Effective Quantum Field Theories

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MENU 2007

Jülich, September 10, 2007

Content

- Introduction
- Chiral Perturbation Theory
- LECs from lattice calculations
- Precision physics
- NREFT
- Comment on K_{e4} decays
- Summary

INTRODUCTION

A comment

About 250 talks at this conference

$\Rightarrow$ I just offer 2 messages which are hopefully of general interest:

- Lattice calculations have come into contact with chiral perturbation theory
- While doing precision physics, we should not forget about Bloch and Nordsieck, and should keep in mind that $m_u \neq m_d$

Still: 250×2 = 500 messages ...

Effective quantum field theories

An effective quantum field theory

- contains the degrees of freedom to describe physical phenomena below a chosen energy scale
- ignores the degrees of freedom at higher energies

EFTs are the counterpart of the Theory of Everything.

Ecker 2005

Consider here 2 simple examples

Beta decay

Effective quantum field theories are as old QFT:
β-decay of neutron:

$$n \to p + e^- + \bar{\nu}_e$$

$$\mathcal{L}_{\rm eff} = \text{G}\, \bar{p}\gamma^\mu n \bar{e}\gamma_\mu \nu + \text{h.c.}$$

Fermi 1933,1934

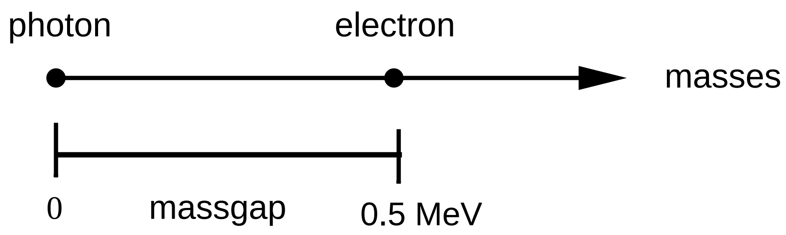
This is an effective, non–renormalizable QFT for the weak interactions at low energies (with due addition of further couplings).

no W, Z

Underlying renormalizable theory: Weinberg 1967; Salam 1968

Photon–photon interactions in QED

Consider interaction between photons at very low energies



- Effective Lagrangian for photon interactions: Write all terms allowed by symmetry (gauge, Lorentz, P, C, T)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e_1}{m_e^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{e_2}{m_e^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \cdots$$

- Amounts to an expansion in powers of ∂_μ / m_e and $F_{\mu\nu}/m_e^2$
- Scale: electron mass
- Low energy constants (LECs) e_i fixed through QED

Euler, Heisenberg 1936

- $\mathcal{L}_{\text{eff}}$ is a non–renormalizable QFT

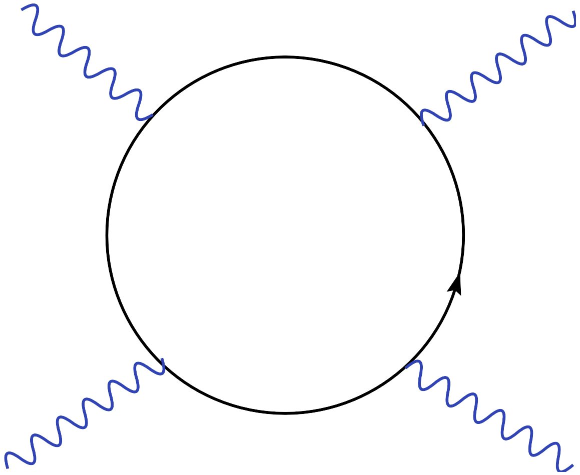
With properly chosen coefficients e_i, the above Lagrangian reproduces the matrix elements

$$n\gamma \rightarrow m\gamma$$

in full QED, to any order in α and in momenta/m_e (for small momenta).
This example contains all the bits and pieces relevant for effective Lagrangian techniques:

- Constructing the Lagrangian $\Leftrightarrow$ symmetries
- Loop calculations $\Leftrightarrow$ unitarity
- Power counting $\Leftrightarrow$ calculations become systematic
- Matching $\Leftrightarrow$ relation to underlying theory
- Validity of expansion

Calculating e_i (matching)



Generates $e_{1,2}$ at leading order in an expansion in α:

$$
\begin{aligned}
e_1 &= \frac{\alpha^2}{90} + O(\alpha^3) \\
e_2 &= \frac{7\alpha^2}{360} + O(\alpha^3), \;\; \alpha = 1/137.036...
\end{aligned}
$$

More efficient version: Schwinger 1951

CHIRAL PERTURBATION THEORY

Carry over the previous construction in QED to the effective Lagrangian of QCD

Effective field theory of QCD

I skip the “chiral symmetry” discussion and refer you to the talk by Uwe–Jens Wiese.

Systematic determination of the structure of matrix elements, as dictated by chiral symmetry:

$$\mathcal{L}_{\mathrm{QCD}} \qquad \stackrel{E \ll M_\rho}{\longrightarrow} \qquad \mathcal{L}_{\mathrm{eff}}$$

$$\boxed{\mathcal{L}_{\mathrm{eff}}}$$

– expressed in observed hadron fields
– same symmetry as QCD

The transition

$$\mathcal{L}_{\mathrm{QCD}} \Longrightarrow \mathcal{L}_{\mathrm{eff}}$$

is non–perturbative $\stackrel{!}{\Longleftrightarrow}$ QED

The leading term

- Consider pions only:

$$\begin{aligned} \mathcal{L}_{\rm eff} &= \mathcal{L}_2 + \mathcal{L}_4 + \cdots \\ \mathcal{L}_2 &= \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + M^2(U + U^\dagger)\rangle \\ & \quad U \in SU(2) \;\;, \;\; \text{contains the pion fields} \end{aligned}$$

- $M^2 = (m_u + m_d)B$
- Is derivative expansion
- F, B: low–energy constants (LECs): not fixed by chiral symmetry

$\mathcal{L}_2$: Euler–Heisenberg Lagrangian for mesons – is a non renormalizable quantum field theory

Higher order Lagrangians

$$\mathcal{L}_4 = \sum_{i=1}^{10} l_i Q_i \; ; \; \mathcal{L}_6 = \sum_{i=1}^{56} c_i P_i \; ; \; \ldots$$

- LECs l_i, c_i not fixed by symmetry. Local polynomials Q_i, P_i (expressed in meson fields) are known.

  J.G., Leutwyler 1985; Bijnens, Colangelo, Ecker 1999

- Calculations with $\mathcal{L}_{\text{eff}}$ generate an expansion in powers of quark masses and of external momenta.
  Scale: $4\pi F \simeq$ 1 GeV.

Chiral perturbation theory (ChPT)

Same keywords as in QED, page 8.

For instructive examples, see Hans Bijnens, this morning

Advantages/disadvantages

Pros

Calculating with this Lagrangian and properly chosen LECs, one reproduces S-matrix elements of QCD, at low energy, in a systematic manner

Weinberg 1979; Leutwyler 1994

Contras

Limited energy range of validity

Many LECs

Note, however: LECs are fixed by QCD, see below

Applications

MANY

see SPIRES, hep-search

In particular: Including in the effective Lagrangian also nucleon fields, one enters the rich field of nuclear physics – see talks at this conference.

LECs FROM LATTICE

Evaluation of LECs in ChPT

Of course, LECs cannot be determined as in Euler-Heisenberg Lagrangian: perturbation theory does not apply. Proper method:

Machines

Main machine: IBM Blue Gene/L at KEK

- 57.6 Tflops peak (10 racks)
- 0.5TB memory/rack
- 8x8x8(16) torus network
- ~30% performance for Wilson kernel
  (Doi and Samukawa, IBM Japan)
- Overlap HMC: 10~15% on one rack

Also using

- Hitachi SR11000 (KEK)
  - 2.15TFlops/0.5TB memory
- NEC SX8 (YITP, Kyoto)
  - 0.77TFlops/0.77TB memory

Hideo Matsufuru, Lattice 2007, 30 July

From Matsufuru, Lattice 2007

Example: l_3

Expansion of the pion mass. Contains chiral logarithms:

$$
\begin{aligned}
M_\pi^2 &= M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\} \\
M^2 &= B(m_u + m_d), \ \bar{l}_3 \equiv \frac{\Lambda_3^2}{M_\pi^2}
\end{aligned}
$$

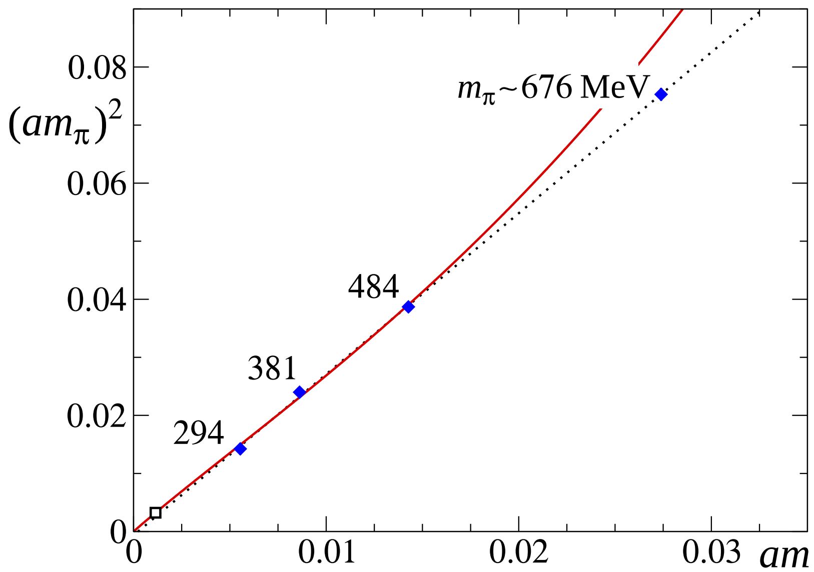
$\bar{l}_3$ is UV–finite part of l_3

Crude result, based on $SU(3)$ formulae:

0.2 GeV $\leq \Lambda_3 \leq$ 2 GeV   Leutwyler, J.G. 1984

Lattice allows more accurate determination of scale Λ_3:
Set $m_u = m_d = m$, determine M_π^2 as a function of quark mass m.

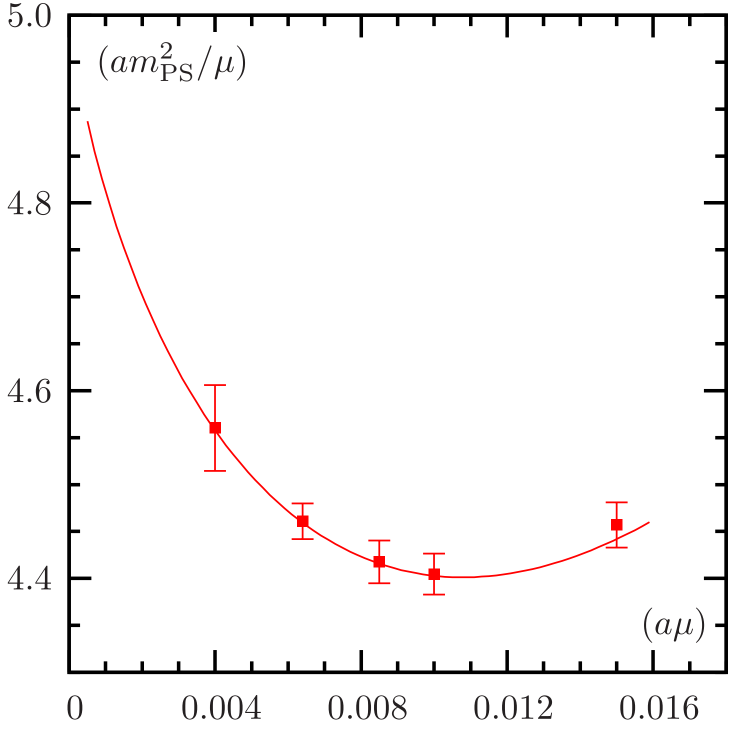
Lattice: M_π^2 versus m



Lüscher, Lattice conference 2005

- Quark masses sufficiently light $\Rightarrow$ ChPT can be used to extrapolate to physical values of m
- Data of very high quality
- Chiral logarithms – are these seen?

Lattice: M_π^2/m versus m



ETM collaboration 2006

$\bar{l}_3 = 2.9 \pm 2.4$ $\Leftrightarrow$ 0.2 GeV $\leq \Lambda_3 \leq$ 2 GeV Leutwyler,J.G. 1984

$\bar{l}_3 = 3.44 \pm 0.28$ ETM 2007

Lattice: F_π versus m

Similarly, lattice can measure quark mass dependence of pion decay constant

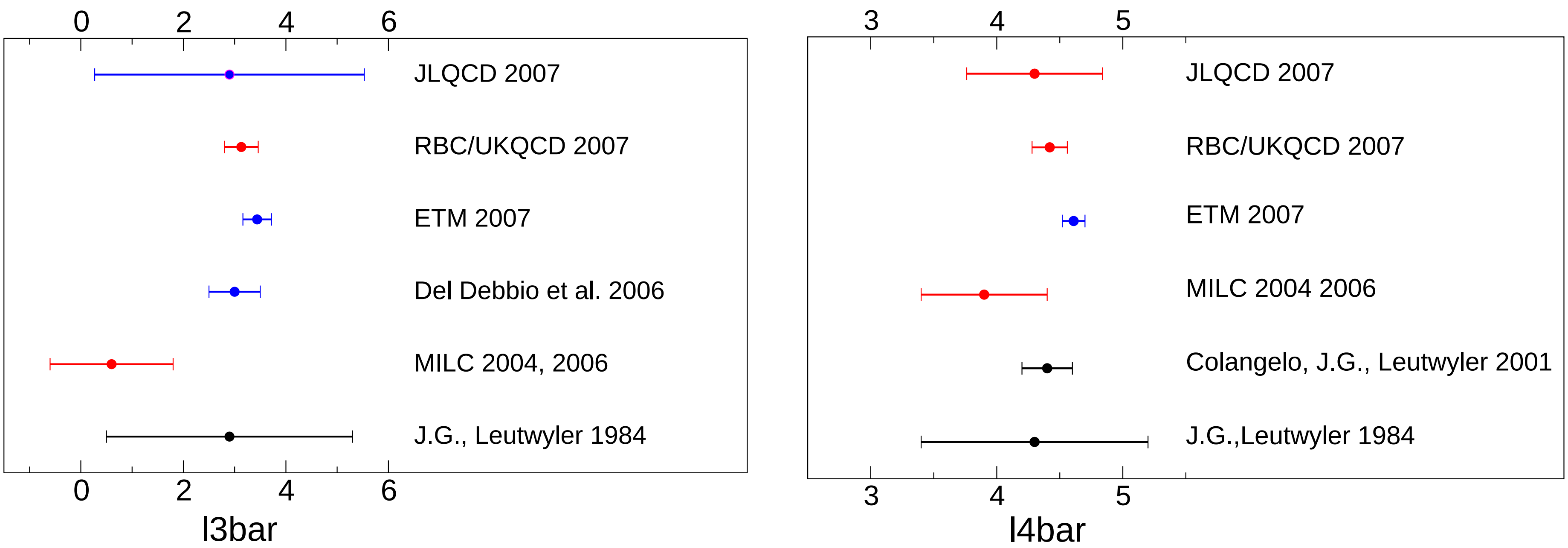
$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$\bar{l}_4 \equiv \frac{\Lambda_4^2}{M_\pi^2}$$

F : Pion decay constant, in the chiral limit $m_u = m_d = 0$

Several measurements of $\bar{l}_{3,4}$ reported at Lattice 2007

Lattice: Values of $\bar{l}_{3,4}$



message 1

$\bar{l}_{3,4}$ also occur in chiral expansion of $a_{0,2}$

PRECISION PHYSICS

$\pi\pi$ scattering lengths

ChPT + Roy equations + data above 800 MeV $\Rightarrow$ precise predictions for $\pi\pi$ S-wave scattering lengths:

$$a_0 = 0.220 \pm 0.005, \;\; a_0 - a_2 = 0.265 \pm 0.004$$

$a_{0 \leftarrow}$isospin

Colangelo, J.G., Leutwyler 2001

Interesting for several reasons:

• Given a_0, a_2, the $\pi\pi$ scattering amplitude can be calculated rather precisely in the low-energy region.

Roy 1971

Ananthanarayan, Colangelo, G., Leutwyler 2001

Descotes, Fuchs, Girlanda, Stern 2002

$\Rightarrow$ Applications in $(g-2)_\mu$ (pion form factor)

• scattering lengths $\Leftrightarrow$ spontaneous chiral symmetry breaking

Stern and collaborators since long time

Other determinations

	[1]	[2]	[3]
a_0	0.228 ± 0.032	0.224 ± 0.013	0.230 ± 0.015
$-10 \cdot a_2$	0.382 ± 0.038	0.343 ± 0.036	0.480 ± 0.046

S. Descotes-Genon, N. H. Fuchs, L. Girlanda and J. Stern 2002 [1]
R. Kaminski, L. Lesniak and B. Loiseau 2003 [2]
J. R. Pelaez and F. J. Yndurain 2005 [3]

See talk Colangelo at KAON07 for comments.

How can one test the predictions?

- Lattice calculations: energy levels of $\pi\pi$ states depend on volume, expansion coefficients contain $\pi\pi$ scattering lengths

  Lüscher 1986

- Pionium lifetime (DIRAC, CERN):
  decay rate $\sim (a_0 - a_2)^2$ Uretsky, Palfrey 1961
- $K \to 3\pi$: cusp effect

  Cabibbo 2004

  See talk by Rainer Wanke, this morning
- $K^+ \to \pi^+\pi^-e^+\nu_e$: final state interactions

  Shabalin 1963

General comment

Predictions as well as measurements are performed with high precision. Need to specify the framework before a meaningful comparison between theory and experiment can be done.

The paradise world

Predictions for $a_{0,2}$ are made in QCD (6 flavours), with

$$m_u = m_d = m, m_s, \Lambda_{\text{QCD}},$$

chosen such that

$$M_\pi = 139.6\,\text{MeV}\ ,\ \ M_K = 493.6\,\text{MeV}\ ,\ \ F_\pi = 92.4\,\text{MeV}$$

Precise values of heavy quark masses are irrelevant here.
No photons: $F_{\mu\nu} = 0$

A paradise world

Lattice calculations of $a_{0,2}$ can also be done in this framework ($N_f = 2, 3$).

Lattice: measurement of a_2

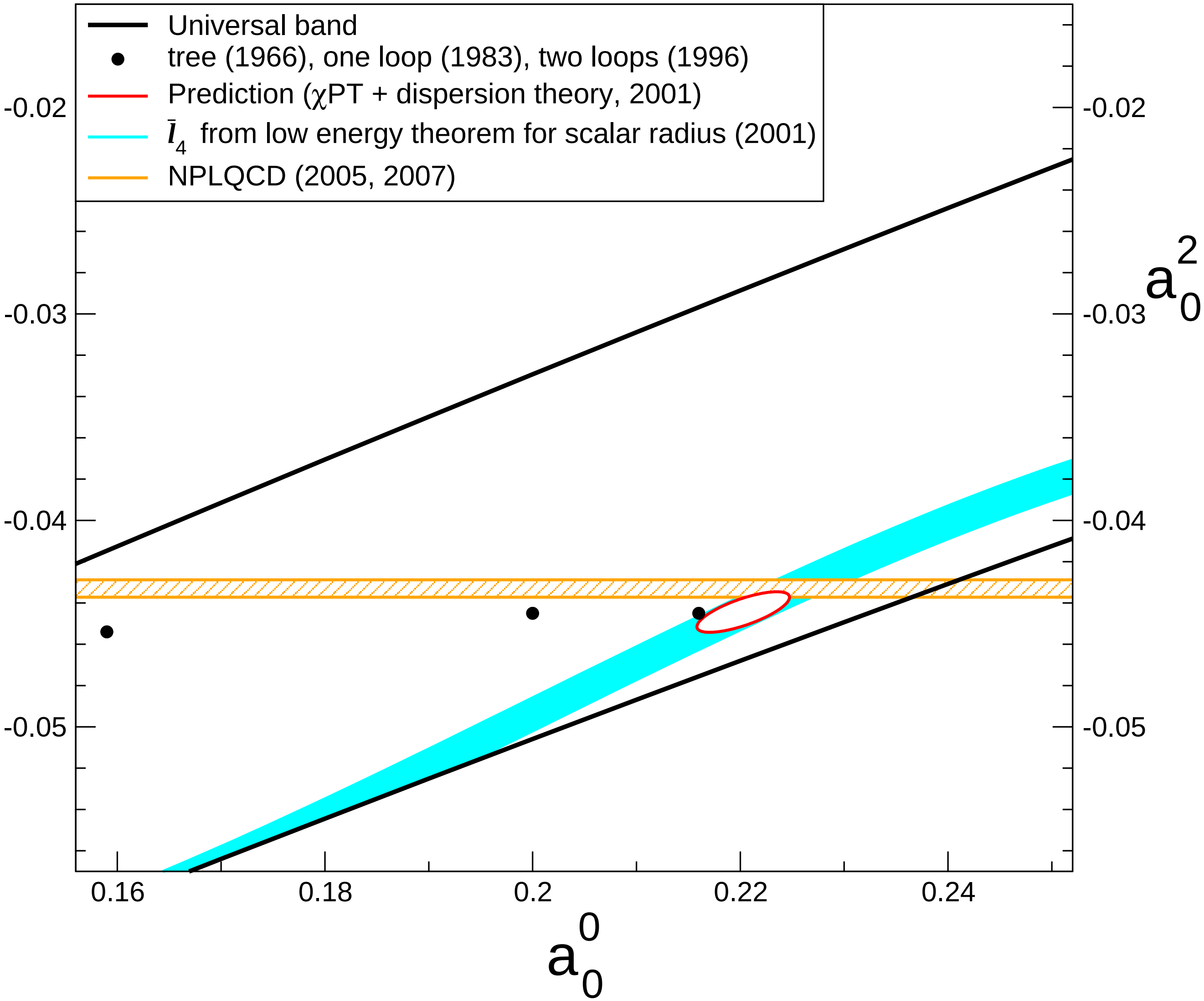


Figure courtesy of H. Leutwyler; $a_0^I = a_I$

Lattice: using $\bar{l}_{3,4}$

The uncertainties in $\bar{l}_{3,4}$ are the main source of uncertainties in the prediction of the scattering lengths. May use lattice determinations

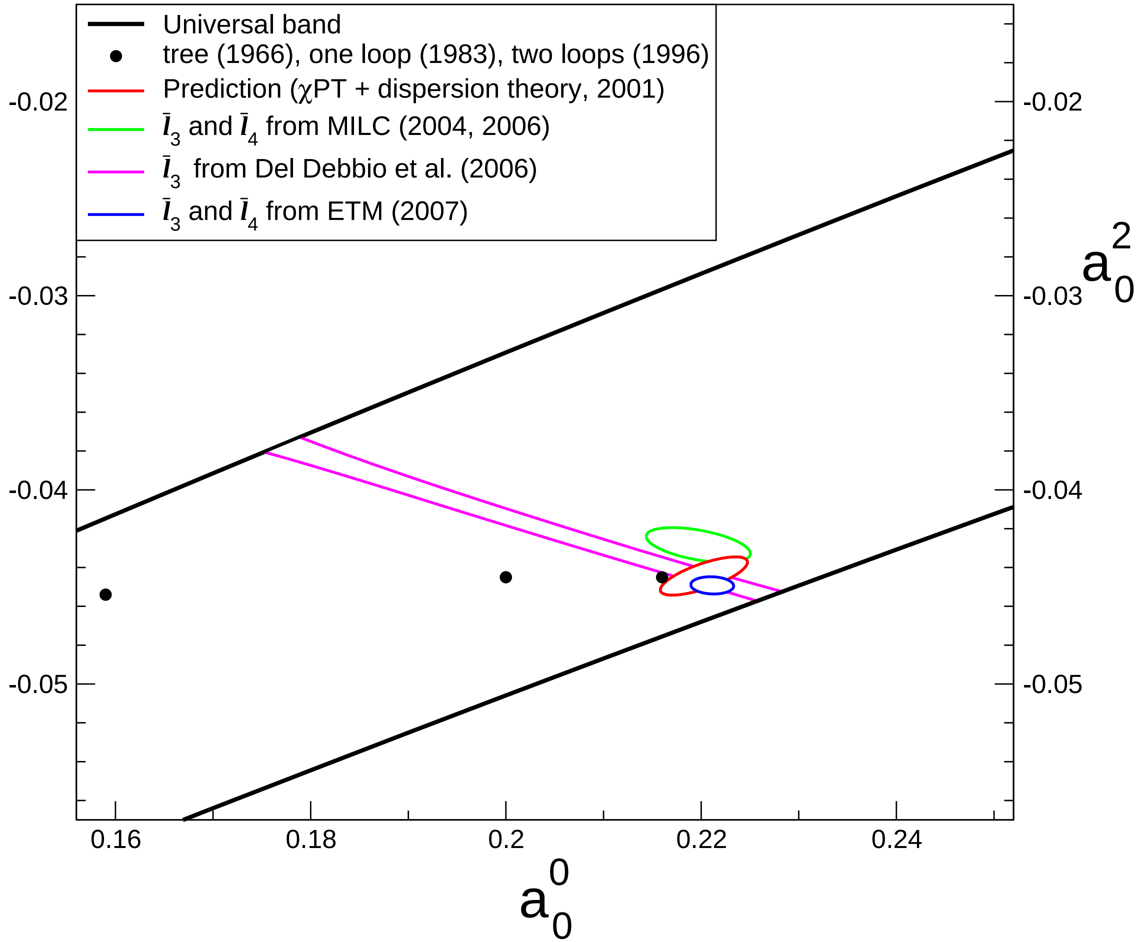


Figure courtesy of H. Leutwyler

Experiments: pionium lifetime, cusps, phases

In the paradise world:

Pionium does not form (no photons)

The kaon is stable (strangeness conservation): $K \not\to 3\pi$

$\Rightarrow$ A direct comparison with experiment is not possible – have to enlarge the framework

Experiments

Experiments are performed in the real world, described by the Standard Model

$$\alpha \neq 0, \; m_u \neq m_d$$

Pionium does form

However:

$K^+ \not\to \pi^+ \pi^0 \pi^0$
$K^+ \not\to \pi^+ \pi^- e^+ \nu_e$

ZERO probability that these processes occur in the laboratory.

Bloch, Nordsieck 1937

$\Leftrightarrow$ message 2 at the beginning

$\Rightarrow$ Need a careful analysis of the situation

HOW?

In some cases, proper method is

NREFT

“Non–relativistic effective field theory”

Pionium

Pionium: bound state of $\pi^+\pi^-$. Unstable: ground state decays into $\pi^0\pi^0, 2\gamma, \ldots$
DGBT-formula:

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^\star (a_0 - a_2)^2$$

Deser, Goldberger, Baumann, Thirring 1954

How to render this formula exact?
Looks very complicated: a_0, a_2 are the scattering lengths in the paradise world. Pionium is bound state $\Rightarrow$

- Bethe-Salpeter equation?
- Potential models?

Clue: pions are non–relativistic – use non–relativistic QFT

Caswell, Lepage for QED 1986

Expand decay rate in powers of isospin breaking parameters,

count $\alpha, (m_d - m_u)^2$ as $O(\delta)$

$$\Rightarrow \Gamma = A\delta^{7/2} + B\delta^{9/2} + O(\delta^5)$$
$$= \Gamma_{2\pi^0} + O(\delta^5)$$

Invoke non-relativistic field theory:

QCD+QED $\Rightarrow$ ChPT $\Rightarrow$ NREFT

Do calculations in NREFT. The pertinent non–relativistic Lagrangian contains unknown LECs. At the end, match these to ChPT:

NREFT $\Rightarrow$ ChPT $\Rightarrow$ QCD+QED

NREFT: number of massive particles conserved by fiat.

Improved DGBT formula

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^\star (a_0 - a_2)^2 (1+\epsilon),$$
$$\epsilon = (5.8 \pm 1.2) \times 10^{-2}$$

Eiras, Gall, G., Ivanov, Jallouli, Kong, Labelle, Lipartia, Lyubovitskij, Raha, Ravndal, Rusetsky, Schweizer, Sazdjian, Soto, Zemp, . . . after 1996

ϵ contains LECs from ChPT. Formula valid in QCD+QED.

Because a_0, a_2 known $\Rightarrow$

$$\tau = (2.9 \pm 0.1) \times 10^{-15} \text{ s}$$

prediction 2001

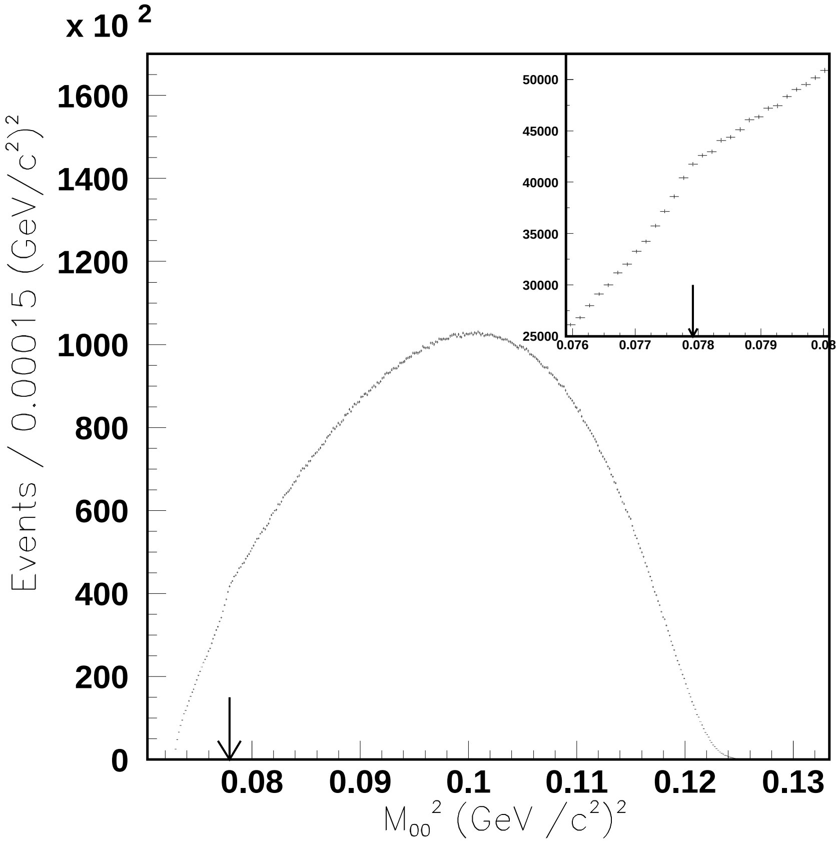
whereas

$$\tau = 2.91^{+0.49}_{-0.62} \times 10^{-15} \text{ s}$$

DIRAC 2005

Potential models: relation to QCD not manifest, unless NRQFT is built in. Superseded by NREFT. Lipartia, Lyubovitskij, Rusetsky 2002

Cusp in $K \to 3\pi$



Histogram for $K^+ \to \pi^+\pi^0\pi^0$

Taken from NA48-collaboration: hep-ex/0511056

- Strength of cusp proportional to $a_0 - a_2$
- Position of cusp dictated by isospin breaking effect

Cabibbo 2004; Cabibbo, Isidori 2005

These authors use S-matrix theory to derive form of transition amplitude.
Alternative framework: NREFT. Advantages:

- Holomorphic properties, unitarity, cluster decomposition built in

  Colangelo, J.G., Kubis, Rusetsky 2006

- Radiative corrections can be performed straightforwardly [QFT].

  Bissegger, Fuhrer, J.G., Kubis, Rusetsky, work in progress

No radiative corrections:

Amplitudes algebraically not identical, their analytic properties differ.
Effect is numerically small.

NA48 collaboration, private communication

COMMENT ON K_{e4} decays

Bloch and Nordsieck

$m_u \neq m_d$

Radiative corrections in K_{e4}

- Real/virtual photons:
  NA48: PHOTOS

  Barberio, Was 1991,...

  Sommerfeld factor (Summing vertex corrections)

  E865: Neveu, Scherk (1968): scalar QED, ϕ^4

Remark: PHOTOS does not know about matrix element
$K \to \pi\pi e\nu\gamma \Rightarrow$ check is mandatory

M. Knecht, L. Mercolli, work in progress

After having taken these corrections into account:

Recent results

Preliminary results from NA48 analyses

$a_0 = 0.256 \pm 0.008_{\rm stat} \pm 0.007_{\rm syst} \pm 0.018_{\rm th}$ Moriond 2007

$a_0 = 0.256 \pm 0.011$ Montpellier 2006

$\text{(stat.+syst. error added)}$

B. Bloch-Devaux

These results are in conflict with the prediction

$$a_0 = 0.220 \pm 0.005$$

Colangelo,Gasser,Leutwyler 2000

Phases

The decay amplitude contains the axial current matrix element

$$\langle \pi^+ \pi^- | A_\mu | K^+ \rangle \quad = \quad \frac{1}{iM_K} [P_\mu F + Q_\mu G + L_\mu R]$$

with

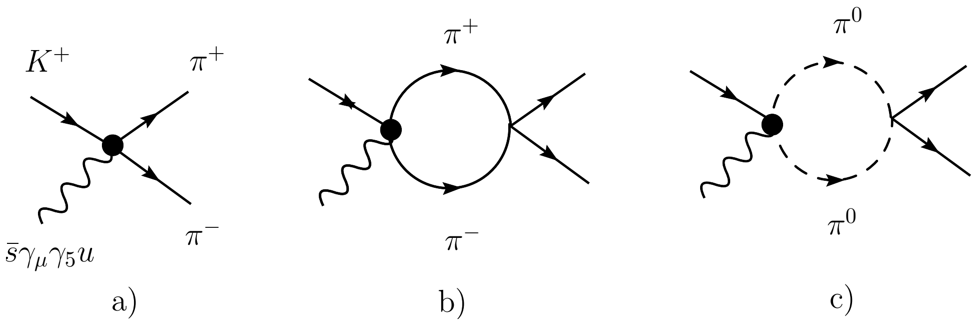
$$F = f_s \exp i\delta_0^0 + f_p \exp i\delta_1^1 + D - \text{waves}$$

This is Watson's theorem, true in the isospin symmetry limit. Allows one to measure the phase difference $\delta_0^0 - \delta_1^1$ and thus to determine the scattering lengths.

Which phase is measured?

$m_u = m_d; e = 0$

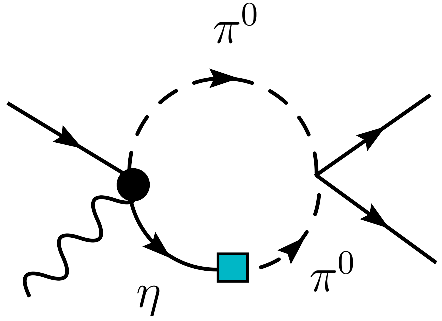
Lowest order:



$\rightarrow \delta_0^0$

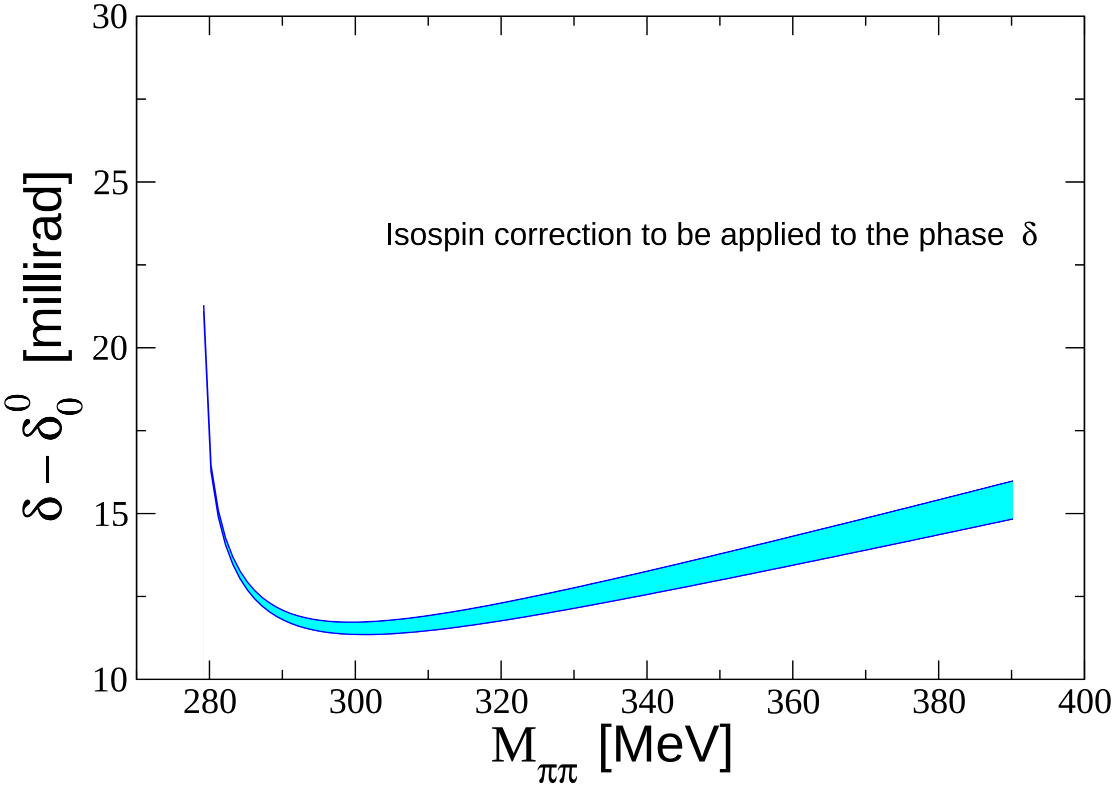
$m_u \neq m_d; e \neq 0$

Analytic structure of diagrams change. Additional diagram:



Phase is changed: $\delta_0^0 \rightarrow \delta$. Experiment measures δ. Effect is large.

The missing piece

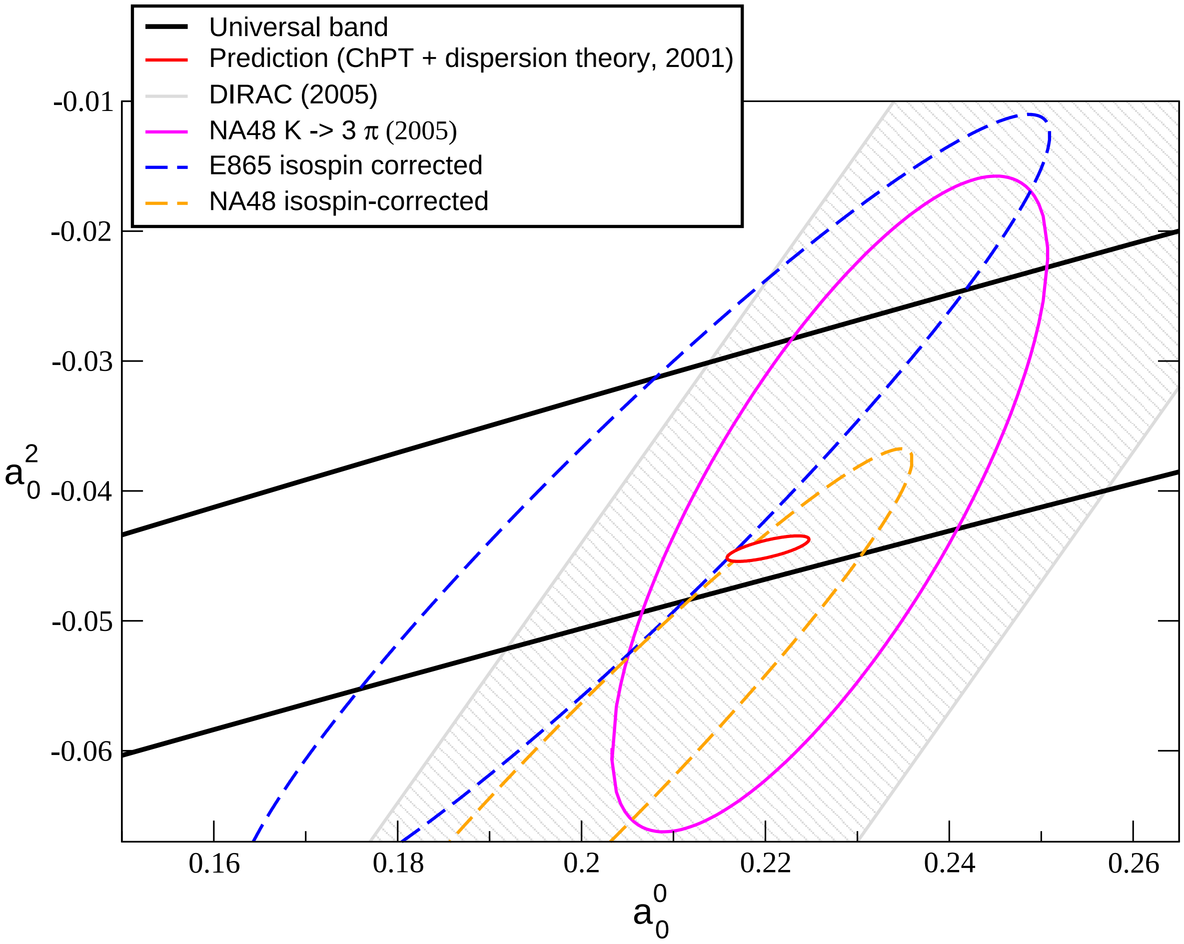


This part must be subtracted from the measured phase before a comparison with the prediction can be made.
Note: the correction displayed stems from a one–loop calculation.

Colangelo, J.G., Rusetsky, work in progress

see also Gevorkyan et al., 2007

Predictions vs. data



SUMMARY

- Lattice calculations have reached a precision that allows one to match the low–energy effective Lagrangian of QCD to the underlying theory, at least in some sectors.
- Some care is needed when translating experimental data into the paradise world. In particular, one should keep in mind that $m_u \neq m_d$ in the real world, and should not forget about Bloch and Nordsieck.