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RECENT NA48/2 RESULTS ON Ke4 AND K3π DECAYS AND DETERMINATION OF THE $\pi\pi$ SCATTERING LENGTHS

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FOR THE NA48/2 COLLABORATION

Large samples of $K^\pm \to 3\pi$ and $K^\pm \to \pi^+\pi^-e^\pm\nu$ (K_{e4}) decays have been collected by the NA48/2 experiment at CERN SPS.
At the $\pi^+\pi^-$ threshold, the $\pi^0\pi^0$ invariant mass spectrum of the decay $K^\pm \to \pi^\pm\pi^0\pi^0$ xhibits a Wigner cusp, from which the S-wave $\pi\pi$ scattering lengths are extracted with high precision.
The same scattering lengths are also independently determined from the accurate measurement of the form factors in the K_{e4} decay $K^\pm \to \pi^+\pi^-e^\pm\nu$.

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1. The NA48/2 Experimental Set-Up and Data Samples

The main goal of the NA48/2 experiment at CERN SPS is the search for direct CP violation in $K^\pm \to 3\pi$ decays[1].

This experiment[a] detects in-flight decays of charged kaons from two simultaneous oppositely charged beams of (60±3) GeV/c momentum, produced by 400 GeV/c protons from CERN SPS impinging on a 40 cm long beryllium target.

The charged particle reconstruction is provided by a magnetic spectrometer,consisting of a dipole magnet and four drift chambers, with a spatial resolution of 100 μm and a momentum resolution $\Delta p/p = (1.0 \oplus 0.044p[\mathrm{GeV/c}])\%$.

The energy and position of photons and electrons are precisely measured by a Liquid Krypton electromagnetic calorimeter, consisting of a $27X_0$ almost homogeneous ionization chamber with high-granularity tower read-out: its energy resolution is $\Delta E/E = 3.2\%/\sqrt{E[GeV]} \oplus 9\%/E[GeV] \oplus 0.42\%$ and its spatial resolution about 1.5 mm.

[a] A sketch and a short description of the experimental apparatus have been presented at this Conference by S. Venditti[2], together with results on radiative decays.

A scintillator hodoscope for fast triggering and precise time measurement, muon and photon veto counters and an iron-scintillator hadron calorimeter complete the experimental apparatus, a detailed description of which can be found in Ref. 3.

The NA48/2 experiment took data during two runs in 2003 and 2004, collecting unprecedented data samples of $\sim 4 \cdot 10^9$ $K^\pm \to \pi^\pm\pi^+\pi^-$, $\sim 10^8$ $K^\pm \to \pi^\pm\pi^0\pi^0$ and $\sim 10^6$ $K^\pm \to \pi^+\pi^-e^\pm\nu$ (K_{e4}) decays.

2. The Cusp Effect in $K^\pm \to \pi^\pm\pi^0\pi^0$ Decays

The analysis of a partial sample of $2.4 \cdot 10^7$ $K^\pm \to \pi^\pm\pi^0\pi^0$ decays[4] showed a cusp anomaly in the $\pi^0\pi^0$ invariant mass (M_{00}) spectrum at $M_{00} \simeq 2m_{\pi^+}$. This anomaly, never observed in previous experiments, was theoretically interpreted[5] as a threshold cusp effect mainly due to final state charge-exchange scattering $\pi^+\pi^- \to \pi^0\pi^0$ in $K^\pm \to \pi^\pm\pi^+\pi^-$ decays. A simple rescattering model has been proposed[5] by Cabibbo, describing the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay amplitude as the sum of two terms:

$$\mathcal{M}(K^\pm \to \pi^\pm\pi^0\pi^0) = \mathcal{M}_0 + \mathcal{M}_1 \ , \tag{1}$$

where $\mathcal{M}_0$ is the "unperturbed" amplitude and $\mathcal{M}_1$ is the contribution from $K^\pm \to \pi^\pm\pi^+\pi^-$ decay through $\pi^+\pi^- \to \pi^0\pi^0$ charge exchange, with the renormalization condition $\mathcal{M}_1 = 0$ at $M_{00}^2 = (2m_{\pi^+})^2$. The contribution $\mathcal{M}_1$ is given by

$$\mathcal{M}_1 = -2a_x m_{\pi^+}\mathcal{M}_+\sqrt{1-\left(\frac{M_{00}}{2m_{\pi^+}}\right)^2} \ , \tag{2}$$

where a_x is the S-wave $\pi^+\pi^-$ charge-exchange scattering length, and $\mathcal{M}_+$ the known $K^\pm \to \pi^\pm\pi^+\pi^-$ decay amplitude at $M_{00} = 2m_{\pi^+}$. As $\mathcal{M}_1$ changes from real to imaginary at $M_{00} = 2m_{\pi^+}$, it interferes destructively with $\mathcal{M}_0$ in the region below $M_{00} = 2m_{\pi^+}$, while it adds quadratically above it. In the limit of exact isospin symmetry $a_x = \frac{1}{3}(a_0 - a_2)$, where a_0 and a_2 are the S-wave $\pi\pi$ scattering lengths in the I=0 and I=2 states, respectively. The experimental measurement of this effect thus provides a precise determination of $a_0 - a_2$ (including its sign).

A more detailed model[6] proposed by Cabibbo and Isidori (CI) takes into account all $\pi\pi$ rescattering processes at the one-loop and two-loop level. In this approach the matrix element of the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay includes several additional terms depending on five S-wave scattering lengths corresponding to the processes $\pi^+\pi^- \to \pi^0\pi^0$, $\pi^+\pi^+ \to \pi^+\pi^+$, $\pi^+\pi^- \to \pi^+\pi^-$, $\pi^+\pi^0 \to \pi^+\pi^0$ and $\pi^0\pi^0 \to \pi^0\pi^0$), which, in the limit of exact isospin symmetry, can be expressed as linear combinations of a_0 and a_2. This model allows to extract from high-precision experimental data both $a_0 - a_2$ and a_2. The theoretical error on $a_0 - a_2$ (due to radiative corrections, isospin symmetry breaking and higher order diagrams, not taken into account in the model) has been evaluated to be about 5%. The small isospin symmetry breaking effect of electromagnetic interactions on the five scattering lengths can be taken

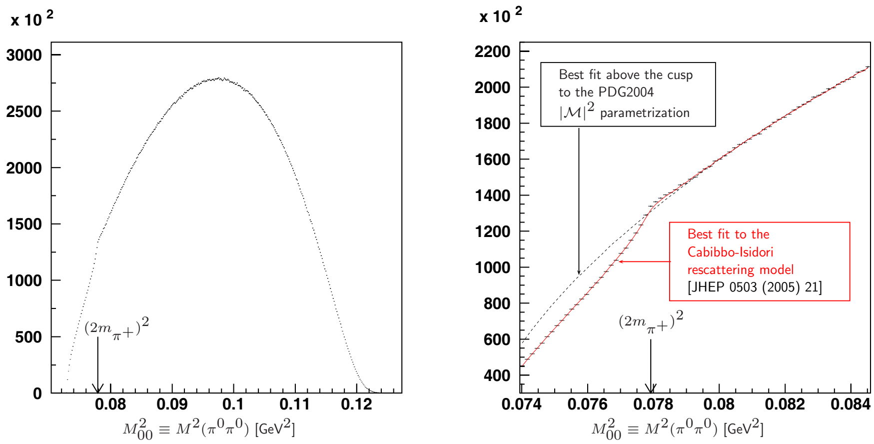

Fig. 1. (Left) $\pi^0\pi^0$ invariant mass distribution in $K^\pm \to \pi^\pm\pi^0\pi^0$ decays. A cusp at $M_{00} \simeq 2m_{\pi^+}$ is clearly visible. (Right) Expanded view of the region around $M_{00} = 2m_{\pi^+}$, with fits to models including[6] (continuous and not including[10] (dashed curve) $\pi\pi$ rescattering effects (see text).

into account (at tree level, omitting one-photon exchange diagrams) as first-order corrections in the parameter $\epsilon = (m^2_{\pi^+} - m^2_{\pi^0})/m^2_{\pi^+} = 0.065$ (Ref. 7).

An effective field theory calculation[8,9] by Colangelo, Gasser, Kubis and Rusetsky (CGKR) obtains results close, but not identical, to those of Ref. 6.

3. Measurement of $\pi\pi$ Scattering Lengths from the Cusp Effect

A dedicated analysis of the full (2003+2004) data sample has been performed, selecting $6 \cdot 10^7$ $K^\pm \to \pi^\pm\pi^0\pi^0$ events. The cusp in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution is clearly visible in Fig. 1 at $M^2_{00} \simeq (2m_{\pi^+})^2$.

A fit (dashed curve) to the parameterization proposed in Ref. 10, not taking into account $\pi\pi$ rescattering processes, clearly disagrees with data at $M_{00} \leq 2m_{\pi^+}$.

A fit (continuous curve) to the rescattering model of Ref. 6, corrected to take into account the small effect of isospin symmetry breaking[7], agrees well with data almost everywhere. As the rescattering model[6] used in the fit does not include radiative corrections, which are particularly important near $M_{00} = 2m_{\pi^+}$ and contribute to the formation of $\pi^+\pi^-$ atoms (pionium), we excluded from the fit a group of seven points centered at $M_{00} = 2m_{\pi^+}$.

The quality of this fit is illustrated in Fig. 2, which also shows the small excess of events in the bins excluded from the fit, peaking at $M^2_{00} \simeq (2m_{\pi^+})^2$.

The measured amount of such excess events, normalized to the number of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays, is

$$\frac{N(\text{excess events at } M_{00} \simeq 2m_{\pi^+})}{N(K^\pm \to \pi^\pm\pi^0\pi^0)} = (5.6 \pm 1.0) \cdot 10^{-5} \,. \quad (3)$$

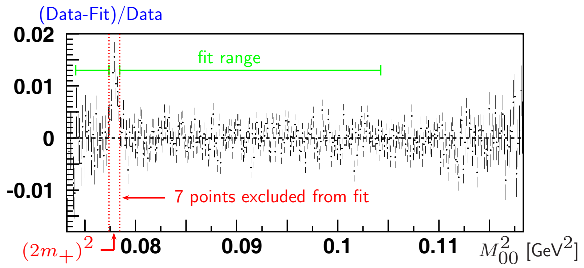

Fig. 2. Δ=(data-fit)/data versus M_{00}^2, obtained from the fit of M_{00}^2 spectrum with the rescattering model of Ref. 6. Seven points centered at $M_{00}^2 = (2m_{\pi^+})^2$ have been excluded from the fit region. In this "pionium region" a small but significant peak is clearly visible.

These "excess events" can be explained as due to electromagnetic effects[11], including those leading to a $\pi^+\pi^-$ bound state $A_{2\pi}$ (pionium)[12] through the decay $K^\pm \to \pi^\pm A_{2\pi}$ (the pionium then decays $A_{2\pi} \to \pi^0\pi^0$).

In order to extract the S-wave $\pi\pi$ scattering lengths, the $\pi^0\pi^0$ invariant mass spectrum of Fig. 1 has been fitted using the theoretical models proposed in Refs. 6 (CI) and 8 (CGKR).

It has been shown[13] that analyticity and chiral symmetry provide a constraint between a_0 and a_2 scattering lengths:

$$a_2 m_{\pi^+} = (-0.0444 \pm 0.0008) + 0.236x - 0.61x^2 - 9.9x^3 \quad [x \equiv (a_0 m_{\pi^+} - 0.22)] \,. \quad (4)$$

By imposing this constraint in the fit, we obtain the following results:

$$\begin{array}{lrcl} \text{(CI):} & (a_0 - a_2)m_{\pi^+} & = & 0.268 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \\ \text{(CGKR):} & (a_0 - a_2)m_{\pi^+} & = & 0.266 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \,. \end{array} \quad (5)$$

If we do not impose the $a_2(a_0)$ constraint, but consider both $a_0 - a_2$ and a_2 as free parameters, we obtain:

$$\begin{array}{lrcl} \text{(CI):} & (a_0 - a_2)m_{\pi^+} & = & 0.266 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \\ & a_2\ m_{\pi^+} & = & -0.039 \pm 0.009_{stat} \pm 0.006_{syst} \pm 0.002_{ext} \\ \text{(CGKR):} & (a_0 - a_2)m_{\pi^+} & = & 0.273 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \\ & a_2\ m_{\pi^+} & = & -0.065 \pm 0.015_{stat} \pm 0.010_{syst} \pm 0.002_{ext} \,. \end{array} \quad (6)$$

The third ("external") error in these results originates from the uncertainty on the branching ratio $\Gamma(K^+ \to \pi^+\pi^+\pi^-)/\Gamma(K^+ \to \pi^+\pi^0\pi^0)$ [14]. The uncertainty of the theoretical model is not included in the errors above: for CI [6] it has been estimated to be about ±5% on $a_0 - a_2$, while no estimate is given for CGKR[8].

4. Measurement of $\pi\pi$ Scattering Lengths from K_{e4} Decays

$K^\pm \to \pi^+\pi^- e^\pm \nu$ decays are of particular interest for the study of $\pi\pi$ phase shift difference $\delta = \delta_0^0 - \delta_1^1$ in absence of any other hadron. The measured variation of this phase with the $\pi\pi$ invariant mass ($M_{\pi\pi}$) near threshold can be related to

a_0 and a_2 scattering lengths using dispersion relations and data at intermediate energies[15,16,17]. In a recently published analysis[18], based on a partial sample of about 670,000 K_{e4} decays collected in 2003, we measured the form factors and the phase δ in ten independent bins of $M_{\pi\pi}$ and extracted the $\pi\pi$ scattering lengths $a_0 m_{\pi^+} = 0.233 \pm 0.016_{stat} \pm 0.007_{syst}$, $a_2 m_{\pi^+} = -0.047 \pm 0.011_{stat} \pm 0.004_{syst}$.

These values are in good agreement with the results of our $K_{3\pi}$ cusp analysis and with the determination of $|a_0 - a_2|$ from pionium lifetime by the DIRAC experiment[19] $|a_0 - a_2| m_{\pi^+} = 0.264^{+0.033}_{-0.020}$. Further theoretical developments[20] suggest that isospin symmetry breaking effects, neglected so far, should be considered when extracting $\pi\pi$ scattering lengths from phase measurements.

The analysis of the full NA48/2 data sample (about one million K_{e4} decays), taking into account isospin breaking corrections[20], is nearly finished and will reach a statistical precision on a_0 and a_2 at the level of the present uncertainty on the corresponding theoretical predictions from Chiral Perturbation Theory[21].

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