

Study of K_{e4} decays at NA48 and tests of ChPT

Dmitry Madigozhin
(JINR, Dubna)

on behalf of the **NA48/2** Collaboration:

**Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara,
Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay,
Siegen, Torino, Vienna**



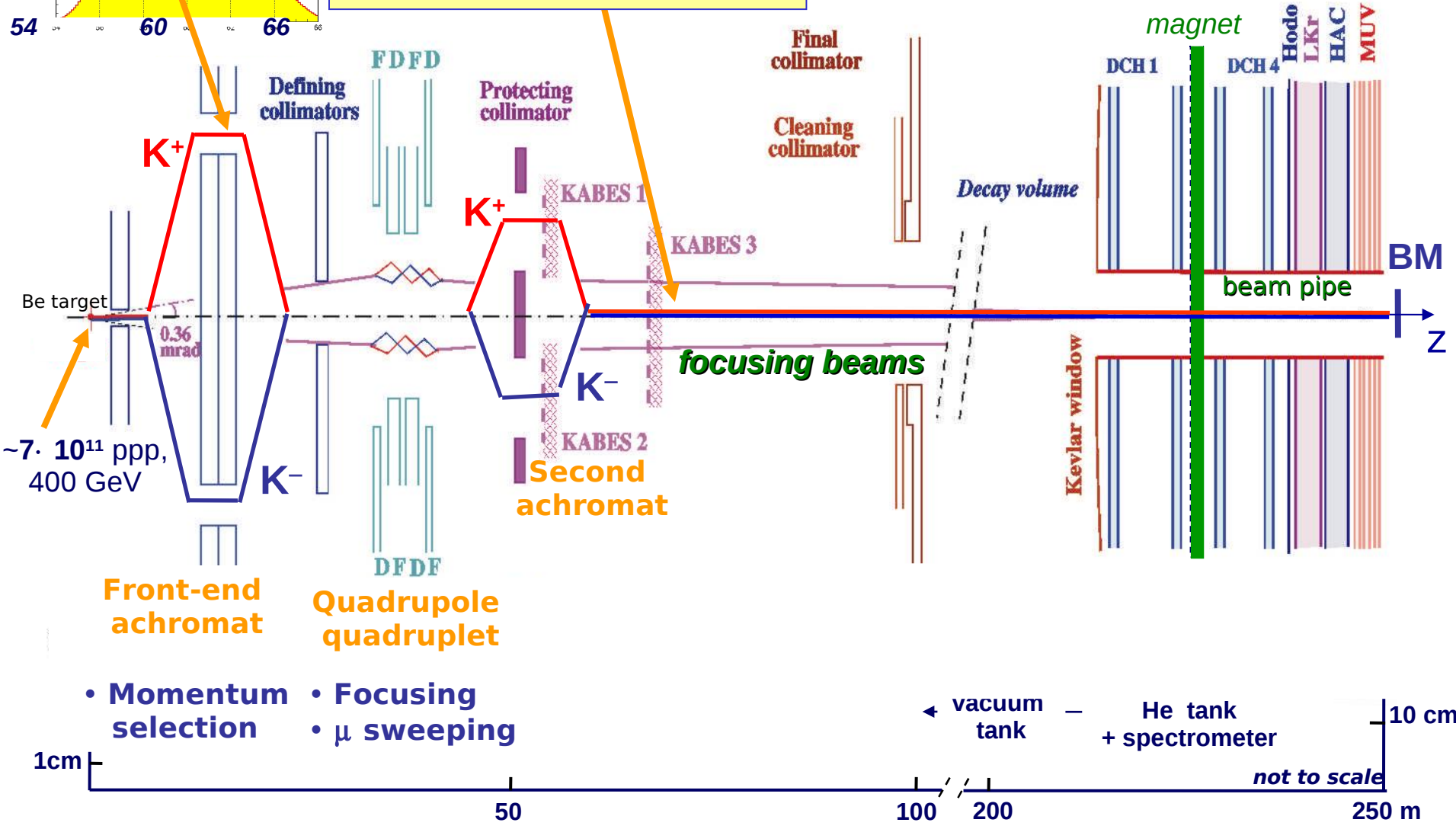
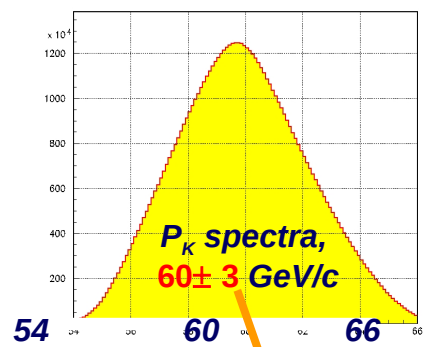
Outline

- Introduction to NA48/2 experiment
- Ke4 : theory
- Ke4 : experimental status
- NA48/2: $\mathbf{K^\pm \rightarrow \pi^+\pi^-e^\pm\nu}$ Form Factors and Branching fraction
- NA48/2: $\mathbf{K^\pm \rightarrow \pi^0\pi^0e^\pm\nu}$ Branching fraction
- Summary

NA48/2 beam line

2-3M K/spill ($\pi/K \sim 10$),
 π decay products stay in pipe.
 Flux ratio: $K^+/K^- \approx 1.8$

Simultaneous K^+ and K^- beams:
 large charge symmetrization of
 experimental conditions

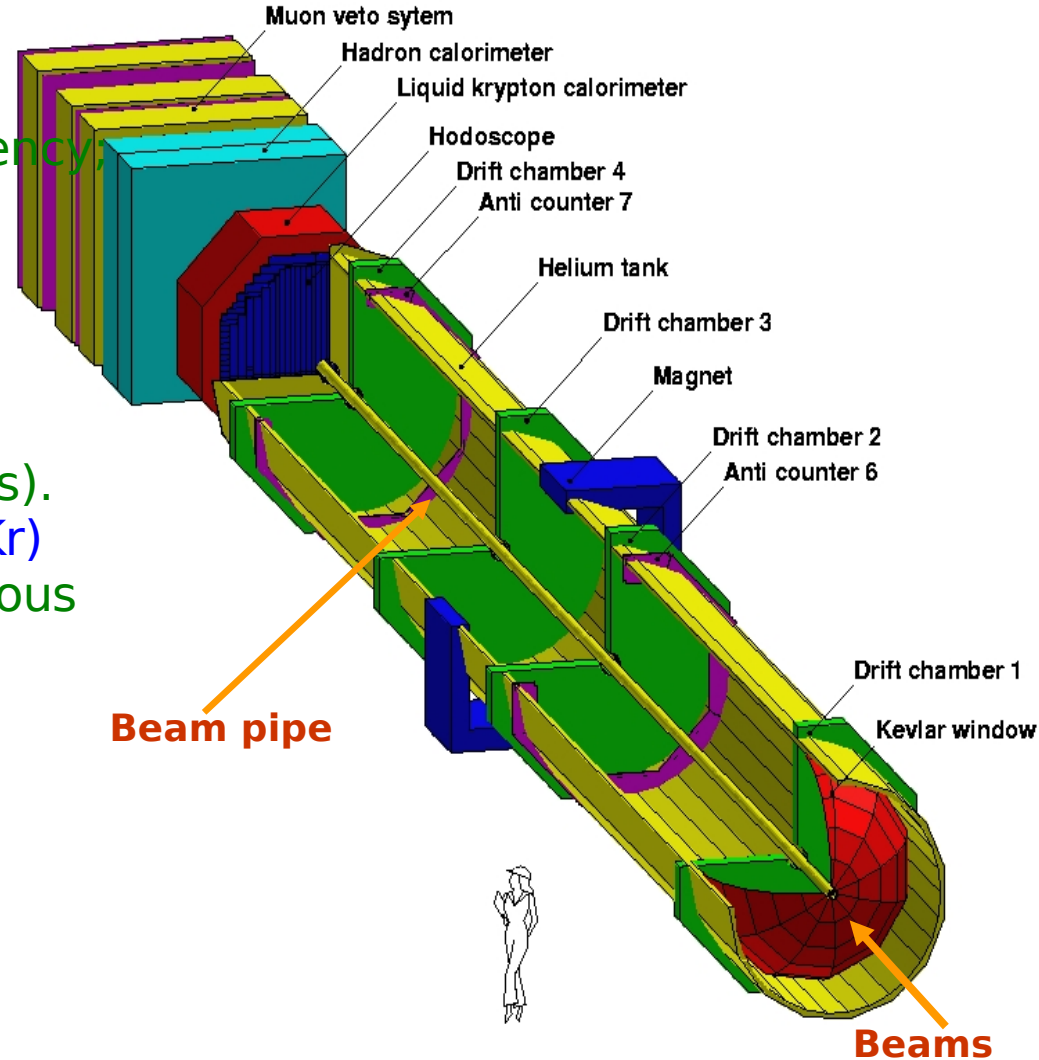


- Momentum selection
- Focusing
- μ sweeping

The NA48 detector

Main detector components:

- Magnetic spectrometer (4 DCHs):
4 views/DCH: redundancy \Rightarrow efficiency;
used in trigger logic;
 $\Delta p/p = 1.0\% + 0.044\%*p$
[p in GeV/c].
- Hodoscope
fast trigger;
precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
High granularity, quasi-homogenous
 $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$
 $\sigma_x = \sigma_y = 0.42/E^{1/2} + 0.06\text{cm}$
[E in GeV].
(0.15cm@10GeV).
- Hadron calorimeter, muon veto
counters, photon vetoes.



Ke4 decays : formalism of $(\pi^+ \pi^-)$ and $(\pi^0 \pi^0)$ modes

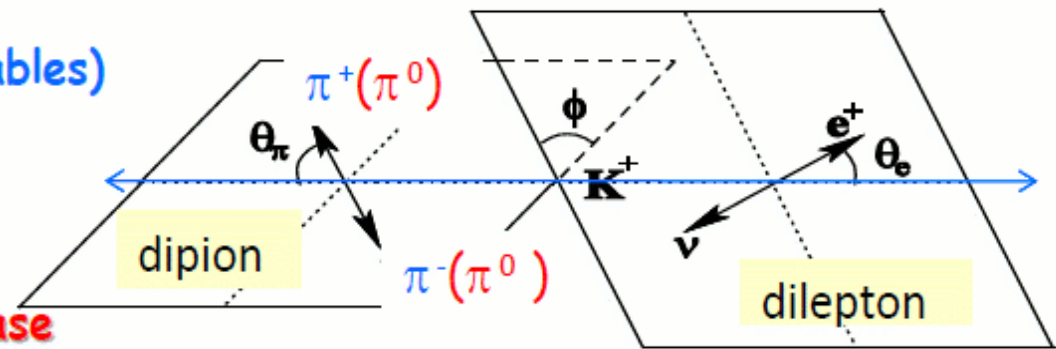
Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

$S_\pi (M_{\pi\pi}^2)$, $S_e (M_{e\nu}^2)$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ

Reduce to 3 variables in the $(\pi^0 \pi^0)$ case

$S_\pi (M_{\pi\pi}^2)$, $S_e (M_{e\nu}^2)$, $\cos\theta_e$



Partial Wave expansion of the amplitude

into s and p waves (Pais-Treiman 1968)

+ Watson theorem (T-invariance) for δ_l^I

$$\delta_0^0 \equiv \delta_s \text{ and } \delta_1^1 \equiv \delta_p$$

F, G = 2 complex Axial Form Factors

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_p$$

$$G = G_p e^{i\delta_g}$$

H = 1 complex Vector Form Factor

$$H = H_p e^{i\delta_h}$$

Reduces to the single F_s Form Factor

Map the distributions of the Ca.Ma. variables in the **five-dimensional space** with **4 real Form factors** and only **one phase shift**, assuming identical phases for the p-wave Form Factors F_p, G_p, H_p

Dalitz plot density proportional to F_s^2

The fit parameters (real) are :

$$F_s \quad F_p \quad G_p \quad H_p \text{ and } \delta = \delta_s - \delta_p$$

reduce to the only F_s

Chiral Perturbative Theory and Ke4 decays

At energy $\ll 1$ GeV an effective theory ChPT is used to describe the physical observables in terms of external momenta and light quark masses (Weinberg 1979).

Isospin symmetry ($m_u = m_d$, $\alpha_{\text{QED}} = 0$) translates into relation between decay modes:

$$\text{Rates (l - lepton): } \Gamma(K_{l4}^{\pm}) = \frac{1}{2} \Gamma(K_{l4}^{0+}) + 2 \Gamma(K_{l4}^{00});$$

Taking into account the lifetimes of K^+ ($1.238 \cdot 10^{-8}$ s) and K_L ($5.116 \cdot 10^{-8}$ s):

$$\text{Branching ratios: } \text{Br}(K_{l4}^{\pm}) = 0.121 \text{Br}(K_{l4}^{0+}) + 2 \text{Br}(K_{l4}^{00})$$

experiments (PDG):

$$K_{e4}^{\pm} (K^{\pm} \rightarrow \pi^+ \pi^- \nu e^{\pm}): \quad \text{Br} = (4.09 \pm 0.10) \cdot 10^{-5} \quad ; \text{ events: } 418000$$

$$K_{e4}^{00} (K^{\pm} \rightarrow \pi^0 \pi^0 \nu e^{\pm}): \quad \text{Br} = (2.2 \pm 0.40) \cdot 10^{-5} \quad ; \text{ events: } 37$$

$$K_{e4}^{0+} (K_L \rightarrow \pi^{\pm} \pi^0 \nu e^{\pm}): \quad \text{Br} = (5.20 \pm 0.10) \cdot 10^{-5} \quad ; \text{ events: } 6131$$

Experimental precision improvement is needed.

Predictions using Form Factor calculations by ChPT at $O(p^2, p^4, p^6)$
(Bijnens, Colangelo, Gasser, Nucl. Phys. B427 1994):

$$\text{Using S118 value as input } K_{e4}^{\pm} : (3160 \pm 140) \text{ s}^{-1} \quad \text{Br: } (3.91 \pm 0.17) \cdot 10^{-5}$$

$$\text{prediction } K_{e4}^{00} : (1625 \pm 90) \text{ s}^{-1} \quad \text{Br: } (2.01 \pm 0.11) \cdot 10^{-5}$$

$$K_{e4}^{0-} : (917 \pm 170) \text{ s}^{-1} \quad \text{Br: } (4.69 \pm 0.87) \cdot 10^{-5}$$

Improved measurement will provide tests of ChPT predictions

K_{e4}^{\pm} ($K^{\pm} \rightarrow \pi^+ \pi^- \nu e^{\pm}$) formfactors in NA48/2

Signal ($\pi^+ \pi^- e^{\pm} \nu$) topology:

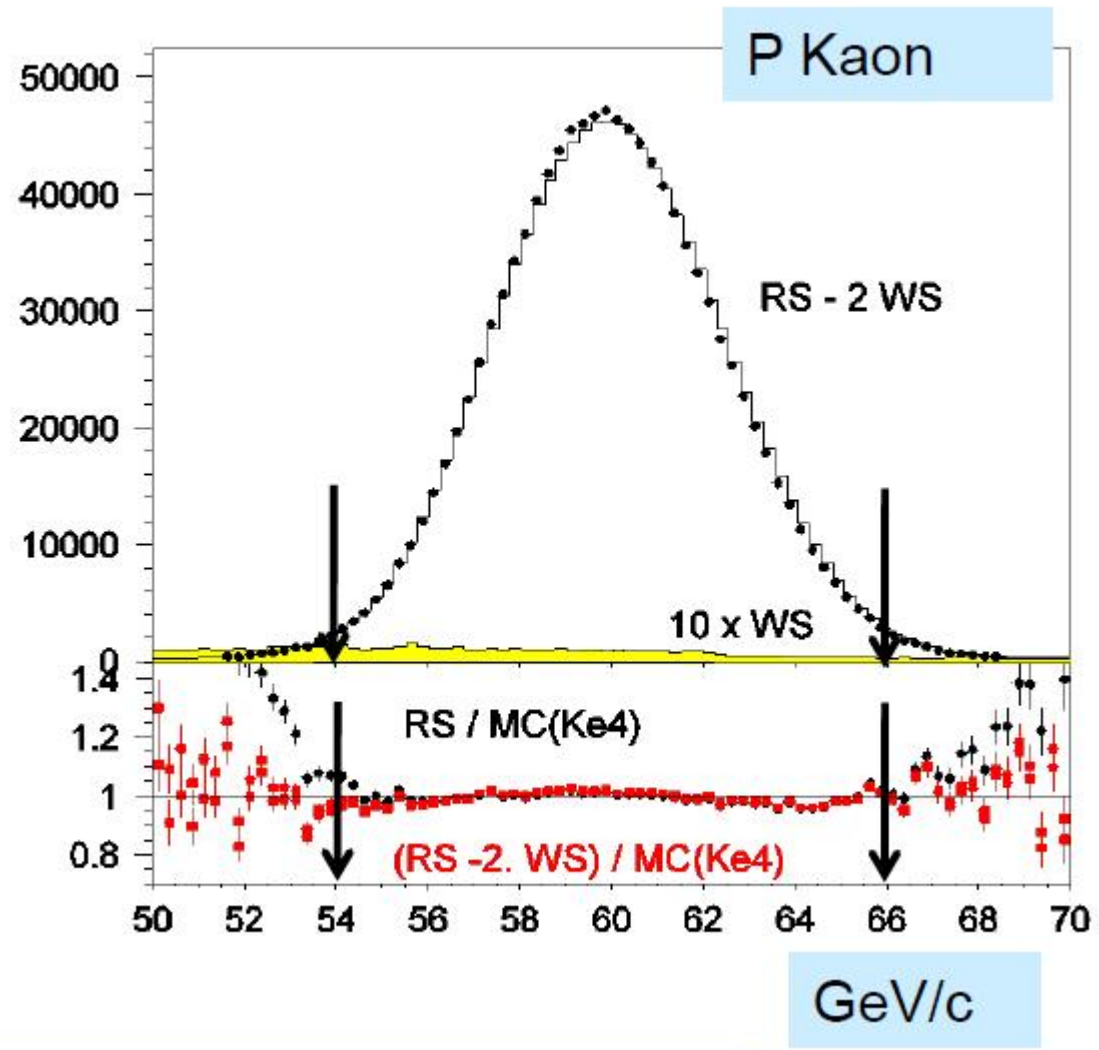
- 3 charged tracks, good vertex
- Opposite sign 2 pions
- 1 electron ($E_{\text{el}} / P \sim 1$)

Main background sources:

- a** $K^+ \rightarrow [\pi^- \text{ misident. as } e^-] \pi^+ \pi^+$
 $K^+ \rightarrow [\pi^- \rightarrow e^- \nu] \pi^+ \pi^+$

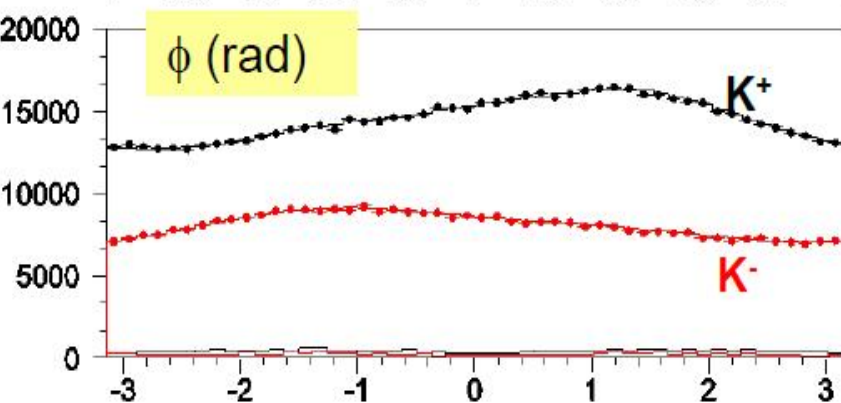
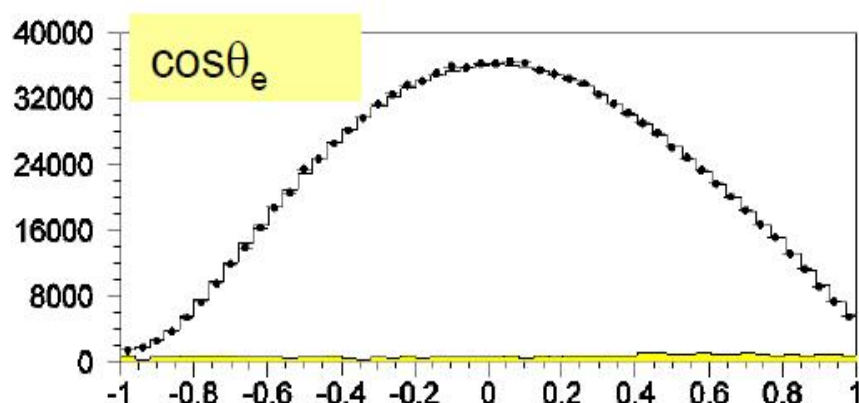
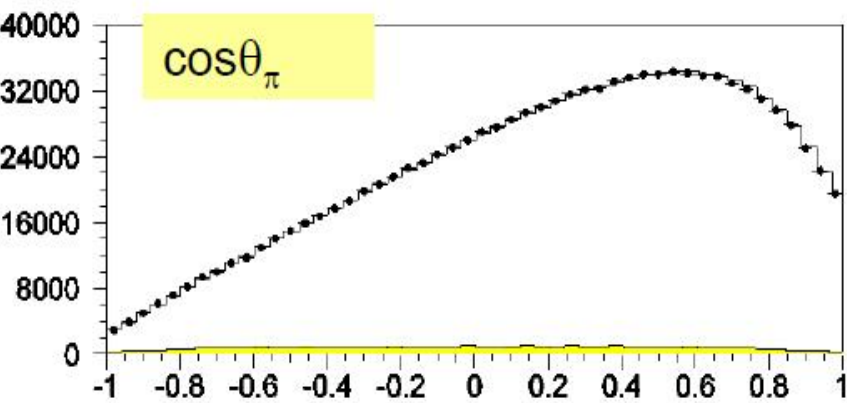
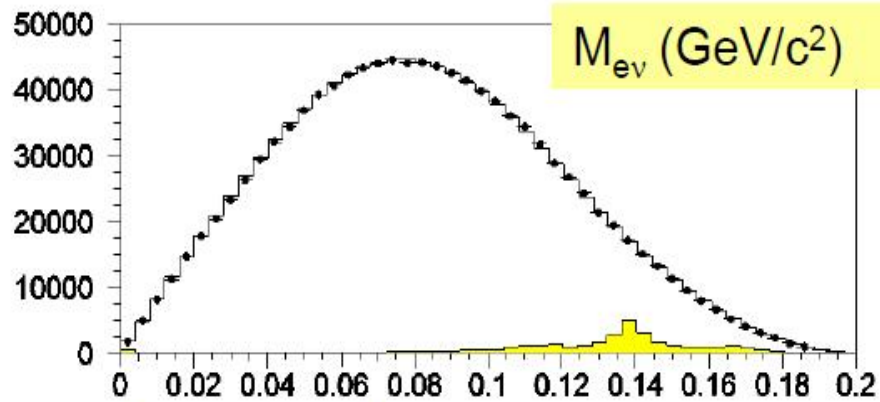
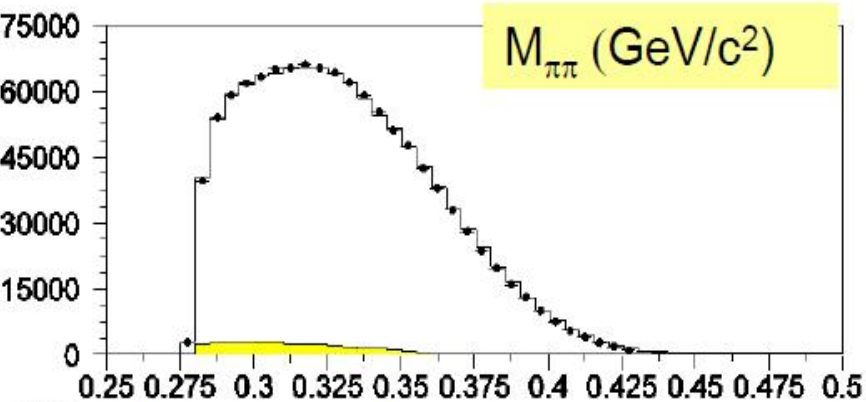
contributes twice more to «right sign» background component (RS) than to «wrong sign» (WS) one (same sign pion charges).

- b** $K^+ \rightarrow [\pi^0 \rightarrow e^+ e^- (\gamma)] (\pi^0) \pi^+$
 almost negligible



Total background is at the level of $2 \times 0.3\%$: estimated from WS events (contribution **a** is dominant) and checked by MC

Ke4 decays : Data/MC comparison after fit



	= Data
	= Simulation after fit
	= WS Background (x 10 to be visible)

CP symmetry :
 $(K^+) \phi$ distribution is opposite of $(K^-) \phi$ distribution

K_{e4} formfactors: fit results

Series expansion with:

- $q^2 = S\pi/4m_\pi^2 - 1$
- $Se/4m_\pi^2$

$$F_s^2 =$$

$$f_s^2(1 + f'_s/f_s q^2 + f''_s/f_s q^4 + f'_e/f_s Se/4m_\pi^2)^2$$

$$G_p/f_s = g'_p/f_s + g''_p/f_s q^2$$

	value	stat	syst
f'_s/f_s	$0.152 \pm 0.007 \pm 0.005$		
f''_s/f_s	$-0.073 \pm 0.007 \pm 0.006$		
f'_e/f_s	$0.068 \pm 0.006 \pm 0.007$		
f_p/f_s	$-0.048 \pm 0.003 \pm 0.004$		
constant			
g_p/f_s	$0.868 \pm 0.010 \pm 0.010$		
g'_p/f_s	$0.089 \pm 0.017 \pm 0.013$		
h_p/f_s	$-0.398 \pm 0.015 \pm 0.008$		
constant			

Published in Eur.Phys. C70 (2010) 635 :

- Form factors
- $\pi\pi$ scattering lengths measurement

Branching ratio value will fix the absolute values of the form factors

K_{e4}^{\pm} branching fraction measurement



$$\text{Br}(K_{e4}) = \text{Br}(K_{3\pi}) \frac{(N(K_{e4}) - N(\text{Bkg}))}{N(K_{3\pi})} \frac{A(K_{3\pi})}{A(K_{e4})} \frac{\epsilon(K_{3\pi})}{\epsilon(K_{e4})}$$

Candidates:

$$N(K_{e4}) = 1.11 \times 10^6 \quad (K^+ : 7.12 \times 10^5, K^- : 3.97 \times 10^5)$$

$$N(\text{Bkg}) : 0.95\% \text{ of } N(K_{e4})$$

$$N(K_{3\pi}) = 1.9 \times 10^9$$

Acceptance - GEANT3 based simulation:

$$A(K_{3\pi}) = 24.18\%$$

$$A(K_{e4}) = 18.22\%$$

Trigger efficiencies (measured using control triggers):

$$\epsilon(K_{3\pi}) = 97.5\%$$

$$\epsilon(K_{e4}) = 98.3\%$$

$$\text{Br}(K_{3\pi}) = (5.59 \pm 0.04)\% \quad (\text{source of } 0.72\% \text{ external error for } \text{Br}(K_{e4}))$$

K_{e4}^{\pm} branching fraction measurement

Systematic uncertainty	%
Acceptance and beam geometry	0.18
Muon veto efficiency	0.16
Accidental activity	0.15
Background	0.14
Particle ID	0.09
Radiative effects	0.08
Independent analysis	0.10

$$\text{Br}(K_{e4}^{+}) = (4.277 \pm 0.009_{\text{stat+trig}}) \times 10^{-5}$$

$$\text{Br}(K_{e4}^{-}) = (4.283 \pm 0.012_{\text{stat+trig}}) \times 10^{-5}$$

$$\text{Br}(K_{e4}^{+-}) = (4.279 \pm 0.004_{\text{stat}} \pm 0.005_{\text{trig}} \pm 0.015_{\text{syst}} \pm 0.031_{\text{ext}}) \times 10^{-5}$$

$$= (4.279 \pm 0.035_{\text{tot}}) \times 10^{-5} \quad (\text{external error is dominating})$$

$$\text{PDG 2010: } (4.09 \pm 0.10_{\text{tot}}) \times 10^{-5}$$

K_{e4}^{00} Branching fraction measurement

$K^\pm \rightarrow \pi^0 \pi^0 \nu e^\pm$ relative to $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$, with $Br = (1.761 \pm 0.022)\%$,
extensively studied by NA48/2

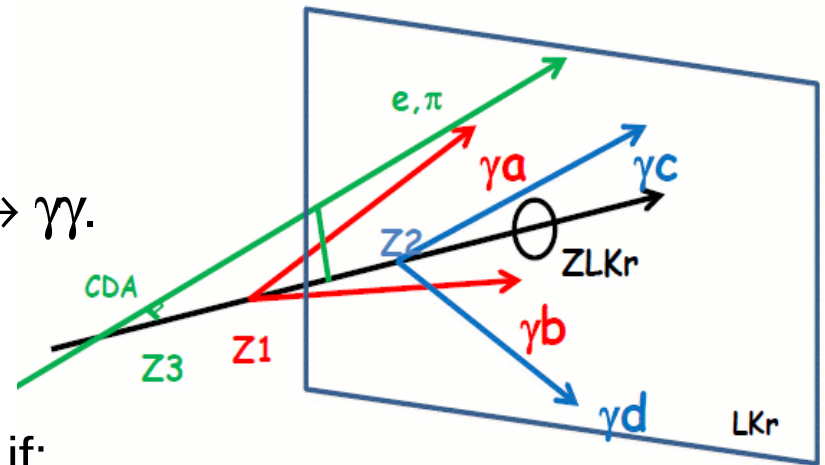
Common part of event reconstruction:

Find LKr γ -cluster pairs (ab) and (cd):

- Decay positions Z_1 and Z_2 assuming $\pi^0 \rightarrow \gamma\gamma$.
- $Z_n = (Z_1 + Z_2)/2$ within $(-16, +90)$ m
- $D_{Z_n} = |Z_1 - Z_2| < 500$ cm

Combined with charged track (Z_3 at CDA) if:

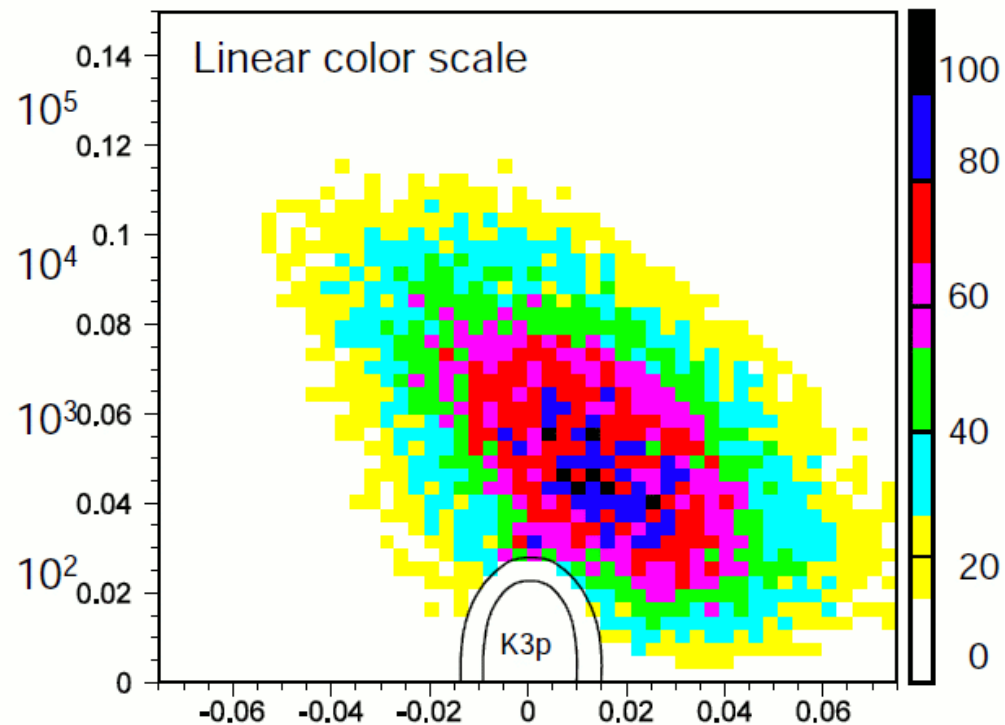
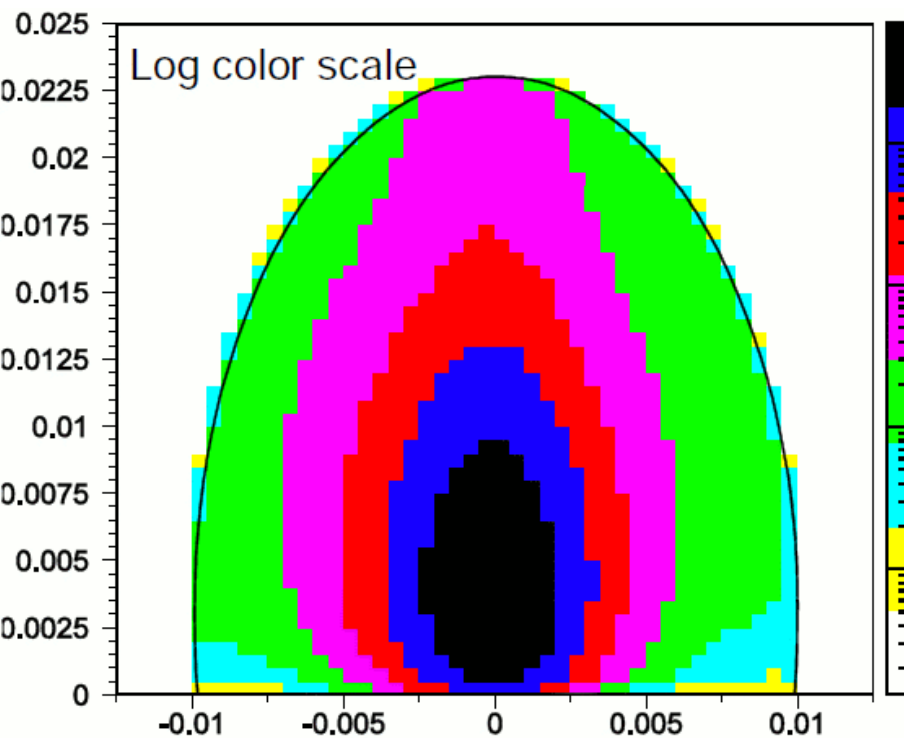
- $D_Z = |Z_3 - Z_n| < 800$ cm



K_{e4}^{00} Branching fraction measurement : signal selection

- Assign m_π to the charged track, plot P_t (to beam) vs invariant mass
- Cut $K3\pi$ events with a small P_t and \sim kaon PDG mass

Elliptic cut separates 70×10^6 $K_{3\pi}$ from 45 000 K_{e4} candidates

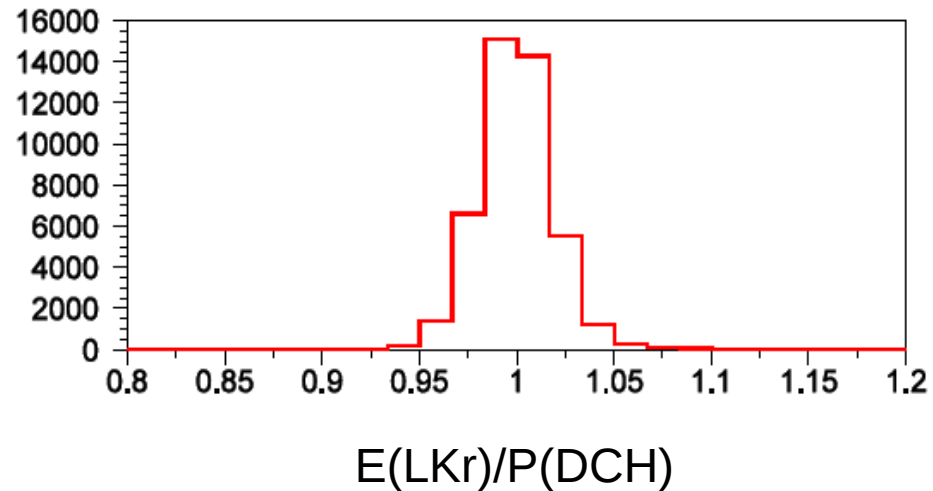


P_t (GeV/c) vs $(M_{3\pi} - M_K)$ (GeV/c²)

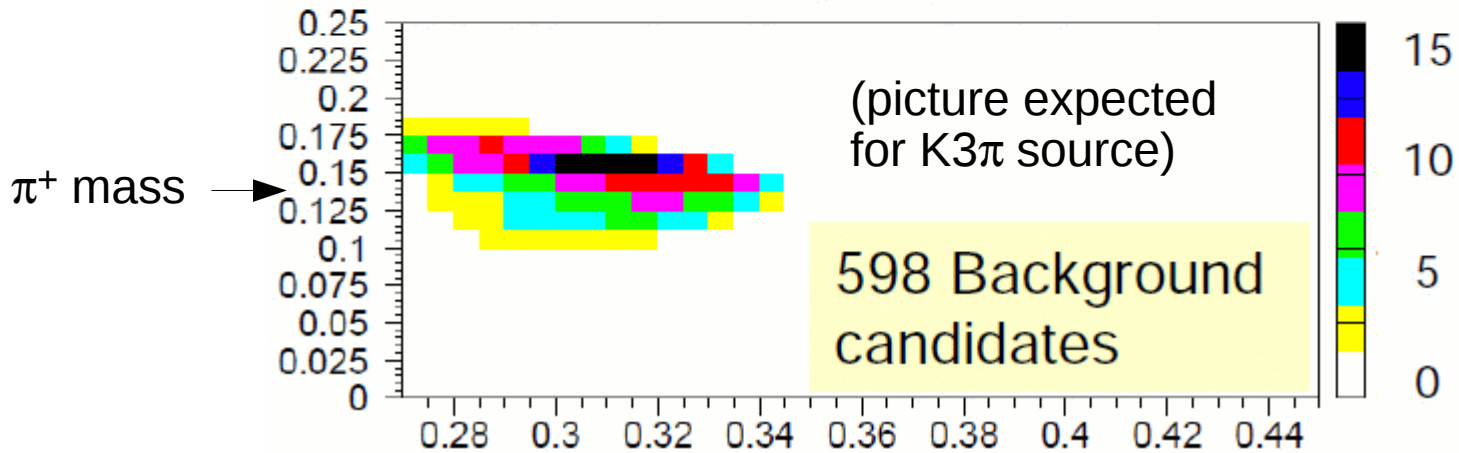
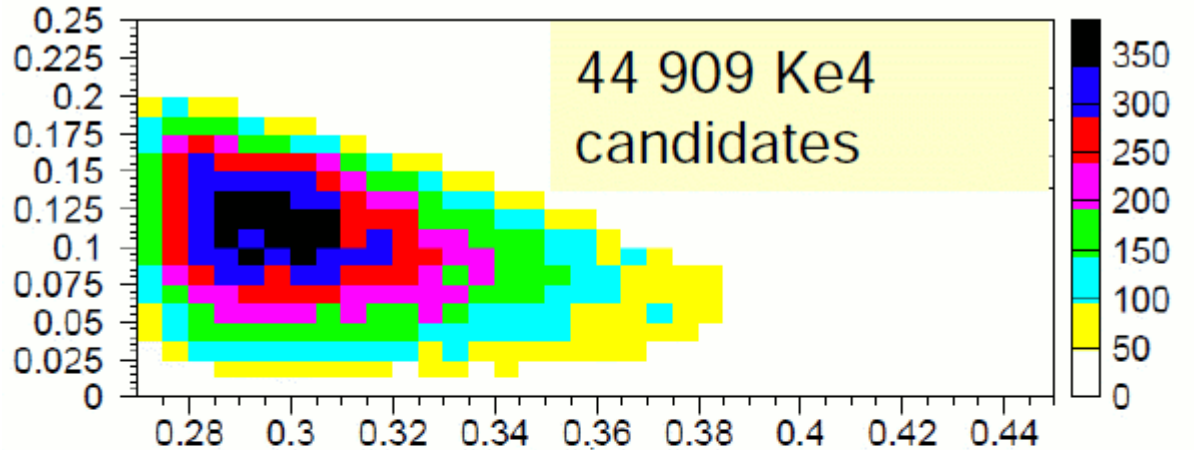
K_{e4}^{00} Branching fraction measurement : background rejection

Electron identification:

- $E(\text{LKr})/P(\text{DCH}) \sim 1$
- Extra rejection using a dedicated variable of **Linear Discriminant Analysis** based on shower properties and trained on real and fake electrons from data
- Residual fake-electron background estimated from control region as 1.3% to K_{e4} signal.
- Background from $K \rightarrow 2\pi^0\pi^+$ with $\pi \rightarrow e\nu$: $\sim 0.1\%$, which is small with respect to the fake-electron background.



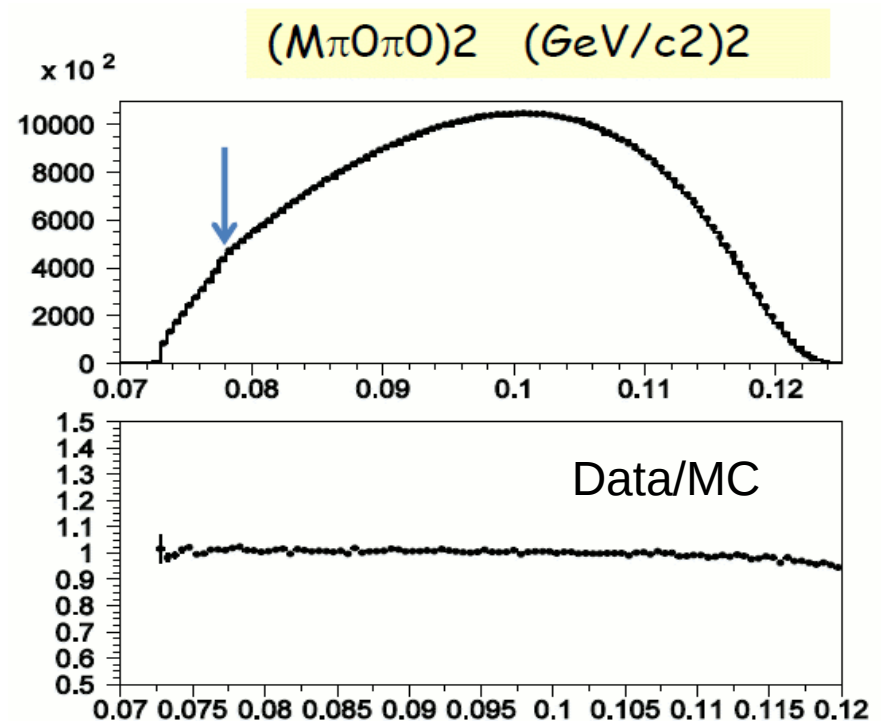
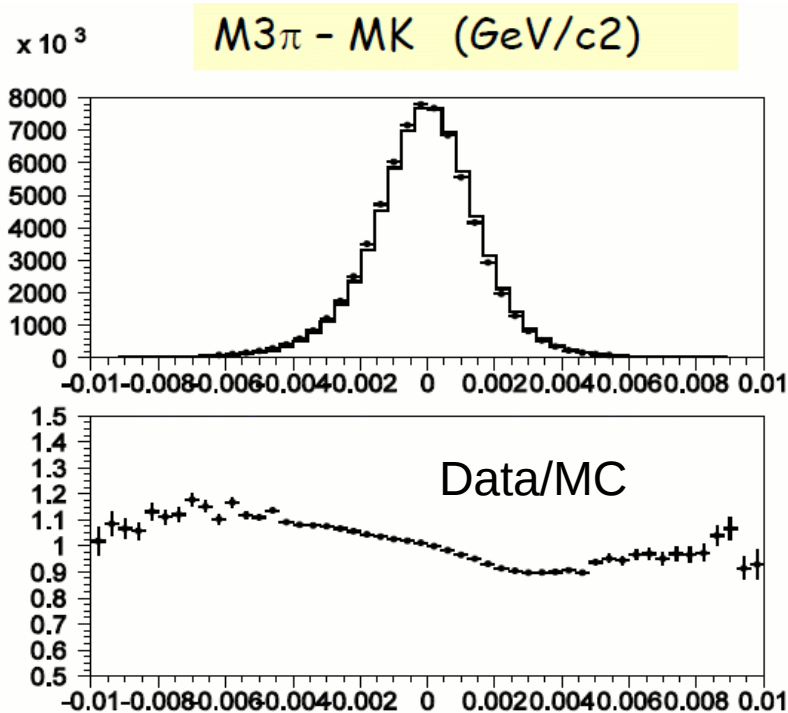
K_{e4}^{00} : Dalitz plot



$M_{e\nu}(\text{GeV}/c^2)$ vs $M_{\pi^0\pi^0}(\text{GeV}/c^2)$

$K_{3\pi}(00)$ calibration control

- Reconstructed kaon mass with expected $1.5 \text{ MeV}/c^2$ resolution
- Small mass shift ($80 \text{ KeV}/c^2$) consistent with beam geometry model control
- $M(\pi^0\pi^0)$ is well modeled (including cusp) by empirical parameterisation of NA48/2 data [[Phys.Lett.B 686 \(2010\) 101](#)]



K_{e4}^{00} : Br measurement

Because of trigger conditions changes in time, Br is measured in successive subsamples and then combined.

$$N(K_{e4} \text{ candidates}) = 44909$$

$$N(\text{bkg}) = 598 \text{ (1.3\% of } K_{e4}\text{)}$$

$$N(K_{3\pi} \text{ candidates}) = 70\,984\,882$$

$$\text{Acceptances: } A(K_{e4}) = 1.77\% \quad A(K_{3\pi}) = 4.11\%$$

$$\text{Trigger efficiency } \varepsilon(K_{e4}) = 92\text{-}98\% \text{ (ratio to } K_{\pi3} \sim 1)$$

$$\text{Normalization: } Br(K_{3\pi}) = (1.761 \pm 0.022)\%$$

Systematic Uncertainty (%)

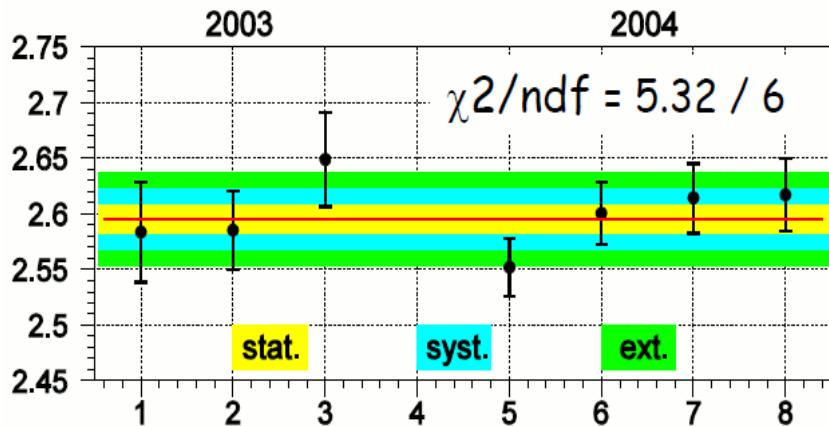
Background	0.35
Simulation stat	0.12
FF dependence	0.20
Rad. Corr.	0.23
Trigger	0.80
Ident. of e	0.10
Beam geometry	0.10

Total **0.94**

$$Br(K_{e4}^{00}) = (2.595 \pm 0.012_{\text{stat}} \pm 0.024_{\text{syst}} \pm 0.032_{\text{ext}})10^{-5}$$

Ke4 Br measurement in consecutive data subsamples (in units of 10^{-5})

Preliminary!

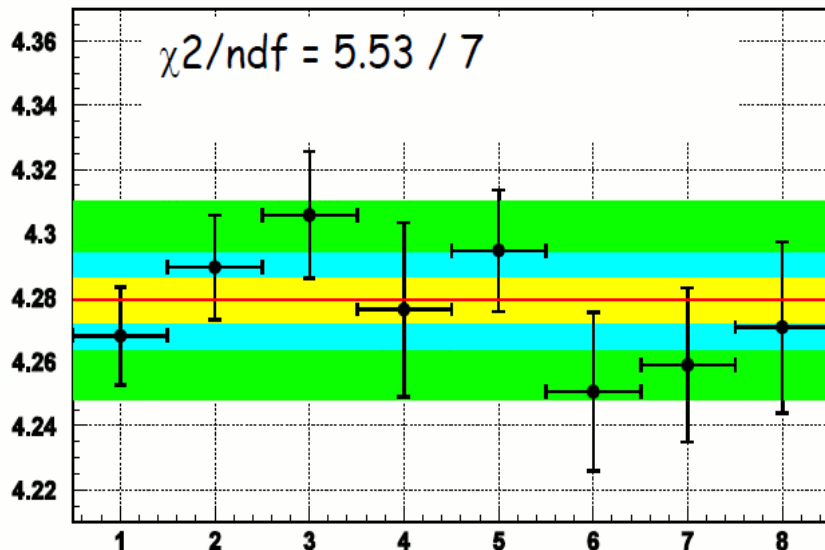


$K_{e4}(00)$ normalized to $K_{3\pi}(00)$

$$(2.595 \pm 0.012 \pm 0.024 \pm 0.032) =$$

Stat Syst Ext

$$(2.595 \pm 0.042) \quad 1.6\% \text{ rel.err.}$$



$K_{e4}(+-)$ normalized to $K_{3\pi}(+-)$

$$(4.279 \pm 0.007 \pm 0.015 \pm 0.031) =$$

Stat Syst Ext

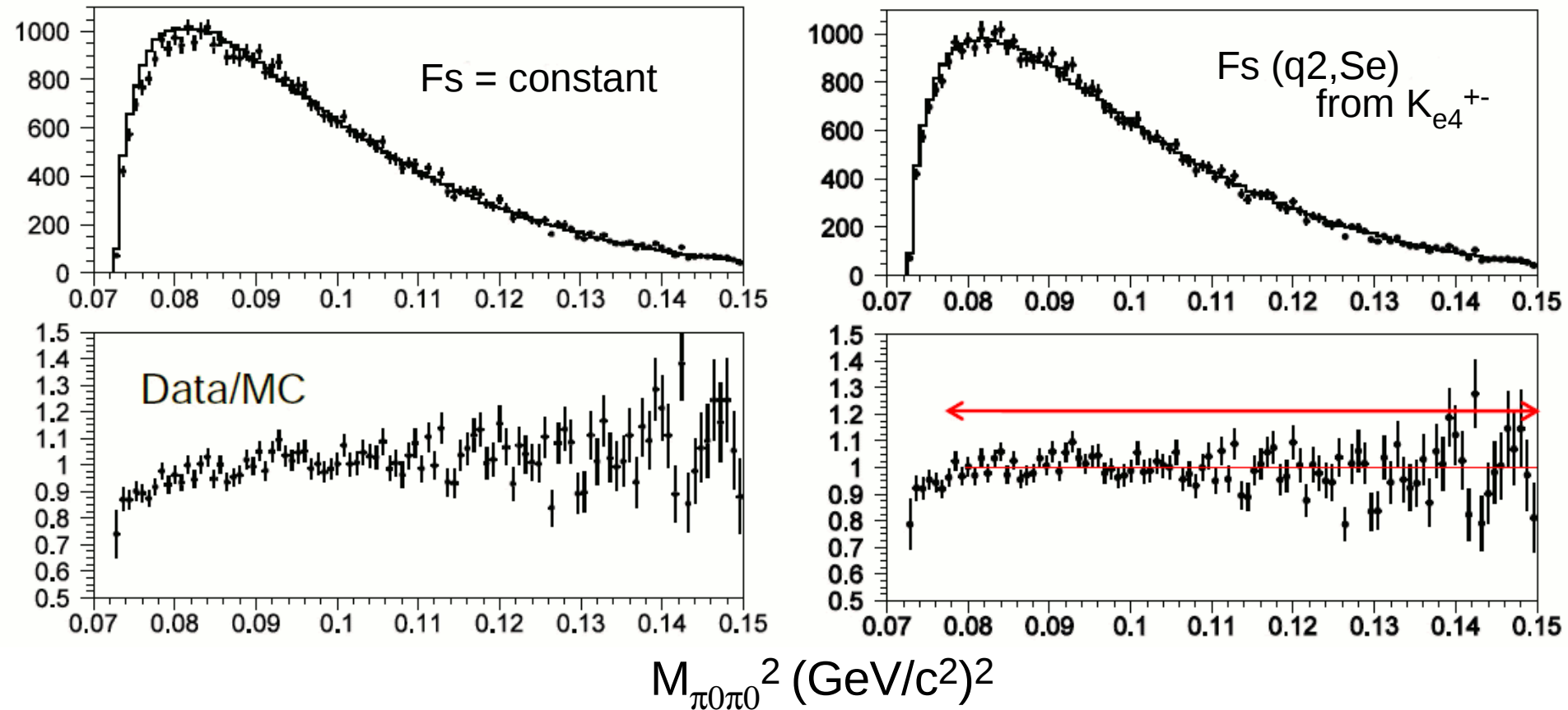
$$(4.279 \pm 0.035) \quad 0.8\% \text{ rel.err.}$$

First look on K_{e4}^{00} Form Factor

Compare $m_{\pi_0\pi_0}^2$ distribution with the MC simulation results with:

Constant Form Factor F_s

Form factor from K_{e4}^{+-} : good agreement + $\pi\pi$ rescattering (cusp) ?



Summary

- 1.11 millions of reconstructed $K^\pm \rightarrow \pi^+ \pi^- \nu e^\pm$ (K_{e4}^\pm) and 44000 of $K^\pm \rightarrow \pi^0 \pi^0 \nu e^\pm$ (K_{e4}^{00}) decays (2003+2004 data).
- Preliminary improved Branching fractions:
 $\text{Br } K_{e4}^\pm = (4.279 \pm 0.035) 10^{-5}$ (3 times better/PDG)
 $\text{Br } K_{e4}^{00} = (2.595 \pm 0.042) 10^{-5}$ (10 times better/PDG)
- First results on $F_s K_{e4}^{00}$ formfactor are consistent with the K_{e4}^\pm one. Drop below $2m_{\pi^+}$ threshold may be due to pions rescattering.
- The study will complete the high-precision picture of K_{e4} decays, and then it may be compared to latest ChPT predictions.