

# QCD Tests at the NA48/2 experiment at CERN

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## Abstract

Very large statistics of charged kaon decays have been accumulated in 2003-2004 by the NA48/2 experiment at the CERN SPS. The analyses of  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$  ( $K_{3\pi}$ ) and  $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$  ( $K_{e4}$ ) decays give complementary approaches to the study of low energy  $\pi\pi$  scattering. From data samples of  $\sim 60$  millions  $K_{3\pi}$  and  $\sim 1$  million  $K_{e4}$  decays, precise values of  $a_0$  and  $a_2$ , the isospin 0 and 2 S-wave  $\pi\pi$  scattering lengths, can be extracted with an unprecedented experimental precision of few percents, allowing accurate tests of Chiral Perturbation Theory predictions.

*Key words:* Ke4 decays, cusp, pi-pi scattering length, pi-pi rescattering, Chiral Perturbation Theory  
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## 1. Introduction

In the past years,  $K_{e4}$  decays were traditionally the cleanest laboratory to study  $\pi\pi$  scattering close to threshold and extract the values of the S-wave scattering lengths. Two experiments [1] collected sizable samples of such decays, the Geneva-Saclay collaboration at the CERN/PS (30000  $K^+$  decays) and the E865 collaboration at BNL ( $\sim 400000$   $K^+$  decays). The NA48/2 collaboration at the CERN/SPS has collected more than one million decays in both charge modes in 2003-2004. Results from a partial sample have been published in [2]. From  $\sim 23$  millions decays collected in the  $K_{3\pi}$  mode, the NA48/2 collaboration has shown experimental evidence of a cusp-like structure in the  $\pi^0 \pi^0$  mass distribution at the  $\pi^+ \pi^-$  threshold [3]. This effect was interpreted as mainly due to charge exchange rescattering of the  $\pi^+ \pi^-$  system to  $\pi^0 \pi^0$ , allowing an independent measurement of the  $\pi\pi$  S-wave scattering lengths. Both analyses triggered new developments in close collaboration between experiment and theory groups. Results based on the full

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data samples will be presented here. The form factors and  $\pi\pi$  phase shift which characterize the  $K_{e4}$  decay are measured simultaneously and the S-wave  $\pi\pi$  scattering lengths are then extracted with an improved precision using recent theoretical work [4] including isospin symmetry breaking mass effects. The slopes of the  $K_{3\pi}$  Dalitz Plot are determined together with the S-wave  $\pi\pi$  scattering lengths and rely also on refined theoretical calculations [5,6]. Both new results are compared to the precise predictions from Chiral Perturbation Theory (ChPT).

## 2. Analysis of $K_{e4}$ decays

The 1.1 million event sample is distributed over a grid of  $(10 \times 5 \times 5 \times 5 \times 12)$  equal population boxes in the five-variable space which describes the decay kinematics: two invariant masses  $M_{\pi\pi}$  and  $M_{e\nu}$  and three angles  $\theta_\pi$ ,  $\theta_e$  and  $\Phi$ . Two axial ( $F, G$ ) and one vector ( $H$ ) complex form factors contribute to the transition amplitude and are further developed in partial waves identified to the phases of the  $\pi\pi$  scattering [7]. The decay is then described by four real form factors (FF =  $F_s, F_p, G_p, H_p$ ) and one phase shift ( $\delta = \delta_s - \delta_p$ ) which are measured in the ten independent  $M_{\pi\pi}$  bins and do not depend upon any particular model. Detailed numerical values of the Taylor expansion coefficients used to describe the FF variations with masses can be found in [8]. After subtraction of isospin mass effects, numerical solutions of Roy equations [9] are used to extract scattering lengths from the phase measurements in a 2-parameter fit:

$$a_0 = 0.218 \pm 0.013_{\text{stat}} \pm 0.007_{\text{syst}} \pm 0.017_{\text{theo}} ,$$

$$a_2 = -0.0457 \pm 0.0084_{\text{stat}} \pm 0.0041_{\text{syst}} \pm 0.030_{\text{theo}} ,$$

as illustrated in Figure 1. Using the ChPT constraint [10] to relate  $a_2$  and  $a_0$ , the value from the 1-parameter fit becomes:  $a_0 = 0.220 \pm 0.005_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.006_{\text{theo}}$ .

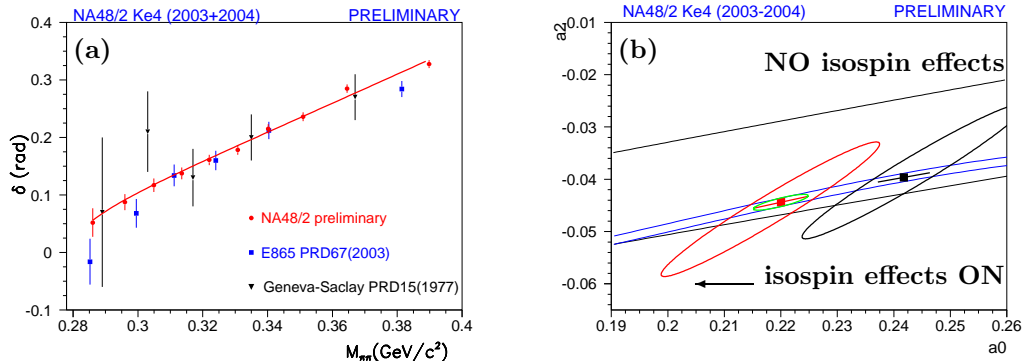


Fig. 1. (a): Phase shift ( $\delta$ ) measurements without mass effects from all  $K_{e4}$  experiments. (b): Fits of the NA48/2  $K_{e4}$  data in the  $(a_0, a_2)$  plane without (black) and with (red) isospin mass effects. The symbols are the result of the one-parameter fit imposing the ChPT constraint. There is impressive agreement with the small (green) ellipse which corresponds to the most accurate prediction from ChPT.

## 3. Analysis of $K_{3\pi}$ decays

Two theoretical approaches have been developed using different formalisms and ingredients [5,6] to describe the decay amplitude together with one and two-loop rescattering

effects. In both approaches, the  $M_{00}^2$  distribution is fitted, using the detector response matrix obtained from a large Monte-Carlo simulation, to extract the scattering lengths  $a_0 - a_2$ ,  $a_2$  and the Dalitz plot slopes  $g_0$ ,  $h'_0$  and  $k'_0$  (details can be found in [8]). Using the ChPT relation [10], both models lead to consistent values from the constrained fit:

$$a_0 - a_2 = 0.267 \pm 0.003_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}} \pm 0.013_{\text{theo}}.$$

The  $M_{00}^2$  distribution and the scattering length results are shown in Figure 2.

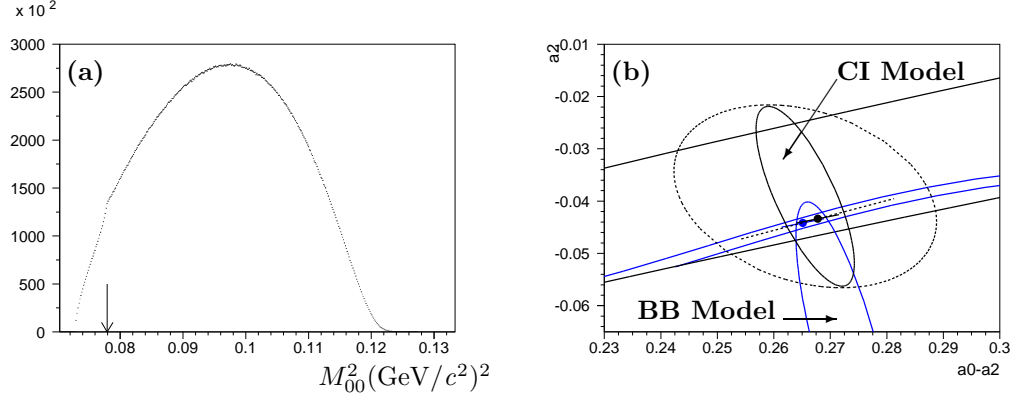


Fig. 2. (a): Invariant  $\pi^0\pi^0$  mass squared. The arrow shows the cusp position. (b):  $K_{3\pi}$  NA48/2 measurements in the  $(a_0 - a_2, a_2)$  plane for the two models CI (Cabibbo-Isidori [5]) and BB (Bern-Bonn [6]). The symbols correspond to the constrained fit and the large ellipse to the conservative theory uncertainty.

#### 4. Conclusion

The NA48/2 experiment has analyzed  $\sim$ one million  $K_{e4}$  decays and  $\sim$ 60 millions  $K_{3\pi}$  decays to extract the isospin 0 and 2 S-wave  $\pi\pi$  scattering lengths with an experimental error of few percents, a factor three better than previous results [1]. With the help of dedicated calculations, developed in close collaboration between experiment and theory, precise results have been obtained:  $a_0 = 0.220 \pm 0.005 \pm 0.006_{\text{theo}}(K_{e4})$  and  $a_0 - a_2 = 0.267 \pm 0.004 \pm 0.013_{\text{theo}}(K_{3\pi})$ , confirming the predictions of Chiral Perturbation Theory computed with similar precision:  $a_0 = 0.220 \pm 0.005_{\text{theo}}$ ,  $a_0 - a_2 = 0.265 \pm 0.004_{\text{theo}}$ .

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