

# Recent **NA48/2** results on **Ke4** and **K3 $\pi$** decays and determination of the **$\pi\pi$** scattering lengths

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On behalf of the **NA48/2** collaboration:

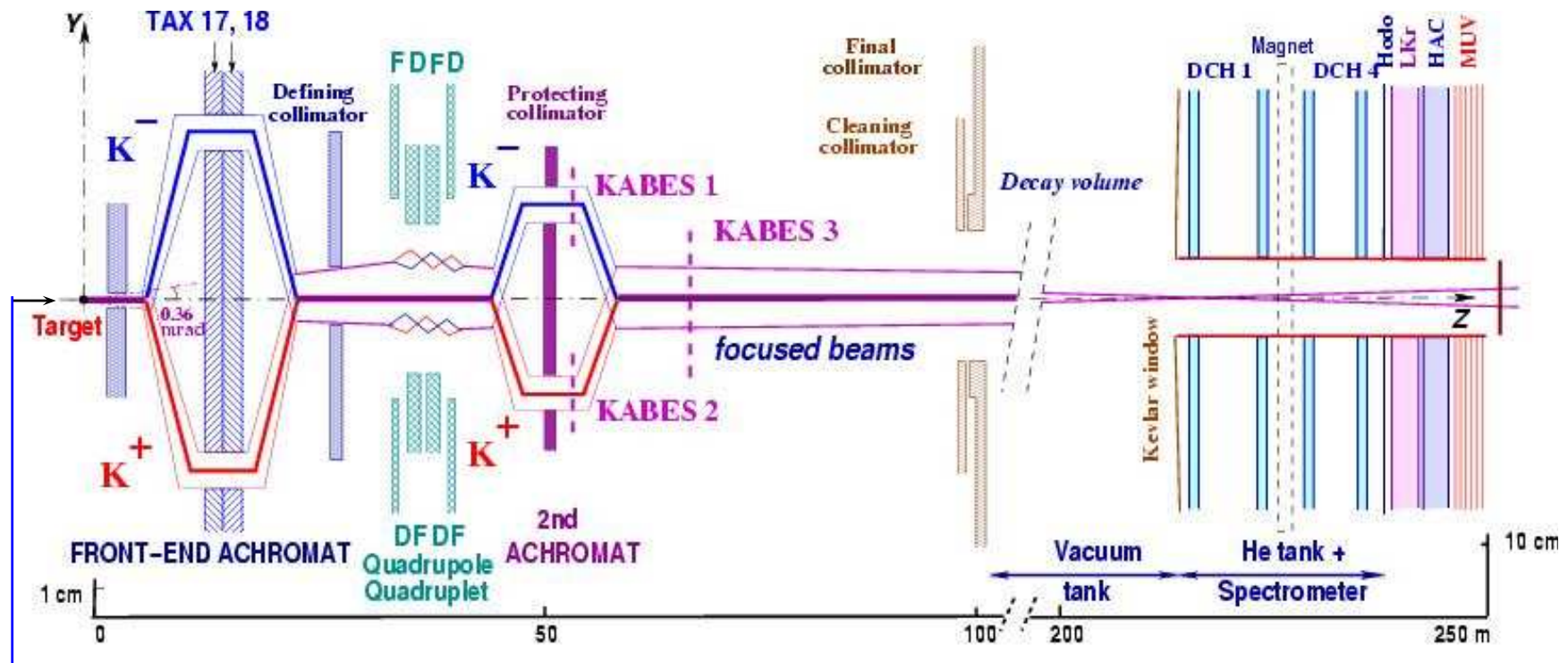
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz,  
Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna



## Outline

- The CERN NA48/2 experiment: setup and data
- Theory: ChPT,  $\pi\pi$  S-wave scattering lengths, cusp(s):  
 $\Rightarrow$  see A. Rusetsky's talk, Plenary Session 1
- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ : new measurement on the “cusp”  
and extraction of the  $\pi\pi$  scattering lengths
- $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ : measurement of form factors  
and pion scattering lengths

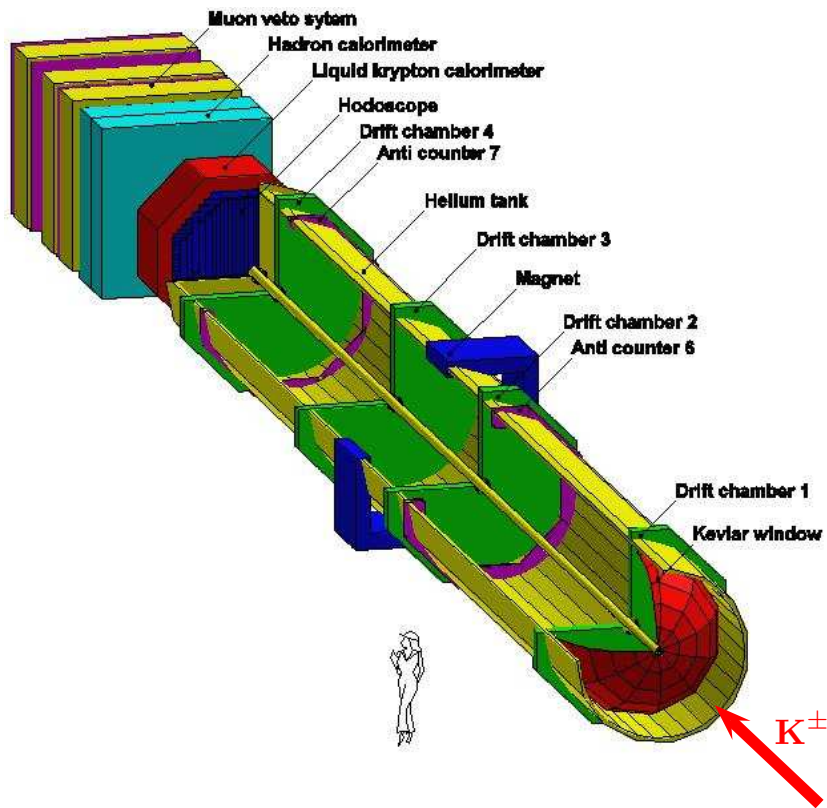
# The NA48/2 experiment



400 GeV/c protons  
from CERN SPS

Simultaneous  $K^+/K^-$  beams:  $P = 60 \pm 3$  GeV/c  
 $K^+/K^- \simeq 1.8$

# The NA48 detectors



Liquid Krypton e.m. calorimeter:

$$\frac{\sigma_E}{E} = \frac{3.2\%}{\sqrt{E/1\text{GeV}}} \oplus \frac{9\%}{E/1\text{GeV}} \oplus 0.42\%$$

Magnetic Spectrometer:

$$\frac{\sigma_P}{P} \simeq 1\% \oplus 0.044\% \frac{P}{(1\text{GeV}/c)}$$

Charged Hodoscope:  $\sigma_t = 150 \text{ ps}$

Hadron calorimeter, muon and photon vetos

- 1997-2001: NA48  $K_L, K_S$  ( $\epsilon'/\epsilon$ )
- 2002: NA48/1  $K_S$
- 2003-2004: **NA48/2**  $K^\pm$  ( $\Delta g$ )
- 2007: NA62  $K^\pm$  ( $K_{e2}/K_{\mu2}$ )
- 2011: NA62  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Main goal of NA48/2: search for CP violation in  $K^\pm \rightarrow 3\pi$  Dalitz plots.

2003+2004 data:  $4 \cdot 10^9 K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  ;  $1 \cdot 10^8 K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  ;  $1 \cdot 10^6 K_{e4}$ .

# $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ event selection

- require 1 track + 4 e.m. clusters
- cluster pairing ( $\pi^0 \rightarrow \gamma\gamma$ ):
  - consider all (3) combinations
  - reconstruct each  $\pi^0 \rightarrow \gamma\gamma$  vertex using  $\pi^0$  mass
  - choose 2-vtx combination with closest vertices
- K decay vertex = average of  $\pi^0$  vertices
- calculate  $M_{00} \equiv M(\pi^0 \pi^0)$  and  $M_{\pm 00} \equiv M(\pi^\pm \pi^0 \pi^0)$  invariant masses

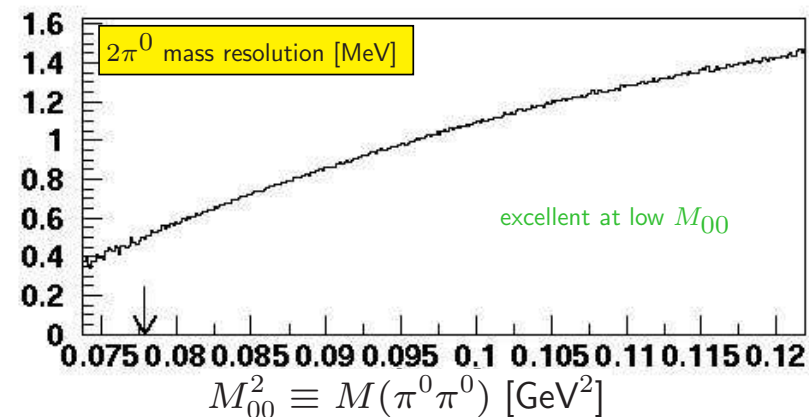
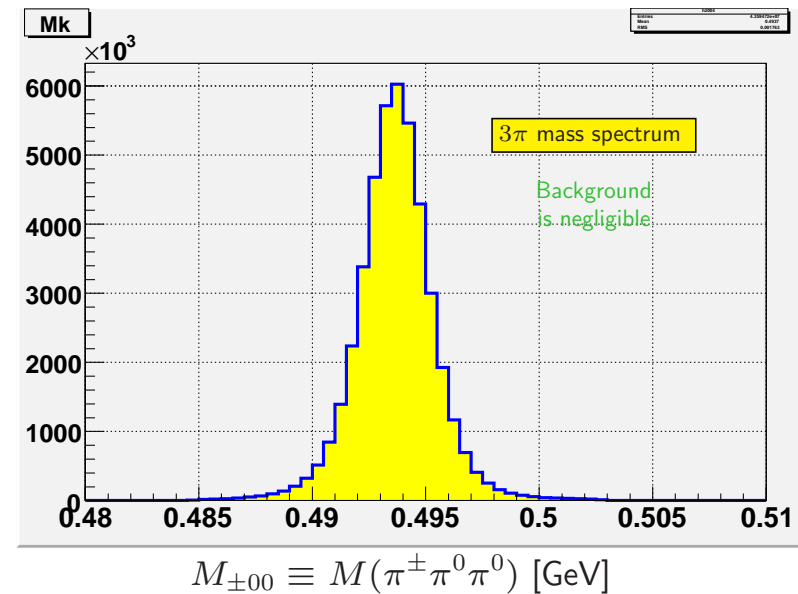
$\sigma(M_{00})$  optimized for low  $M_{00}$  values:

$$\sigma(M_{00}) = 0.56 \text{ MeV at } M_{00} = 2m_{\pi^+}$$

First cusp result (2003 data,  $2.3 \cdot 10^7$  events):

→ published on PLB 633 (2006) 173

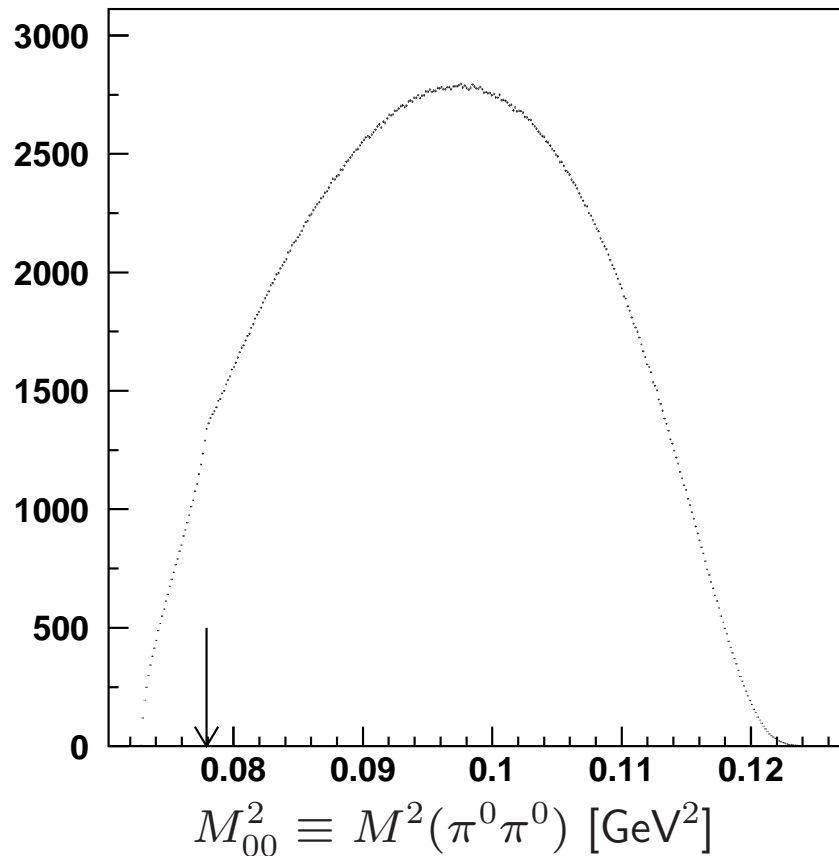
This analysis (2003 + 2004 data):  $6 \cdot 10^7$  events



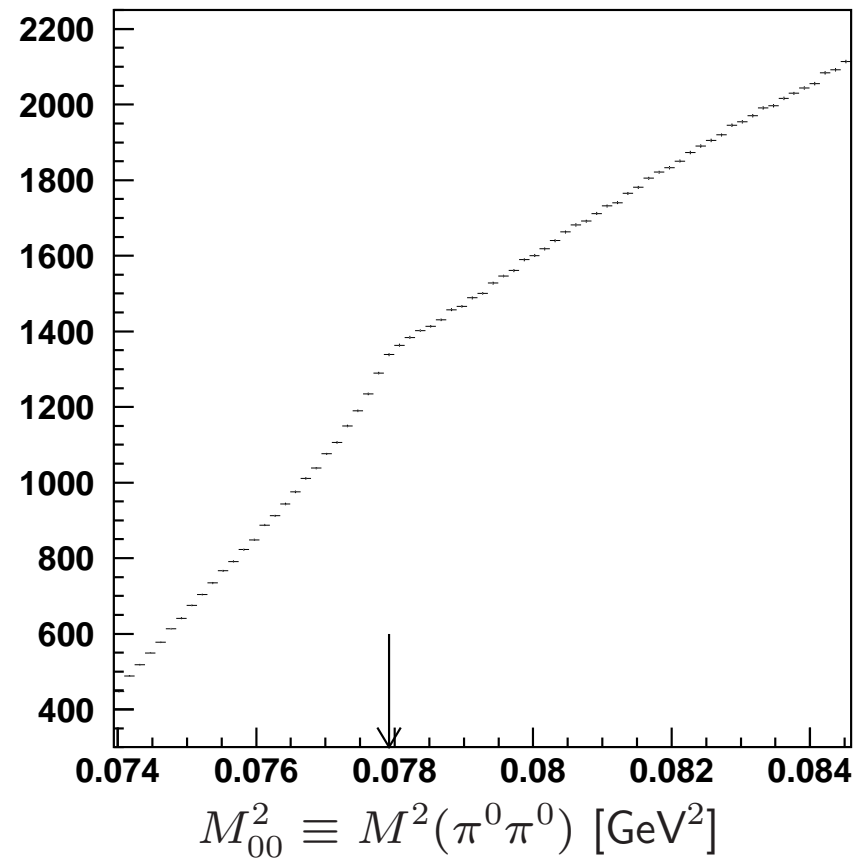


# $M_{\pi^0\pi^0}^2$ distribution in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays

$\times 10^2$  NA48/2 (2003+2004 data)



$\times 10^2$  Zoom on the cusp region



Evidence of a **cusp** at  $M_{00}^2 = (2m_{\pi^+})^2 \simeq 0.078 \text{ GeV}^2$

# Cusp interpretation

$\pi\pi$  rescattering: one -loop calculation

[N. Cabibbo, PRL 93(2004) 12181]

$$\mathcal{M}(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = \mathcal{M}_0 + \mathcal{M}_1$$

$\mathcal{M}_0$ :  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  (direct)

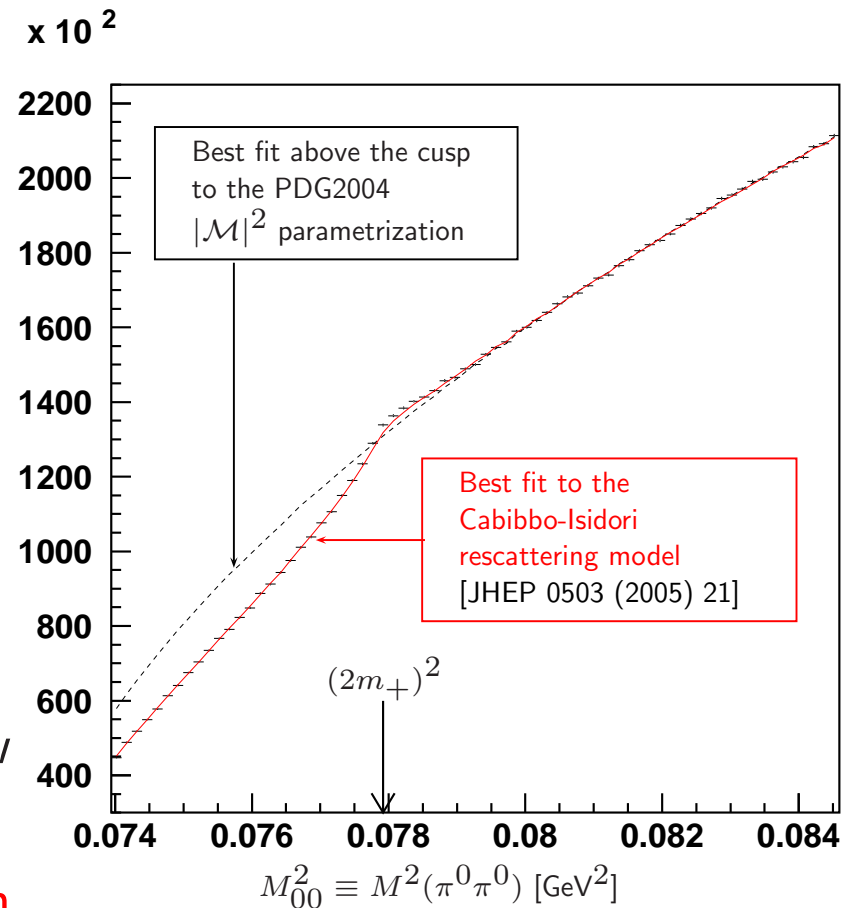
$\mathcal{M}_1$ :  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ ,  $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$

$$\mathcal{M}_1 = -\frac{2}{3}(a_0 - a_2)m_+ \mathcal{M}_{+,thr} \sqrt{1 - \frac{4m_+^2}{M_{00}^2}}$$

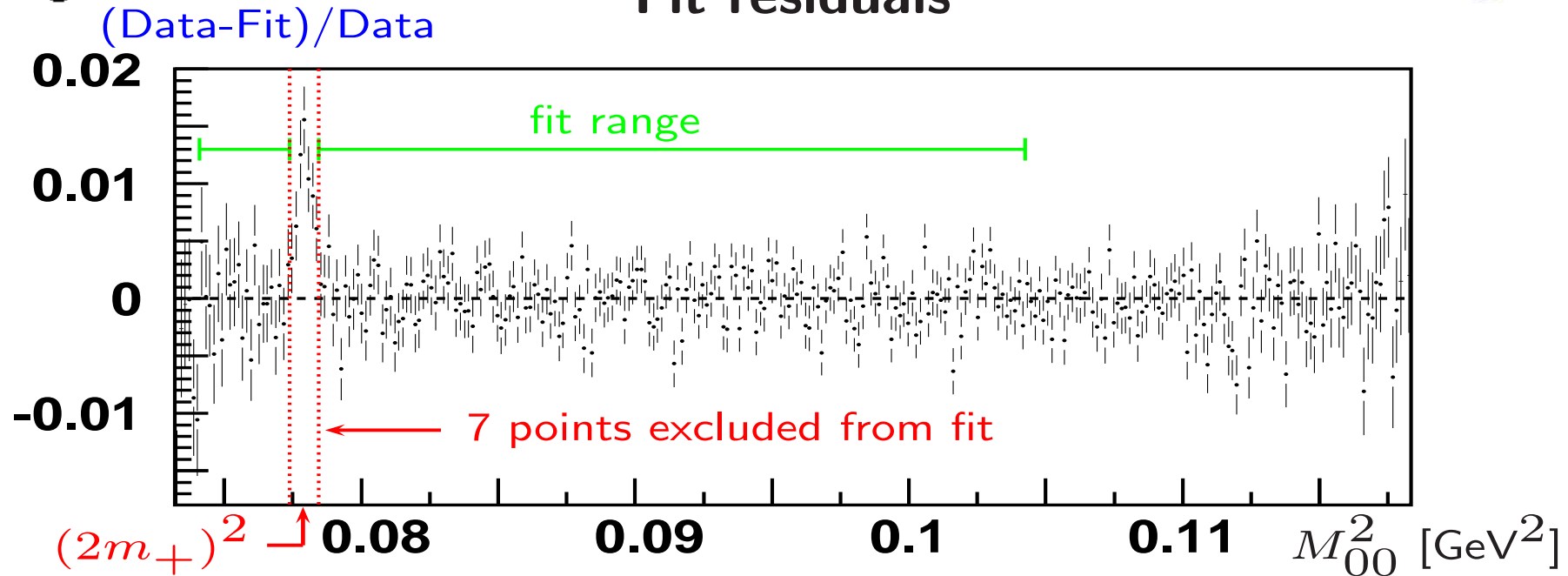
[ $m_+ \equiv m_{\pi^+}$ ;  $\mathcal{M}_+ \equiv \mathcal{M}(K^+ \rightarrow \pi^+ \pi^+ \pi^-)$ ]

interferes destructively with  $\mathcal{M}_0$  below  $\pi^+ \pi^-$  threshold: integrated deficit  $\approx 13\%$

Possibility to determine  $(a_0 - a_2)$  and its sign



## Fit residuals



Observe an excess of events in the  $M_{00}^2$  interval excluded from fit.

Rate =  $(5.6 \pm 1.0) \cdot 10^{-5}$  per  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  decay

(adding a delta-function at  $M_{00} = (2m_+)$  before taking into account detector resolution and using its integral as a free parameter)

Two contributions to the peak in the  $M_{00} \simeq 2m_+$  region:

- ponium formation:  $(\pi^+ \pi^-)_{atom} \rightarrow \pi^0 \pi^0$  (Silagadze, JETP Lett.60 (1994) 689)
- additional  $\pi^+ \pi^-$  unbound states with resonant structure  $\rightarrow \pi^0 \pi^0$  (Gevorkian, Tarasov, Voskresenskaya, hep-ph/0612129)



## Cusp: fit results (I)

Data fitted using two different theoretical models:

- Cabibbo-Isidori (CI) rescattering model [JHEP 0503 (2005) 21]
- Colangelo-Gasser-Kubis-Rusetsky (CGKR) effective field theory [PLB 638 (2006) 187]

Fit with **analyticity and chiral symmetry constraint** between  $a_2$  and  $a_0 - a_2$

$$a_2 m_+ = (-0.0444 \pm 0.0008) + 0.236(a_0 m_+ - 0.22) - 0.61(a_0 m_+ - 0.22)^2 - 9.9(a_0 m_+ - 0.22)^3$$

[Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008]

$$\text{(CI): } (a_0 - a_2)m_+ = 0.268 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

$$\text{(CGKR): } (a_0 - a_2)m_+ = 0.266 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

**External uncertainty:** from the uncertainty on the ratio (PDG 2006)

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0 \pi^0)} = 3.182 \pm 0.047 \Rightarrow \frac{A(K^+ \rightarrow \pi^+ \pi^+ \pi^-)}{A(K^+ \rightarrow \pi^+ \pi^0 \pi^0)} = 1.975 \pm 0.015$$

at the Dalitz plot centres (**exact isospin symmetry predicts 2**)

**Theoretical uncertainty** on  $(a_0 - a_2)m_+$ :  $\pm 0.013$  (CI), not given (CGKR)  
(estimated from neglecting higher order diagrams and radiative corrections)



## Cusp: fit results (II)

Fits imposing chiral symmetry constraint:

$$(CI): \quad (a_0 - a_2)m_+ = 0.268 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

$$(CGKR): \quad (a_0 - a_2)m_+ = 0.266 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

Fits **not** imposing the chiral symmetry constraint: possibility to measure  $a_2$

$$(CI): \quad (a_0 - a_2)m_+ = 0.266 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$
$$a_2 m_+ = -0.039 \pm 0.009_{stat} \pm 0.006_{syst} \pm 0.002_{ext}$$

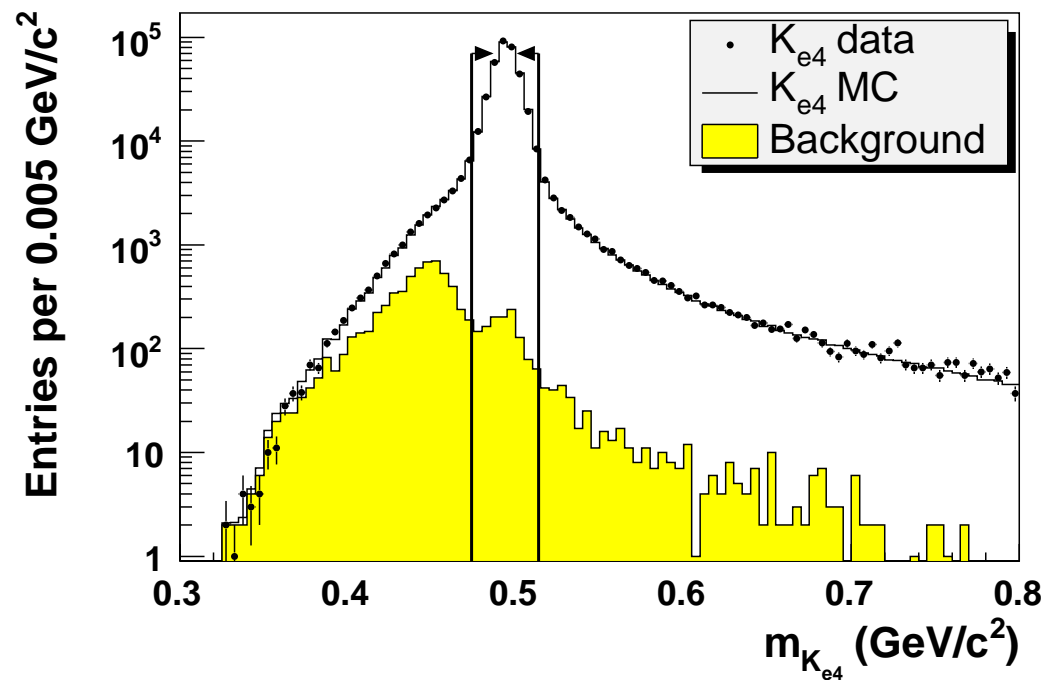
$$(CGKR): \quad (a_0 - a_2)m_+ = 0.273 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$
$$a_2 m_+ = -0.065 \pm 0.015_{stat} \pm 0.010_{syst} \pm 0.002_{ext}$$

Theoretical uncertainty:  $\pm 5\%$  (CI); not given (CGKR).



# Ke4 ( $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ )

Analysis of 2003 data (Eur. Phys. J. C 54 (2008) 411)



Rare decay:

$$\text{B.R.} = (4.09 \pm 0.09) \cdot 10^{-5}$$

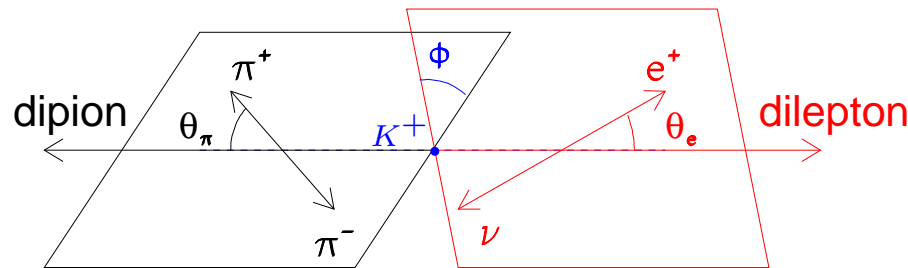
Event selection:

- 3 tracks
- missing energy and  $p_t$
- $E/p$  for  $e$  and  $\pi$  ID

$6.8 \cdot 10^5$  selected events

Background estimated from “wrong sign” events ( $\pi^\pm\pi^\pm e^\mp$ ):  $\approx 0.5\%$

## Ke4 kinematics



Kinematics described by 5 independent Cabibbo-Maksymowicz variables:

$$M_{\pi\pi}^2, M_{e\nu}^2, \theta_\pi, \theta_e, \phi$$

Assuming CP symmetry:

$$K^+ \rightarrow K^- \Rightarrow \phi \rightarrow \phi + \pi, \\ \theta_e \rightarrow \pi - \theta_e$$

Decay amplitude is described by two axial ( $F$ ,  $G$ ) and one vector ( $H$ ) form factors, which can be **partial wave** expanded ( $s$  and  $p$  waves;  $d$  neglected):

$$F = F_s(q^2)e^{i\delta_s} + F_p(q^2)e^{i\delta_p} \cos \theta_\pi$$

$$G = G_p(q^2) e^{i\delta_p}$$

$$H = H_p(q^2) e^{i\delta_p}$$

$$\delta(q^2) \equiv \delta_s - \delta_p$$

$$q^2 \equiv (M_{\pi\pi}^2/4m_\pi^2) - 1$$

and further expanded in **powers of  $q^2$** :

$$F_s(q^2) = f_s + f'_s q^2 + f''_s q^4 + f_e (M_{e\nu}^2/4m_\pi^2) + \dots$$

$$F_p(q^2) = f_p + f'_p q^2 + \dots$$

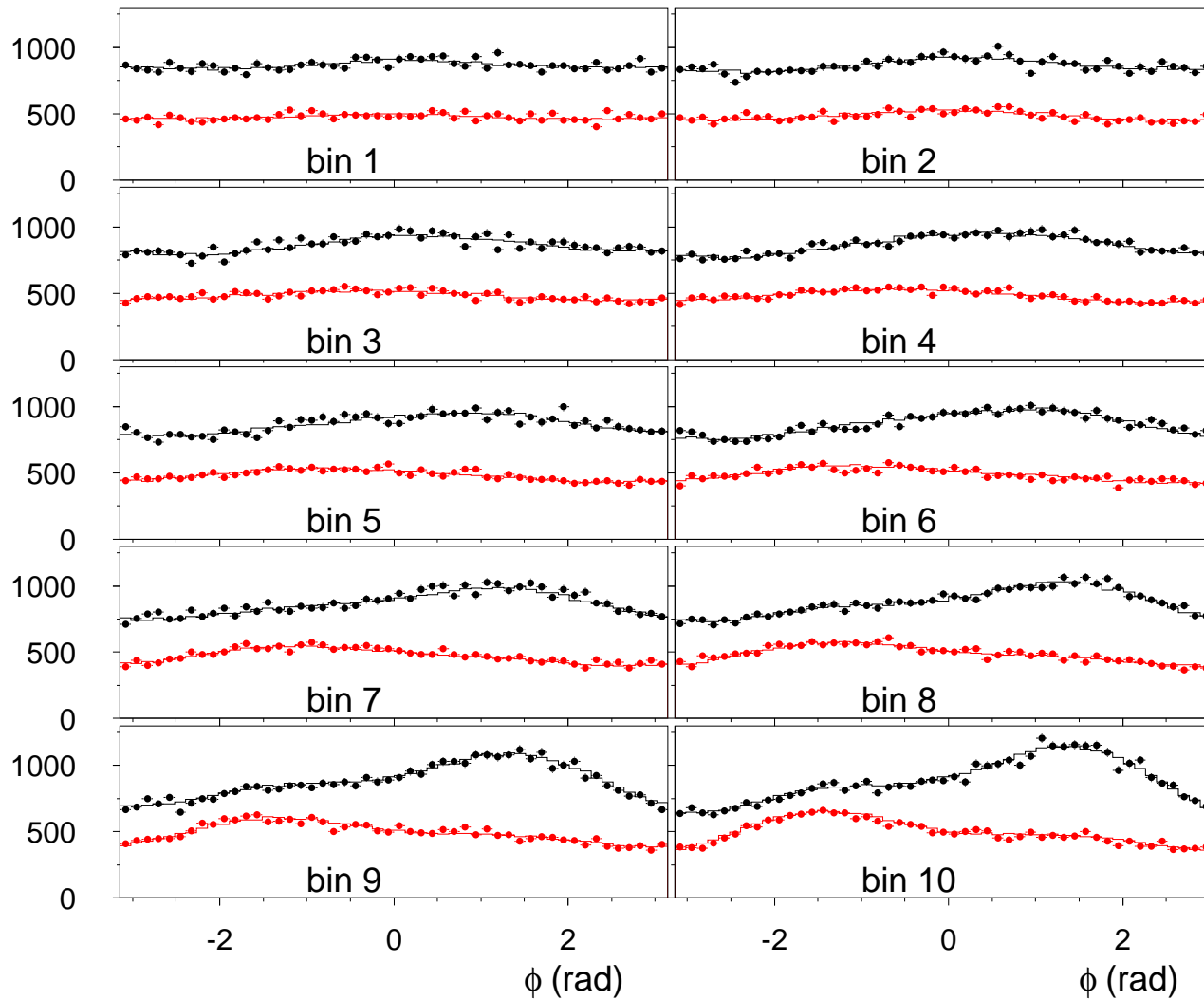
$$G_p(q^2) = f_p + g'_p q^2 + \dots$$

$$H_p(q^2) = h_p + h'_p q^2 + \dots$$

$\delta \neq 0 \Rightarrow$  **asymmetry in  $\phi$  distribution, increasing with  $M_{\pi\pi}$**



## Ke4 $\phi$ distributions



one plot  
per  
 $M_{\pi\pi}$  bin

$K^+$  (black)  
and  $K^-$  (red):

points = data

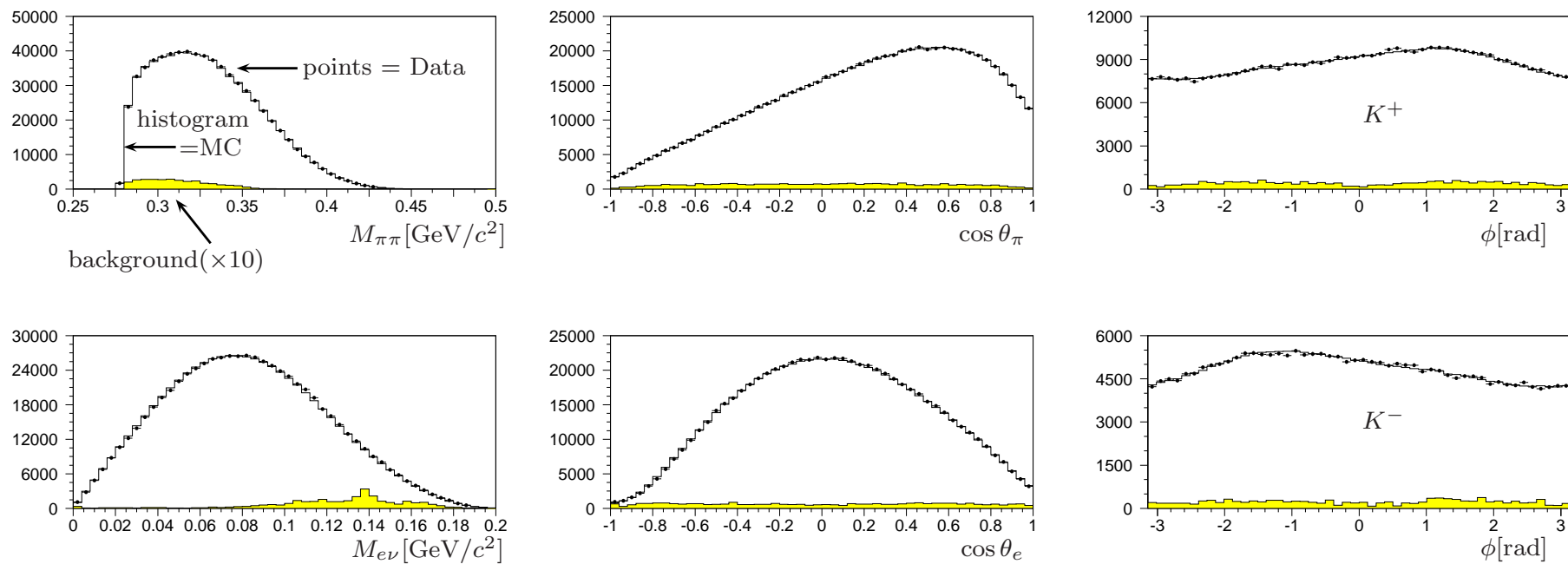
histogram = MC



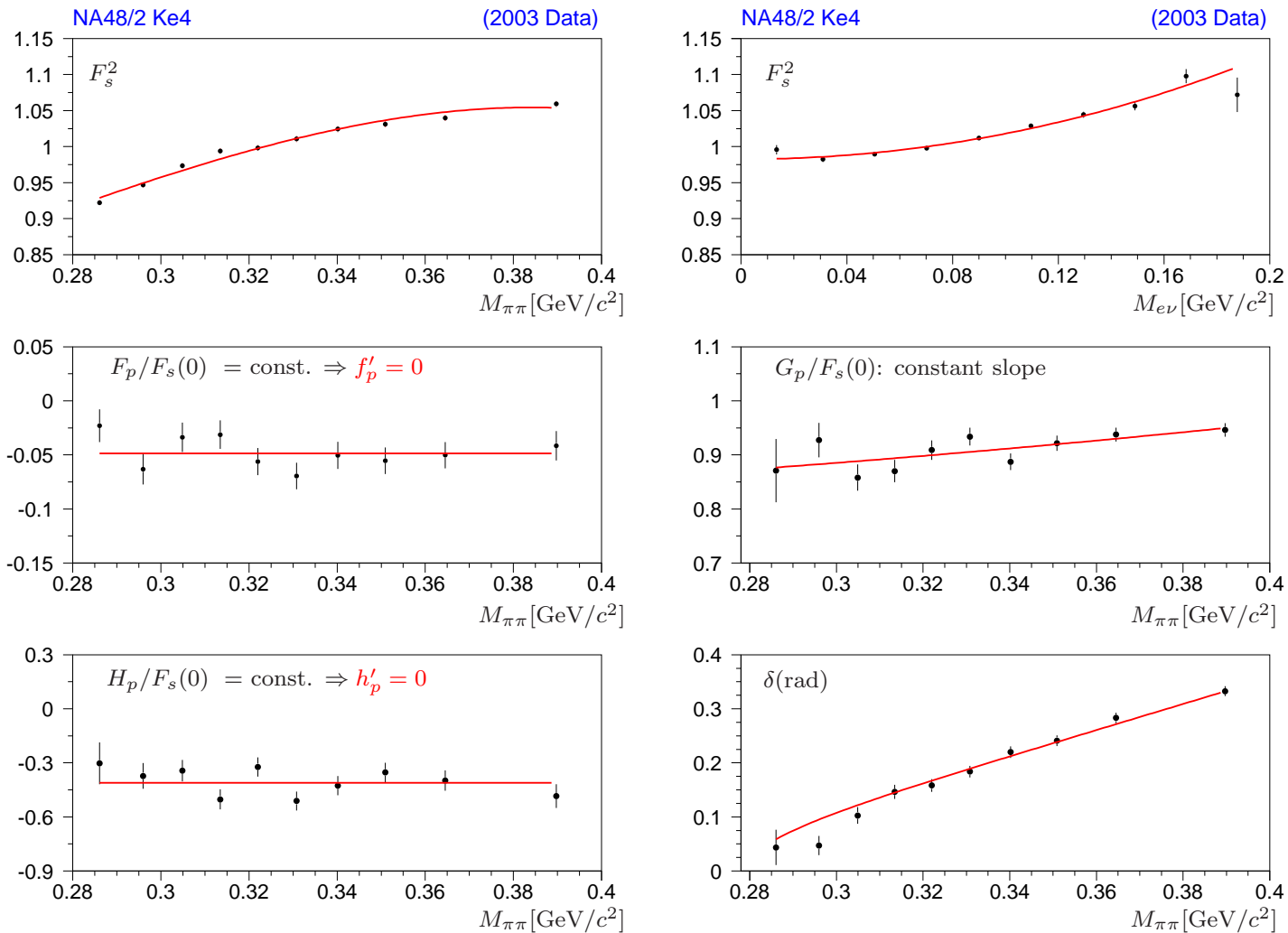
## Ke4 form factors fit (I)

$F_s, F_p, G_p, H_p$  and  $\delta$  are extracted from a fit to  
 $10(M_{\pi\pi}) \times 5(M_{e\nu}) \times 5(\cos\theta_e) \times 5(\cos\theta_\pi) \times 12(\phi) = 15000$  equi-populated bins.

Event distributions in Cabibbo-Maksymowicz variables (2003 data,  $6.8 \cdot 10^5$  events)



# Ke4 form factors fit (II)





## Ke4 form factors results (2003 data)



$$f'_s/f_s = 0.172 \pm 0.009_{stat} \pm 0.006_{syst}$$

$$f''_s/f_s = -0.090 \pm 0.009_{stat} \pm 0.007_{syst}$$

$$f'_e/f_s = 0.081 \pm 0.008_{stat} \pm 0.009_{syst}$$

$$f_p/f_s = -0.048 \pm 0.004_{stat} \pm 0.004_{syst}$$

$$g_p/f_s = 0.873 \pm 0.013_{stat} \pm 0.012_{syst}$$

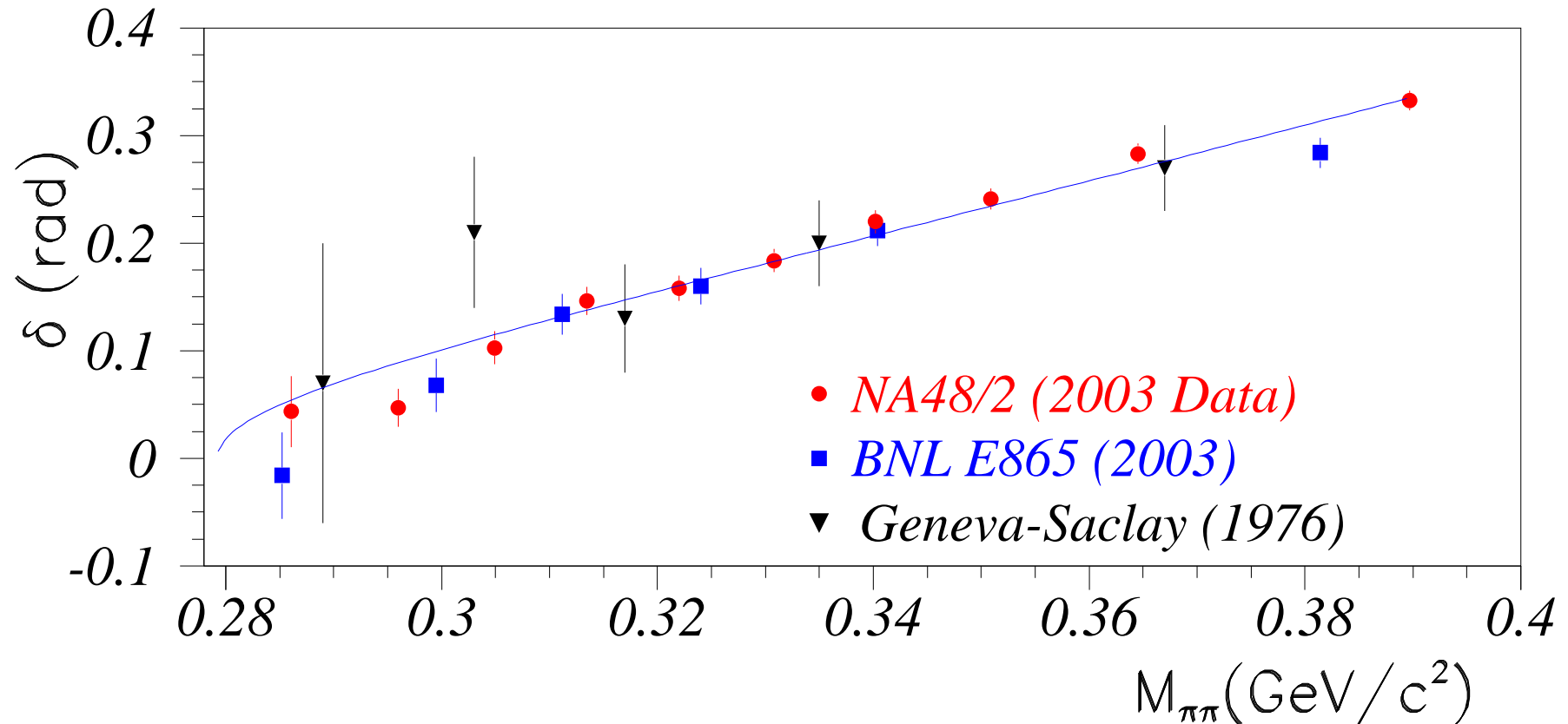
$$g'_p/f_s = 0.081 \pm 0.022_{stat} \pm 0.015_{syst}$$

$$h_p/f_s = -0.411 \pm 0.019_{stat} \pm 0.008_{syst}$$

Relative form factors (normalized to  $f_s$ )

Separate measurements for  $K^+$  and  $K^-$ , then combined

## Ke4: $\delta(M_{\pi\pi})$ and scattering lengths



Fit to Universal Band (UB) central line in  $(a_0, a_2)$  space

(UB) = limits from experimental data at higher energies, extrapolated using Roy equations

[Ananthanarayan, Colangelo, Gasser, Leutwyler, Phys. Rep. 353 (2001) 207]



## Ke4: $\pi\pi$ scattering lengths

Fit to Universal Band (UB) central line in  $(a_0, a_2)$  space:

$$a_0 m_+ = 0.256 \pm 0.006_{stat} \pm 0.002_{syst}^{+0.018}_{-0.017_{ext}}$$

$$a_2 m_+ = -0.0312 \pm 0.0011_{stat} \pm 0.0004_{syst}^{+0.0129}_{-0.0122_{ext}}$$

(“External” error: from central line to limits of UB)

Fit with 2 free parameters ( $a_0$  and  $a_2$ ):

$$a_0 m_+ = 0.233 \pm 0.016_{stat} \pm 0.007_{syst}$$

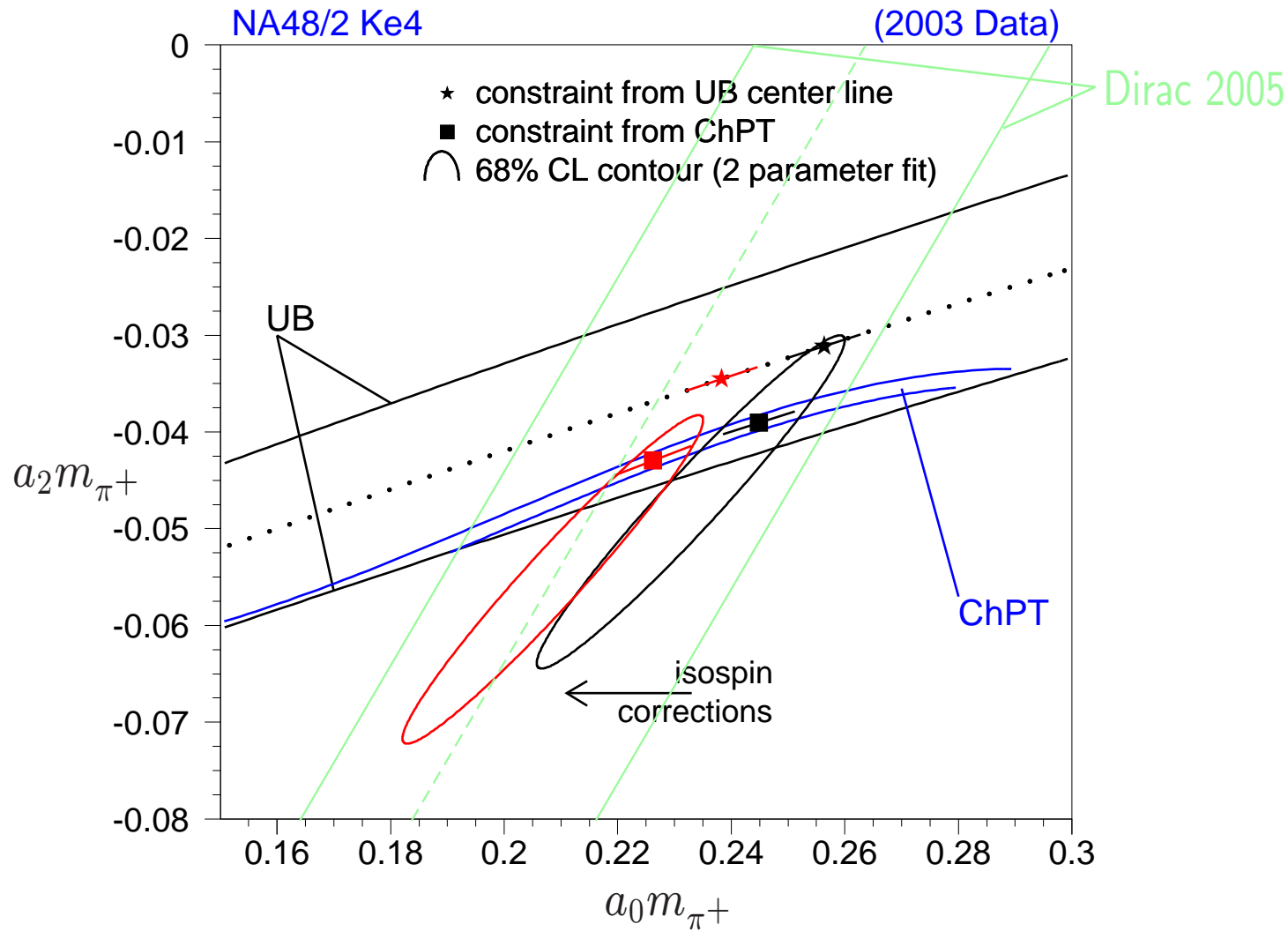
$$a_2 m_+ = -0.047 \pm 0.011_{stat} \pm 0.004_{syst}$$

Fits performed assuming isospin symmetry

Expected corrections for isospin symmetry breaking:  $10 \sim 15$  mrad on  $\delta$  ;  
 $\approx -0.02$  on  $a_0 m_+$  .

The analysis of full (2003+2004) data sample ( $10^6 K_{e4}$  decays), including isospin symmetry breaking effects, is nearly finished.

# Ke4: $\pi\pi$ scattering lengths





## Conclusions and outlook

- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ : full 2003+2004 sample (60 million events) analysed.
- $a_0 - a_2$  extracted from cusp analysis using both [Cabibbo-Isidori](#) and [Colangelo-Gasser-Kubis-Rusetsky](#) formulae, with reasonable agreement, especially in the ChPT-constrained fit.
- Results consistent with  $a_0$  and  $a_2$  determined from  $K_{e4}$  analysis of 2003 data [Eur.Phys.C 54 (2008) 411]
- $K_{e4}$  analysis of 2003+2004 data (1 million events) nearly finished, will reach a statistical precision at the level of theoretical precision.