ke4 decays
- **systematic** uncertainties from different origins : mostly independent
 - calorimeter + trigger \Leftrightarrow background + electron identification
- different **theoretical inputs** :
 - (1- + 2-loop) re-scattering models \Leftrightarrow Roy equations + isospin mass effects
- Large overlap in the (a_0, a_2) plane or $(a_0 - a_2, a_2)$ plane with **different correlations** suggests to combine the two results in a single one with improved uncertainties
- similar combination can be done for the results using the **same ChPT constraint**

Combining cusp and Ke4 results : method

Ke4:	Δ_{stat}	Δ_{syst}	Δ_{theo}	Correlation
$a_0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$				$\rho_{12} = 0.967$ (stat only)
$a_2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$				(0.969 with all errors)

Cusp:	Δ_{stat}	$\Delta_{\text{syst+ext}}$	Δ_{theo}	
$a_0 - a_2 = 0.2571 \pm 0.0048 \pm 0.0029 \pm 0.0088$				x3 $\rho_{34} = -0.839$ (stat only)
$a_2 = -0.0241 \pm 0.0129 \pm 0.0096 \pm 0.0149$				x4 $\rho_{34} = -0.879$ (all errors)

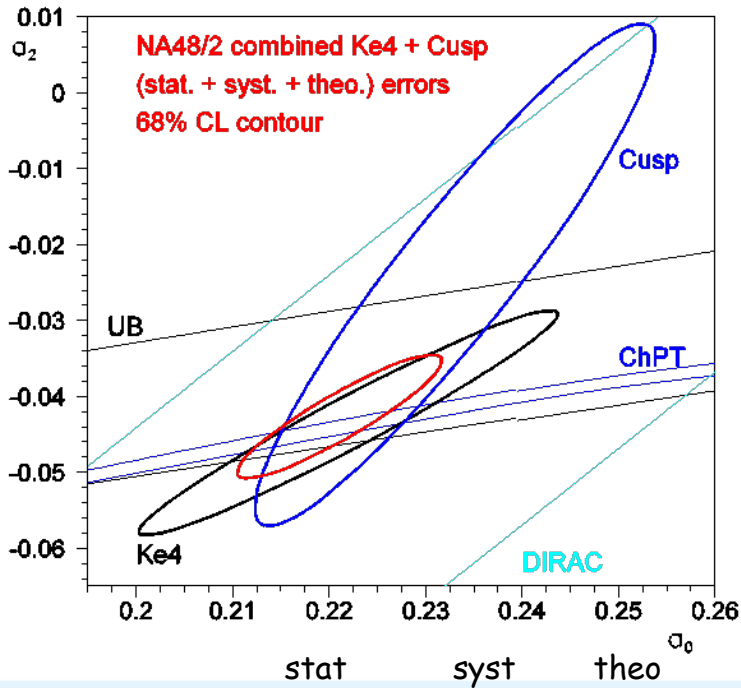
• Four measurements : (x1, x2, x3, x4)

• Two fitted parameters : (a₀, a₂) or (a₀-a₂, a₂)

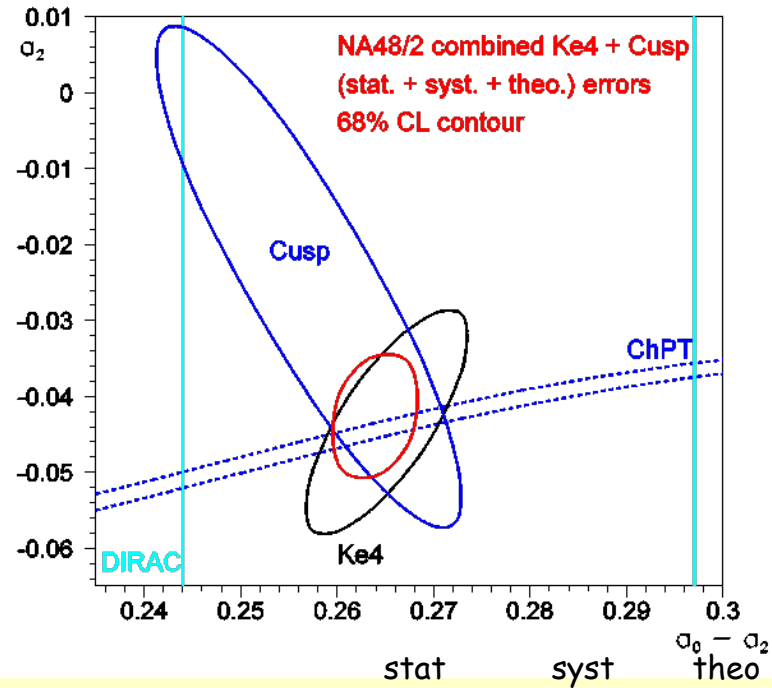
• Error matrix is $\begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 & 0 \\ 0 & 0 & \sigma_{33} & \sigma_{34} \\ 0 & 0 & \sigma_{43} & \sigma_{44} \end{pmatrix}$ where $\sigma_{ii} = \Delta x_i^2$
 block-diagonal if no correlated error cusp-Ke4 and $\sigma_{ij} = \sigma_{ji} = \rho_{ij} \Delta x_i \Delta x_j$

Minimize $\chi^2 = \sum_{i,j=1}^4 (x_i - y_i(a))^T (V_{ij})^{-1} (x_j - y_j(a))$ where V_{ij} is the covariance matrix

Combined results from cusp and Ke4



$a_0 = 0.2210 \pm 0.0047 \pm 0.0015 \pm 0.0049$
 $a_2 = -0.0429 \pm 0.0044 \pm 0.0016 \pm 0.0030$
Correlation 0.912
 Total errors $\Delta a_0: \pm 0.0070$ (3% rel. precision)
 $\Delta a_2: \pm 0.0055$ (13% rel. precision)



$a_0 - a_2 = 0.2639 \pm 0.0020 \pm 0.0004 \pm 0.0021$
 $a_2 = -0.0429 \pm 0.0044 \pm 0.0016 \pm 0.0030$
Correlation 0.277
 Total errors $\Delta (a_0 - a_2): \pm 0.0030$ (1% rel. precision)
 $\Delta a_2: \pm 0.0055$ (13% rel. precision)

Including the ChPT constraint:

	stat	syst	theo	
				$a_2 = -0.0444 \pm 0.007 \pm 0.005 \pm 0.0012$
$a_0 = 0.2196 \pm 0.0027 \pm 0.0021 \pm 0.0048$	or			$a_0 - a_2 = 0.2640 \pm 0.0020 \pm 0.0017 \pm 0.0035$
Total error $\Delta a_0: \pm 0.0059$		$\Delta a_2: \pm 0.0015$		$\Delta (a_0 - a_2): \pm 0.0044$

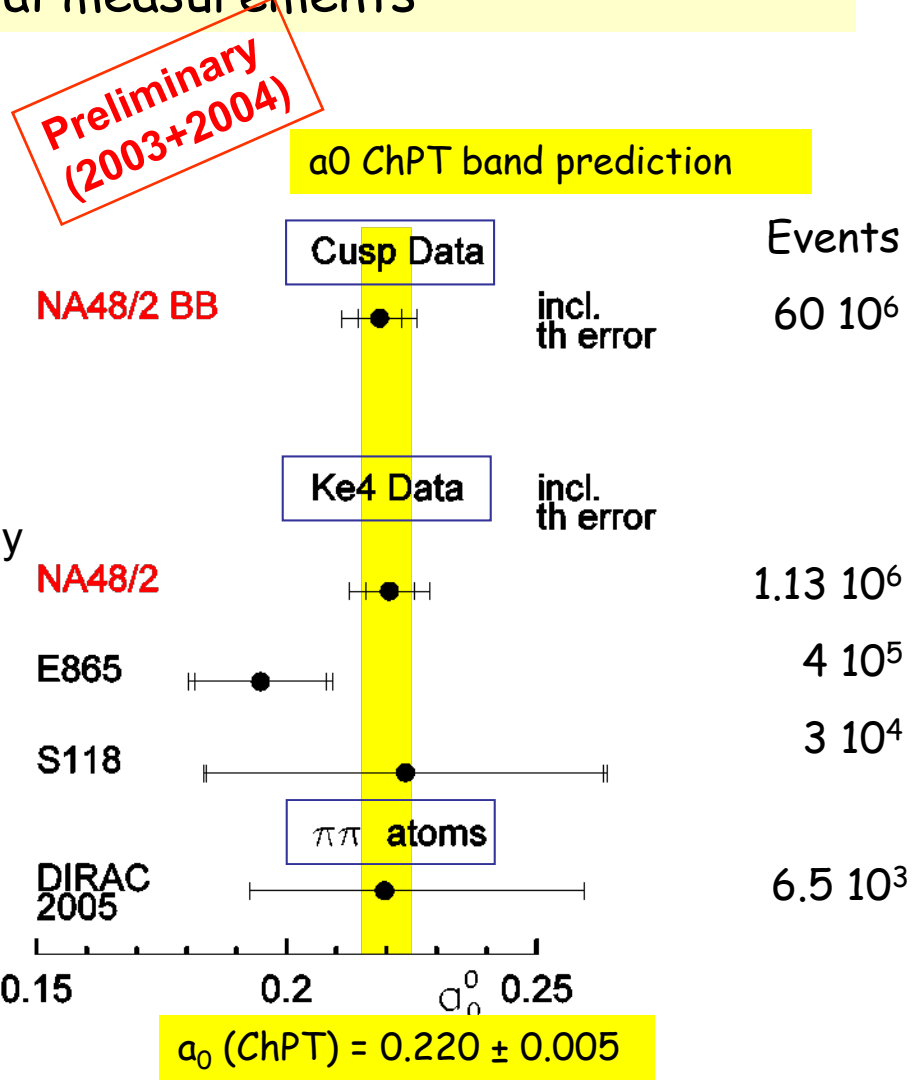
Comparison with other experimental measurements

Cusp : (a_0 - a_2) ChPT fit with 2 models, BB is the most complete in terms of radiative corrections

Ke4 : apply **isospin corrections** to published phase points of all experiments and perform a_0 ChPT fit

Note : E865 result dominated by highest energy data point, otherwise compatible

$\pi\pi$ atoms DIRAC: $|a_0 - a_2|$ errors from PLB619 (2005), use ChPT constraint (only 40% Data analyzed)



Yellow band: best ChPT prediction

NA48/2 experimental precision and most precise theory prediction now at the same level!

Summary

NA48/2 has analyzed ~1.13 M Ke4 events recorded in (2003+2004)

- Ke4 Form Factors measured with an improved precision
- Scattering lengths, extracted in Ke4 and K3 π Cusp analyses, give a consistent experimental picture and an impressive agreement with predictions from ChPT, both for free (a_0, a_2) and constrained fits :

ChPT best prediction $a_0 = 0.220 \pm 0.005$, $a_2 = -0.0444 \pm 0.0008$, $a_0 - a_2 = 0.264 \pm 0.004$

(ChPT cusp) $a_0 - a_2 = 0.2633 \pm 0.0024_{\text{stat}} \pm 0.0014_{\text{syst}} \pm 0.0019_{\text{ext}} \pm 0.0053_{\text{th}}$

(ChPT Ke4) $a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{th}}$

(ChPT combined) $a_0 = 0.2196 \pm 0.0027_{\text{stat}} \pm 0.0021_{\text{syst}} \pm 0.0048_{\text{th}}$

$a_2 = -0.0444 \pm 0.0007_{\text{stat}} \pm 0.0005_{\text{syst}} \pm 0.0012_{\text{th}}$

$a_0 - a_2 = 0.2640 \pm 0.0020_{\text{stat}} \pm 0.0017_{\text{syst}} \pm 0.0035_{\text{th}}$

- The achieved experimental precision (stat + syst) of combined result for a_0 and $a_0 - a_2$ is now smaller than the theoretical precision .

The collaboration with many theory groups was/still is invaluable in understanding how to extract scattering lengths from NA48 Data. Thanks to all groups in Bern, Bonn, Dubna, Orsay, Madrid, Marseille, Rome .. !

Two final publications (Cusp and Ke4) in progress

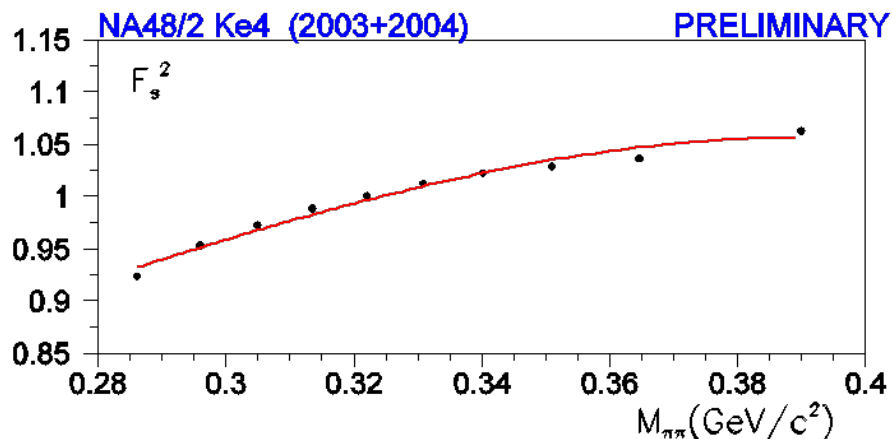
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Ke4 Form Factors : getting F_s

- **variations** of the normalization with q^2 , q^4 and $S_e/4m_\pi^2$ are investigated by a 4-parameter fit in the plane $[M_{\pi\pi}, M_{ev}]$
- Introducing a slope with S_e improves the fit χ^2 (211./83 \rightarrow 94./82)

$$F_s^2 = f_s^2 (1 + f'_s/f_s q^2 + f''_s/f_s q^4 + f'_e/f_s S_e/4m_\pi^2)^2$$

$$q^2 = (S_e/4m_\pi^2 - 1)$$

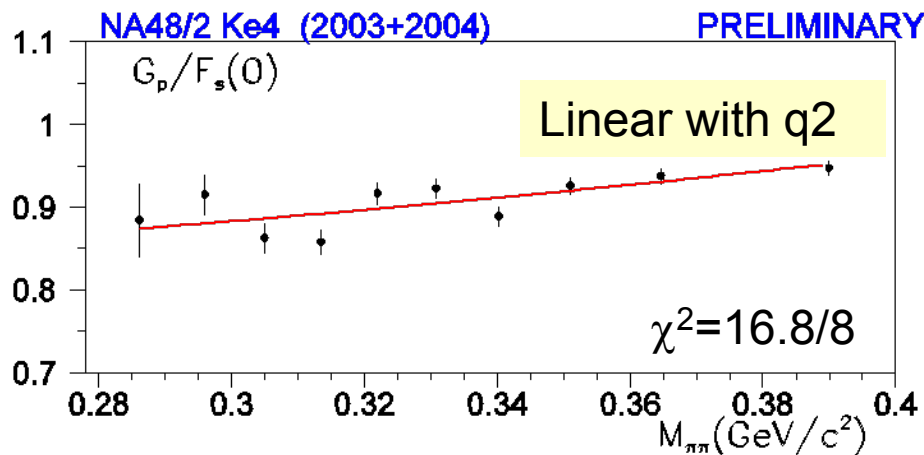
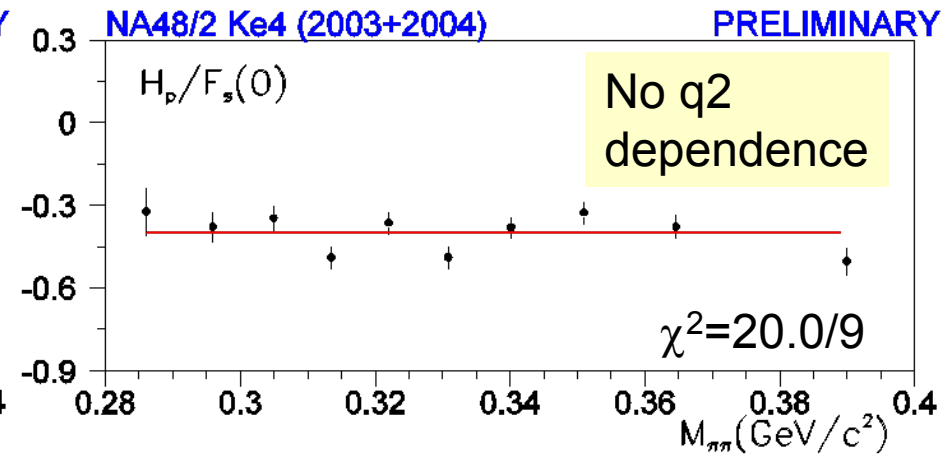
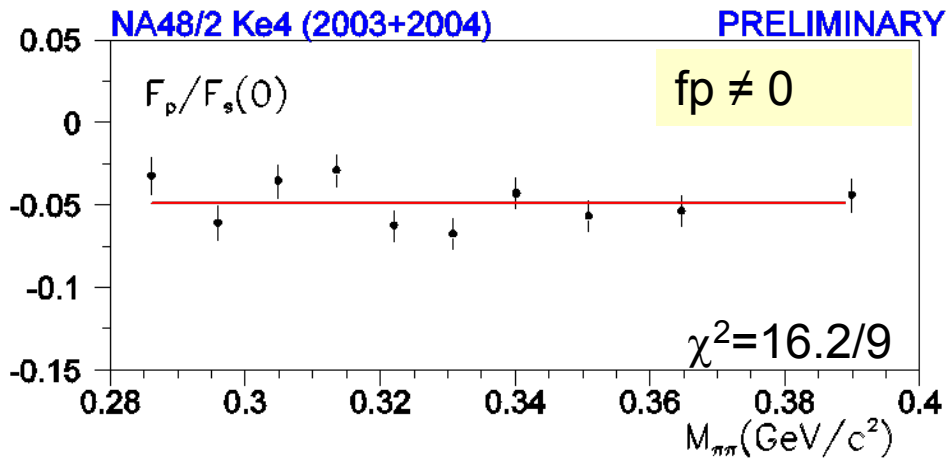


Correlations:

	f''_s/f_s	f'_e/f_s
f'_s/f_s	-0.95	0.08
f''_s/f_s		0.02

Other parameterizations could be easily tried

Ke4 Form Factors: getting F_p , G_p , H_p



$$G_p/f_s = g_p/f_s + g'_p/f_s q^2$$

Correlation

	g'_p/f_s
g_p/f_s	-0.91

Common errors to Cusp and Ke4 results ?

- Statistical errors from cusp and Ke4 measurements are independent
- Systematic errors : if (partly) correlated, contribute to the off-diagonal terms of the error matrix but very few common items
- Theoretical uncertainties come from different sources

Cusp units 10^{-4}	$a_0 - a_2$	a_2
Acceptance (z)	4	20
Acceptance (v)	1	8
PK spectrum	13	32
Trigger eff.	10	39
LKr resol	9	29
LKr nonlinear.	12	67
LKr shower	10	18
MC (time)	5	1
k0 control	2	6
Total	25	94

Ke4 units 10^{-4}	a_0	a_2
Acceptance (z) + acceptance (r) + PK spectrum ($\sim 2/3$)	20	13
Trigger eff. (but not the same)	6	4
Fit method	9	5
Bkg shape	31	18
e-ident	14	19
Radiative corrections	16	10
Bkg level	19	10
Se control	13	7
Total	50	34

Ke4 analysis : theoretical uncertainties

Work done in collaboration with Colangelo + Gasser

Ingredients to extract (a_0, a_2) from phase δ

- Roy equations : two formulations ACGL(Bern)/ DFGS(Orsay)

Orsay can vary the solutions with phases @ matching point (0.8 GeV)

Reference : $\delta_0^0 = 82.3^\circ$ and $\delta_1^1 = 108.9^\circ$

Vary : δ_0 by $\pm 3.4^\circ$

Vary : δ_1 by $\pm 2.0^\circ$

- Isospin corrections

Vary $R = 37 \pm 5$

Vary $F_\pi = (86.2 \pm 0.5) \text{ MeV}$

Neglected Higher orders

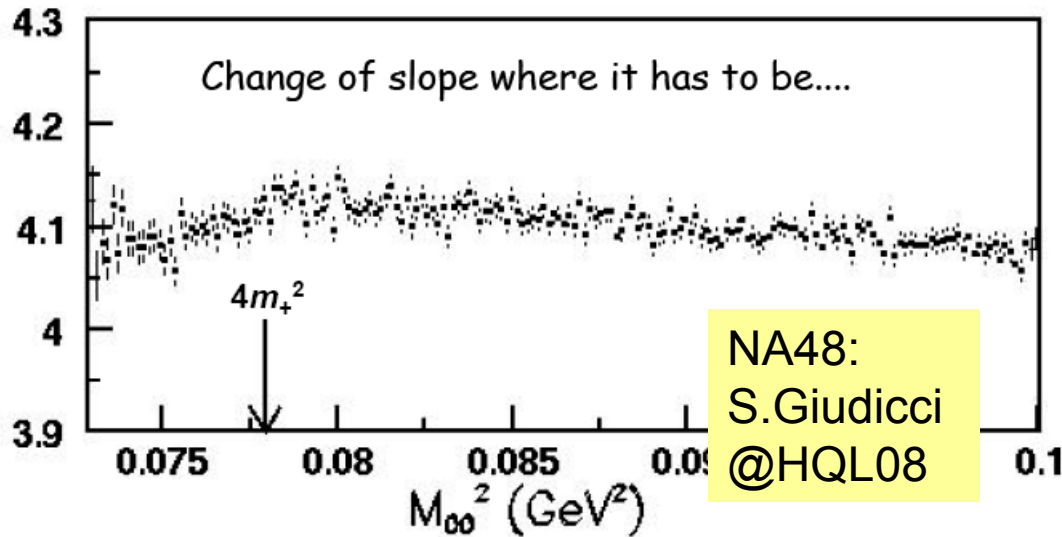
- CHPT constraint

Vary by $a_2 \pm 0.0008$

	2p-fit		1p-fit
	a_0	a_2	a_0
Roy equation solutions			
BERN-ORSAY	0.0000	0.0006	0.0013
$\delta_0^0 \pm 3.4^\circ$	0.0010	0.0027	0.0043
$\delta_1^1 \pm 2.0^\circ$	0.0000	0.0002	0.0003
Isospin corrections			
$R \pm 5$.	0.0005	0.0000	0.0008
$F_\pi \pm 0.5 \text{ MeV}$	0.0003	0.0001	0.0003
Higher Orders	0.0035	0.0005	0.0042
ChPT constraint ± 0.0008	—	—	0.0017
quadratic sum	0.0037	0.0028	0.0064

The cusp in $K_L \rightarrow 3\pi^0$ decays: work in progress

Ratio data / prediction



$$(a_0 - a_2)m_\pi$$

ChPTH

DIRAC $\pi^+\pi^-$ atom (2005)

NA48 $K^+ \rightarrow \pi^+\pi^0\pi^0$ (2006)

KTeV $K_L \rightarrow 3\pi^0$ (2008)

0.16 0.18 0.2 0.22 0.24 0.26 0.28

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June 10, 2009

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