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Pion scattering lengths from NA48

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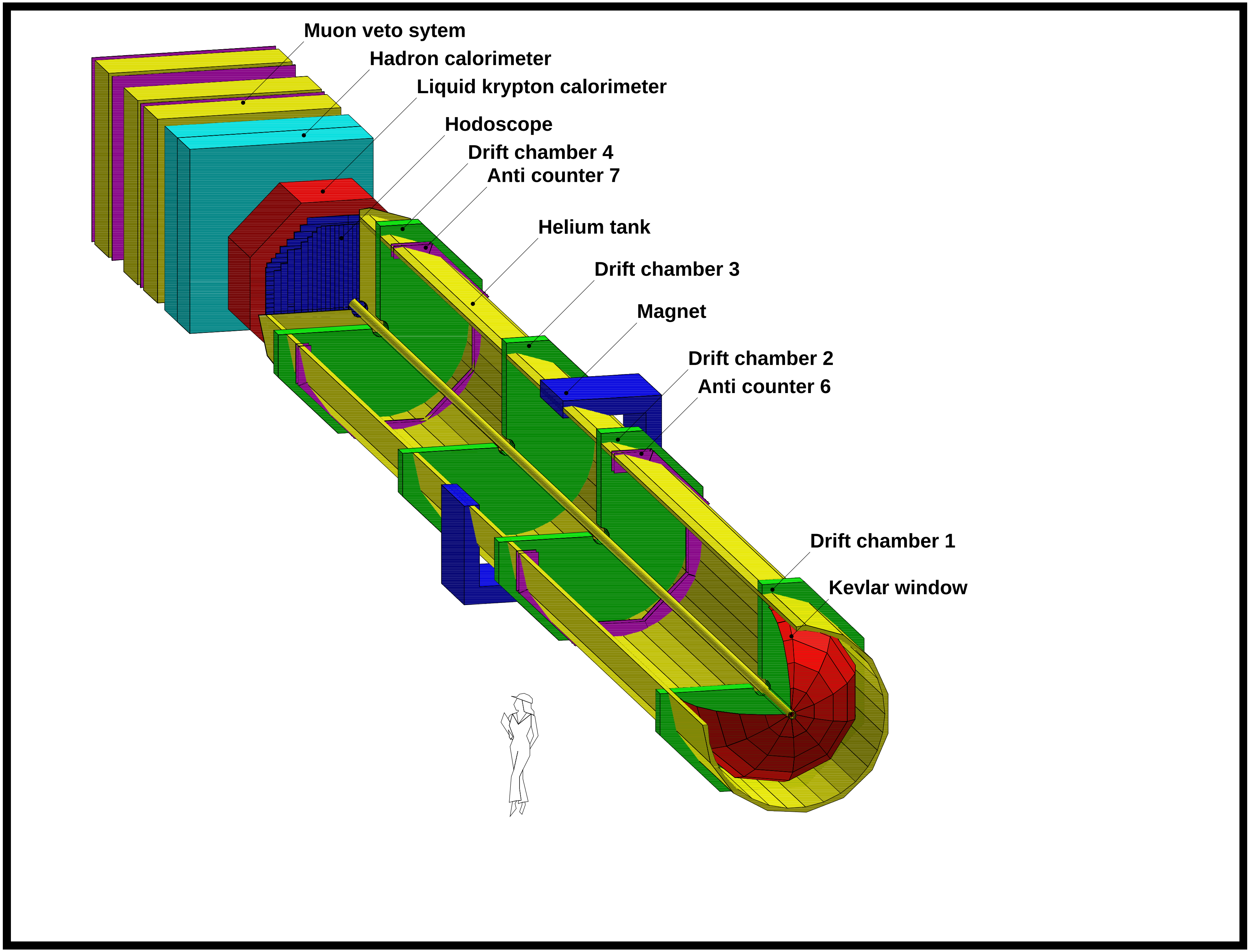
for NA48/2 collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Outlook:

- NA48/2 experiment and data
- Events selection
- Theoretical background
- Fitting
- Results and conclusions


Muon veto sytem
Hadron calorimeter
Liquid krypton calorimeter
Hodoscope
Drift chamber 4
Anti counter 7
Helium tank
Drift chamber 3
Magnet
Drift chamber 2
Anti counter 6
Drift chamber 1
Kevlar window

The key detector for this task is Liquid Krypton calorimeter (LKr). It measures the position and energy of showers, initiated by γ-quanta from π^0 decays.

13248 projective cells $2 \times 2\ cm^2$.

Multi π^0 detection capability.

$$\frac{\sigma(E)}{E} = \frac{0.032}{\sqrt{E}} + \frac{0.09}{E} + 0.0042 \tag{1}$$

$$\sigma_x = \sigma_y = \frac{0.42}{\sqrt{E}} + 0.06cm \tag{2}$$

1 NA48/2 kaon statistics

Why does NA48/2 collect so many $K^{\pm}$? The main task - charged kaons decay asymmetry (see Wednesday Cristina Biino presentation).

$$\frac{d\Gamma}{dU} \propto 1 + gU$$
$$\delta g = \frac{g^{+} - g^{-}}{g^{+} + g^{-}} \tag{3}$$

Data taking: 2003-2004, preliminary result for $3\pi^{\pm}$, based on 2003 year data: $\delta g = (0.5 \pm 3.8) \times 10^{-4}$. Statistics collected:

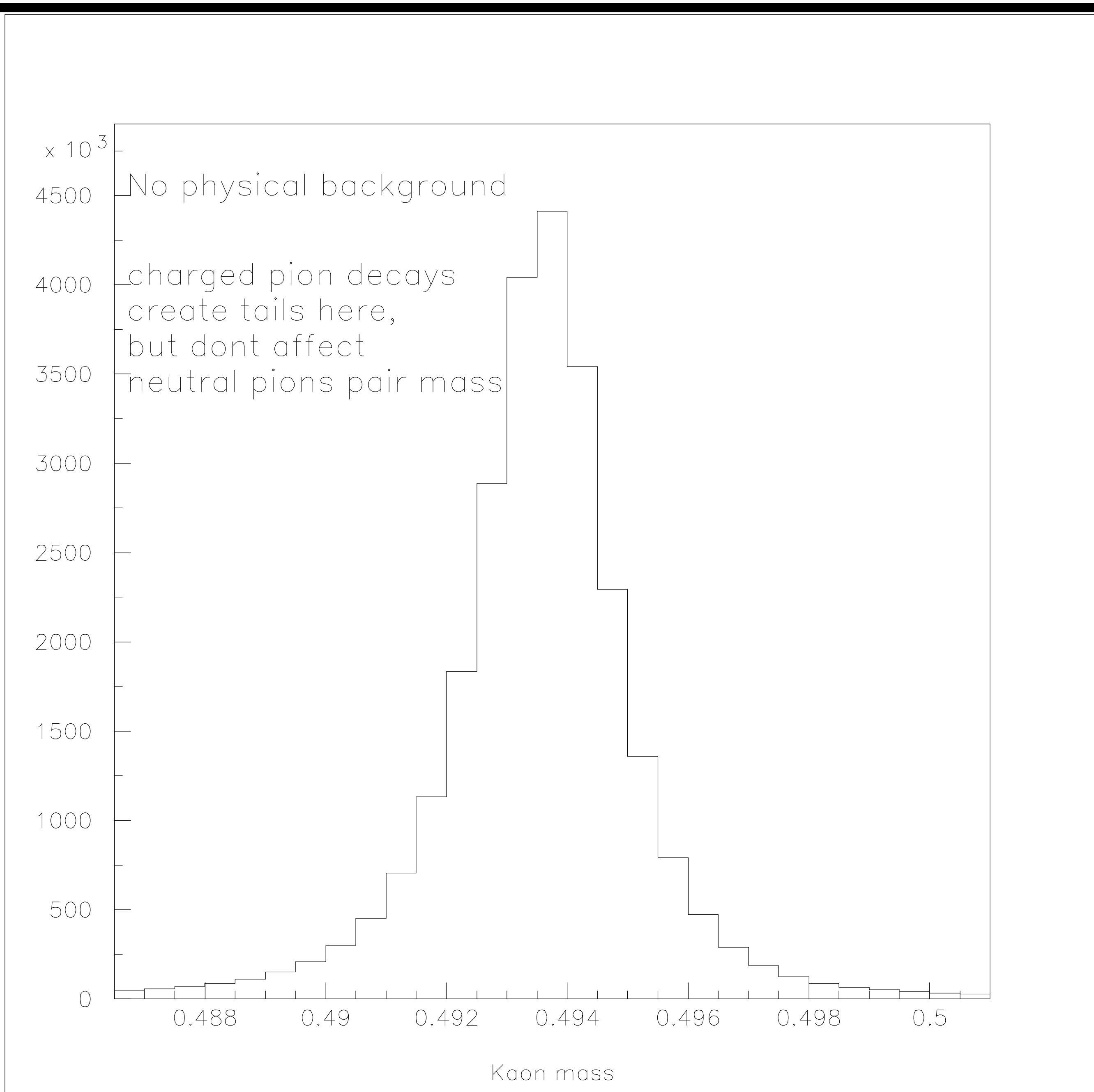
- $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0} : 2 \times 10^{8}$
- $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-} : 4 \times 10^{9}$

For the present analysis we use a part of data, collected in 2003:
$\approx 0.28 \times 10^8$ $K^\pm \to \pi^\pm\pi^0\pi^0$ events after final cuts.

2 Events selection

Essential cuts: (common for data and MC):

- Events selected to be compatible with $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$.
- Distance between electromagnetic calorimeter cluster and dead cell > 2 cm.
- Distance between spectrometer track and cluster > 15 cm.
- Distance between clusters > 10 cm.
- Track momentum > 5 GeV.
- Cluster energy > 3 GeV.


x 10^3
4500
4000
3500
3000
2500
2000
1500
1000
500
0
No physical background
charged pion decays
create tails here,
but dont affect
neutral pions pair mass
0.488
0.49
0.492
0.494
0.496
0.498
0.5
Kaon mass

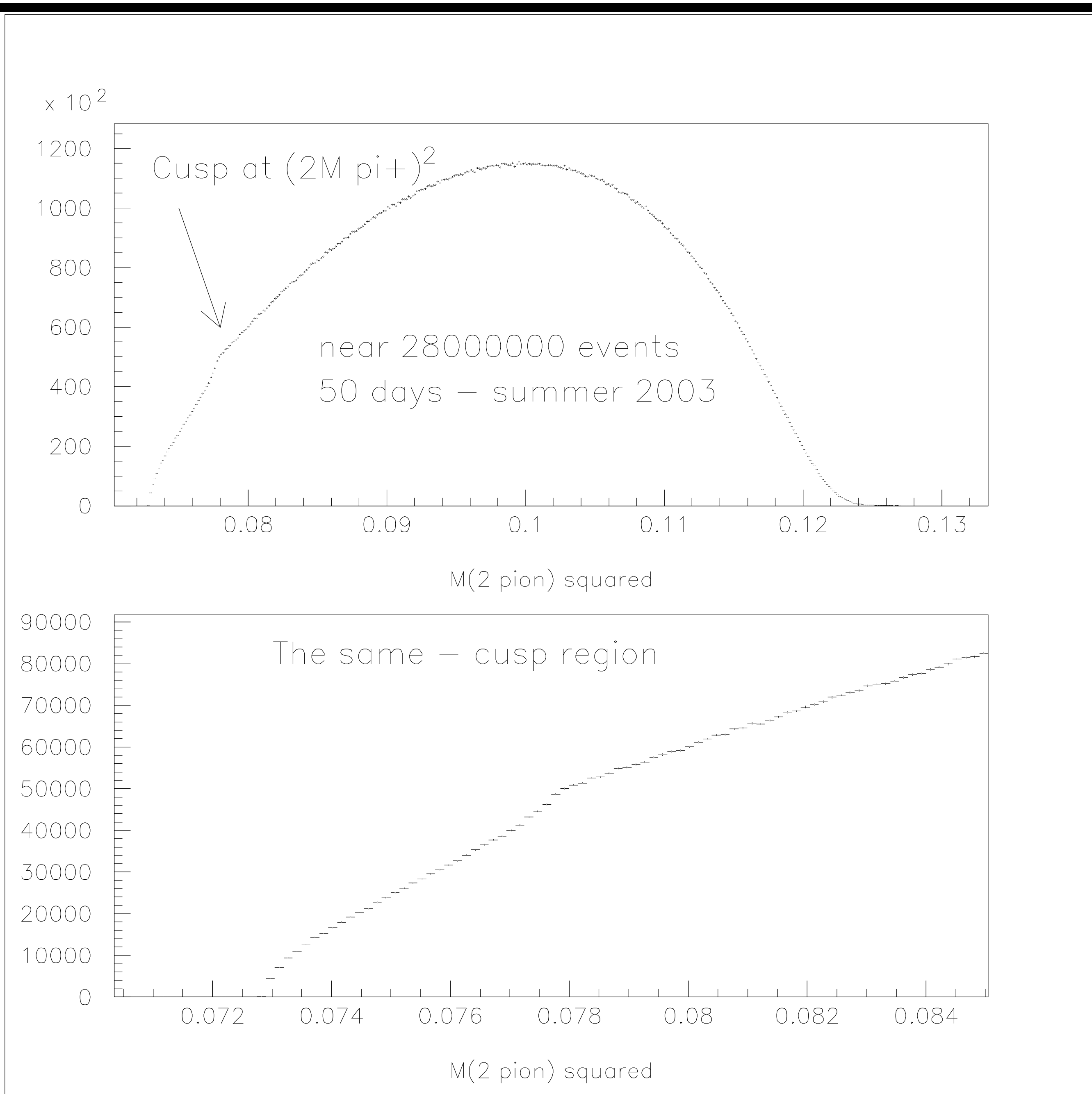
The discussed effect is measured in the distribution of $2\pi^0$ mass in decays $K^\pm \to \pi^\pm\pi^0\pi^0$. It is calculated from the LKr data only as follows:

$$z = \frac{\sqrt{E_{\gamma 1}E_{\gamma 2}}\frac{d_{12}}{m_{\pi^0}} + \sqrt{E_{\gamma 3}E_{\gamma 4}}\frac{d_{34}}{m_{\pi^0}}}{2} \tag{4}$$

$$m_{2\pi^0} = \sqrt{\Sigma E_{\gamma i}E_{\gamma j}d_{ij}^2}\frac{1}{z} \tag{5}$$

So if we calculate Z from PDG m_{π^0}, the combination mass $m_{2\pi^0}$ does not depend on the LKr energy scale:

$$m_{2\pi^0} = 2\frac{\sqrt{\Sigma E_{\gamma i}E_{\gamma j}d_{ij}^2}}{\sqrt{E_{\gamma 1}E_{\gamma 2}d_{12}^2} + \sqrt{E_{\gamma 3}E_{\gamma 4}d_{34}^2}} m_{\pi^0}^{PDG} \tag{6}$$


x 10^2
1200
1000
800
600
400
200
0
Cusp at (2M pi+)^2
near 28000000 events
50 days – summer 2003
0.08
0.09
0.1
0.11
0.12
0.13
M(2 pion) squared
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
The same – cusp region
0.072
0.074
0.076
0.078
0.08
0.082
0.084
M(2 pion) squared

3 Theoretical background

a_0 and a_2 means $a_0 m_{\pi^+}$ and $a_2 m_{\pi^+}$. The invariant mass $s_{\pi^0\pi^0} = s_3$, $3s_0 = \Sigma_{i=1,3} s_i$;

First approximation formula: N.Cabibbo, Phys.Rev.Lett. 93(2004) 121801.

$$M(K^+ \to \pi^+\pi^0\pi^0) = M_0 + M_1, \tag{7}$$

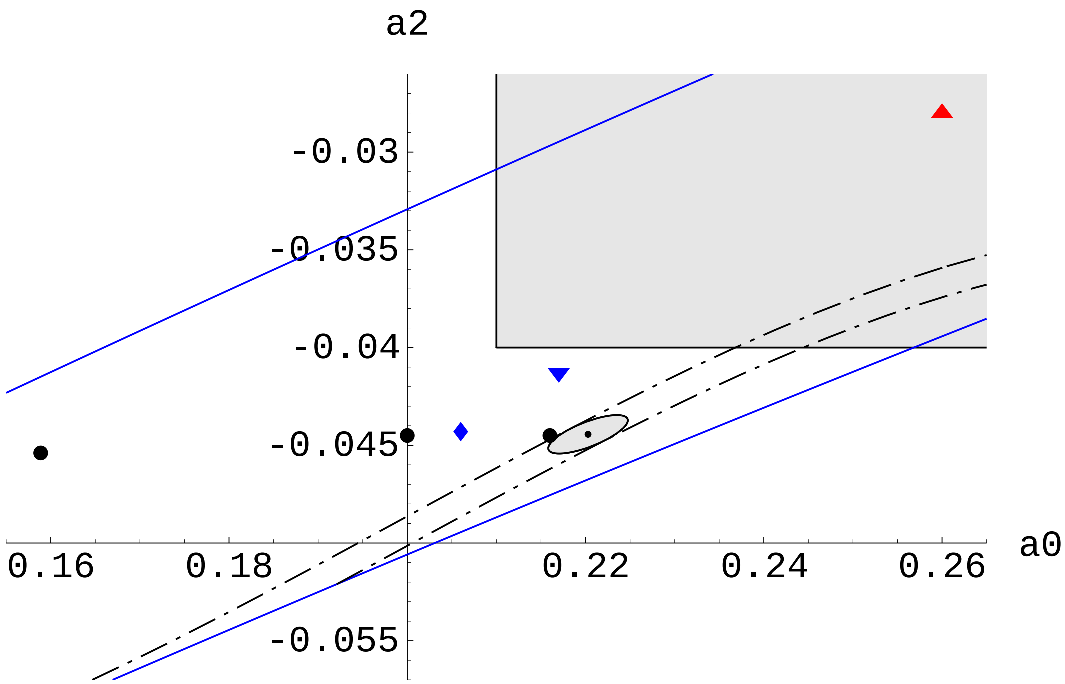
where

$$M_0 = A^0(1 + g^0(s_3 - s_0)/2m_{\pi^+}^2) \tag{8}$$

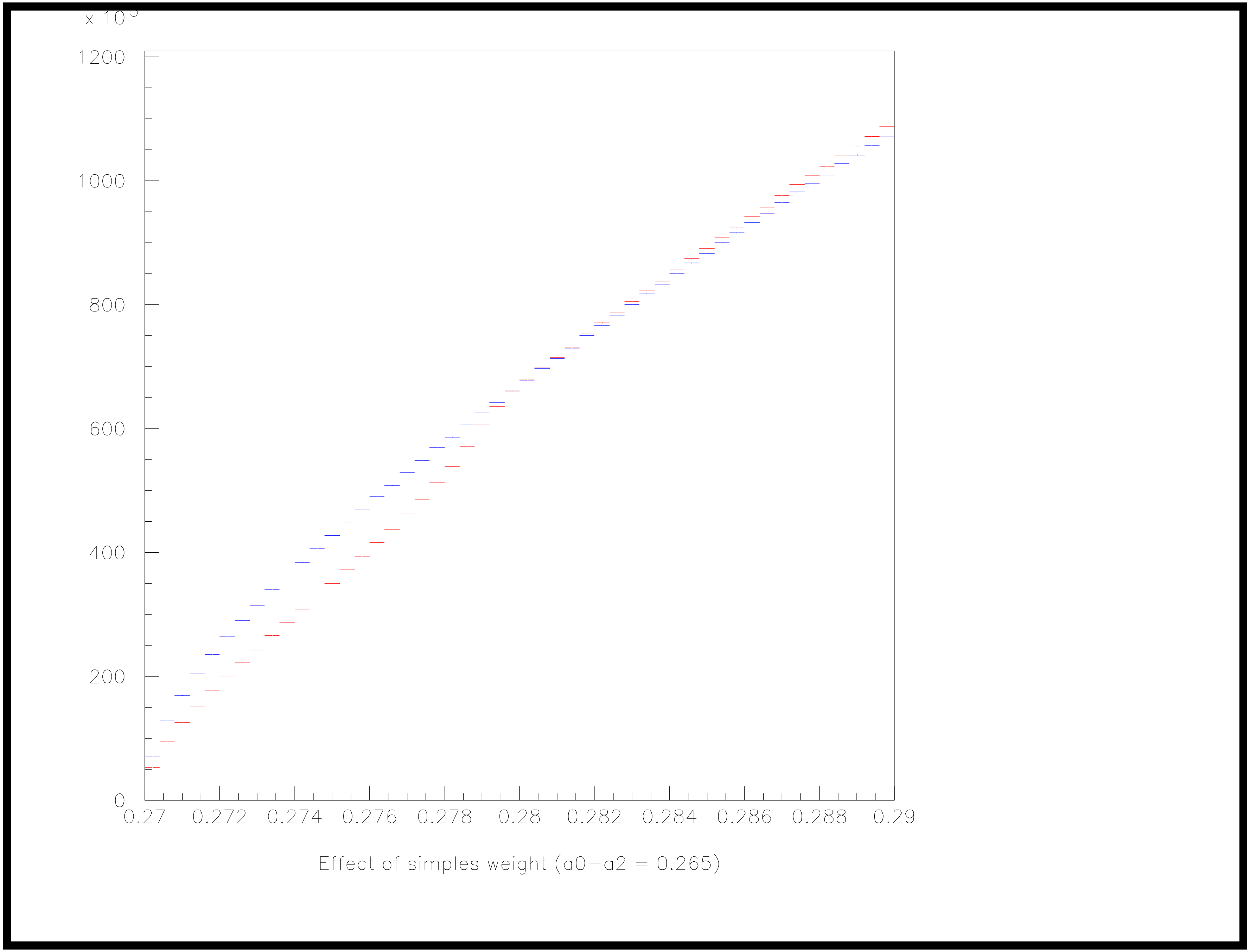
$$M_1 = -\frac{2(a_0 - a_2)}{3} 2A^0(1 + g^+ \frac{(M_K^2 - 9m_{\pi^+}^2)}{12m_{\pi^+}^2}) \sqrt{\frac{4m_{\pi^+}^2 - s_{\pi\pi}}{s_{\pi\pi}}} \tag{9}$$

"$\pi^+\pi^- \to \pi^0\pi^0$ re-scattering from $K^+ \to \pi^+\pi^+\pi^-$ causes a cusp singularity in the $\pi^0\pi^0$ spectrum at the $\pi^+\pi^-$ threshold. The cusp is proportional to $(a_0 - a_2)$"

The ChPT results for the scattering length parameters are: $a_2 = -0.0444 \pm 0.0010$, $a0 - a2 = 0.265 \pm 0.004$. Illustration from the $\pi\pi$ scattering paper (G.Colangelo, J. Gasser and H.Leutwyler, Phys. Lett. **B 488** 261 (2000)) :



The effect of the simplest approximation for a0-a2=0.265 :



x 10^3
1200
1000
800
600
400
200
0
0.27
0.272
0.274
0.276
0.278
0.28
0.282
0.284
0.286
0.288
0.29
Effect of simples weight (a0−a2 = 0.265)

For two-loop approximation (N.Cabibbo, G.Isidori CERN-PH-TH-2005-012) the unperturbed matrix elements for $K \to 3\pi$ include the quadratic term:

$$R^0(s_3) = A^0(1 + \frac{g^0}{2}U + \frac{h^0}{2}U^2) \quad (10)$$

$$R^+(s_3) = 2A^0(1 + \frac{g^+}{2}U + \frac{h^+}{2}U^2) \quad (11)$$

$$(12)$$

Note that it is written for amplitudes itself, not the modules squared, so approximately $g \approx g_{PDG}$, but $h \approx h_{PDG} - (g/2)^2$.

Two-loop approximation produces much longer formulae, that depends not only on $(a_0 - a_2)$ and $s_{\pi^0\pi^0} = s_3$, but also on other combinations of a_0 and a_2 as well as on other two invariant masses $s_{\pi^0\pi^\pm}$. So we will write only the general expression:

$$M_{00+} = A_{00+} + B_{00+} V_{\pm}(s_3) \quad s > 4m_{\pi^+}^2, \tag{13}$$

$$M_{00+} = A_{00+} + iB_{00+} V_{\pm}(s_3) \quad s < 4m_{\pi^+}^2 \tag{14}$$

$$V_{\pm}(s_3) = \sqrt{\frac{|s - 4m_{\pi^+}^2|}{s_3}} \tag{15}$$

Assuming the unperturbed amplitude parameters and the scattering length values (calculated in ChPT):

$$g^0 = 0.638 \tag{16}$$

$$h^0 = -0.051 \tag{17}$$

$$g^+ = -0.2154 \tag{18}$$

$$h^+ = 0.0004 \tag{19}$$

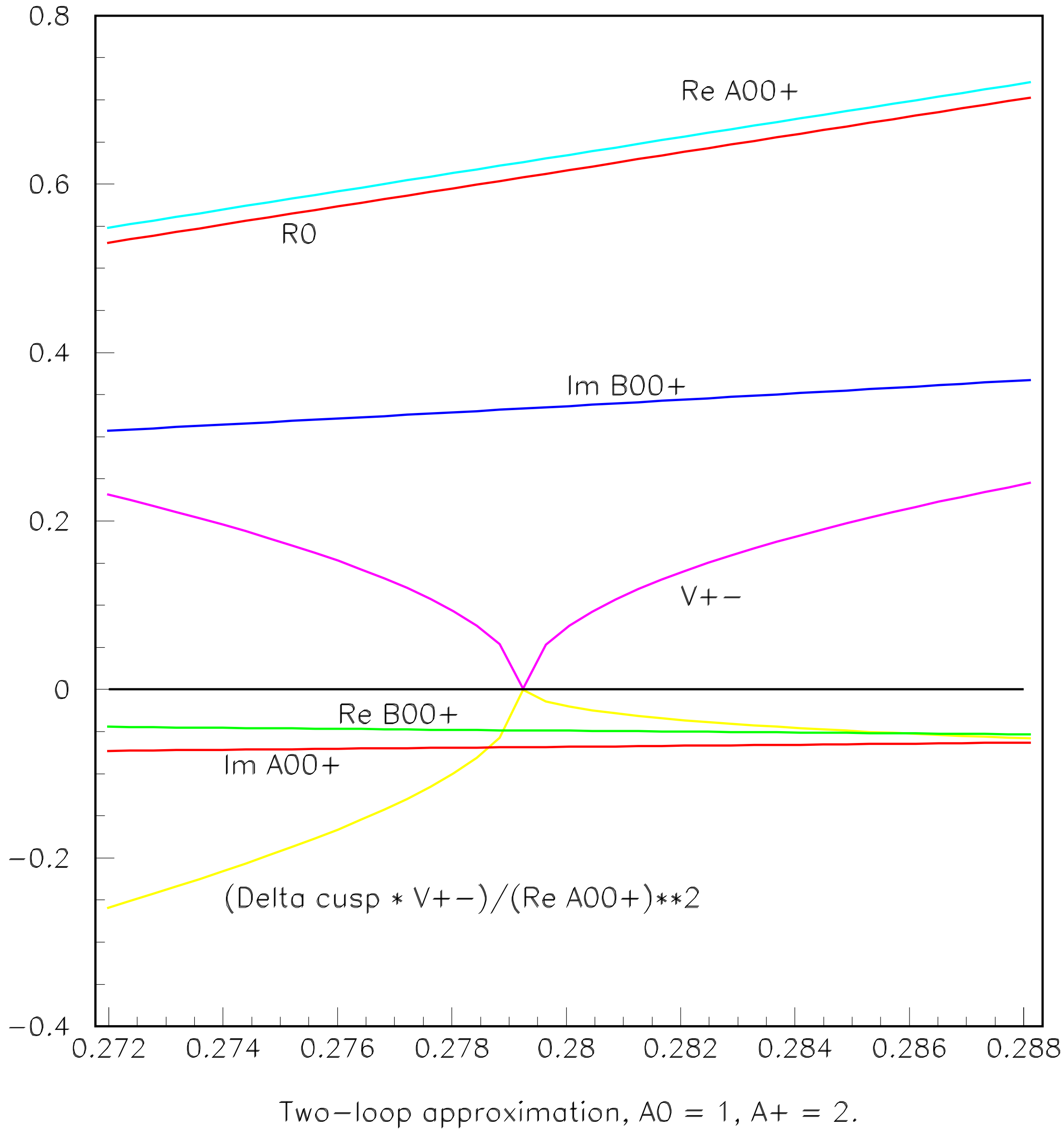
$$a_0 - a_2 = 0.265 \tag{20}$$

$$a_2 = -0.0444, \tag{21}$$

and replacing the $s_{\pi^0\pi^\pm}$ masses with the average values

$$< s_1 >=< s_2 >= \frac{3s_0 - s_3}{2} \tag{22}$$

one can plot the amplitude components:


0.8
0.6
0.4
0.2
0
−0.2
−0.4
0.272
0.274
0.276
0.278
0.28
0.282
0.284
0.286
0.288
Re A00+
R0
Im B00+
V+−
Re B00+
Im A00+
(Delta cusp ∗ V+−)/(Re A00+)∗∗2
Two−loop approximation, A0 = 1, A+ = 2.

One more theoretical ingredient is pionium, a bound state $\pi^+\pi^-$ with the dominant decay mode $\pi^0\pi^0$. We use the Silagadze paper (arXiv:hep-ph/9411382), where the ratio is calculated:

$$\frac{\Gamma(K^+ \to \pi^+ pionium)}{\Gamma(K^+ \to 3\pi^\pm)} \approx 10^{-5}. \tag{23}$$

Now it is recalculated with the current PDG input: $\approx 0.8 \times 10^{-5}$. We need the width with respect to $\pi^+2\pi^0$ decay, so using the PDG branching ratios we have

$$\frac{\Gamma(K^+ \to \pi^+ pionium)}{\Gamma(K^+ \to \pi^+\pi^0\pi^0)} \approx 2.584 \times 10^{-5}. \tag{24}$$

We add it to the s_3 bin (of width 0.00015), with the center at $(2m_{\pi^+})^2$. The predicted contribution in the bin is 2.57 %. The sensitivity of the result to this value is small, so currently we fix it in our formulae.

Currently there are still some theoretical uncertainties that are under investigation by theorists: radiative corrections near the cusp and at the minimum $M_{2\pi^0}$, isospin breaking effects. It leads to uncertainty even in the choise of the best fitting region of this mass distribution.

4 Experimental status for pion scattering length

- 1977: Geneve/Saclay experiment - 20% accuracy.
- 2003: BNL E865 - 5% based on $K \to \pi\pi e\nu$ form factors: $a_0 m_\pi = 0.216 \pm 0.013(stat) \pm 0.002(syst) \pm 0.002(theor.)$
- DIRAC aims to measure pionium lifetime with 10% accuracy and to extract $a_0 - a_2$ with a 5% precision.

5 Fitting

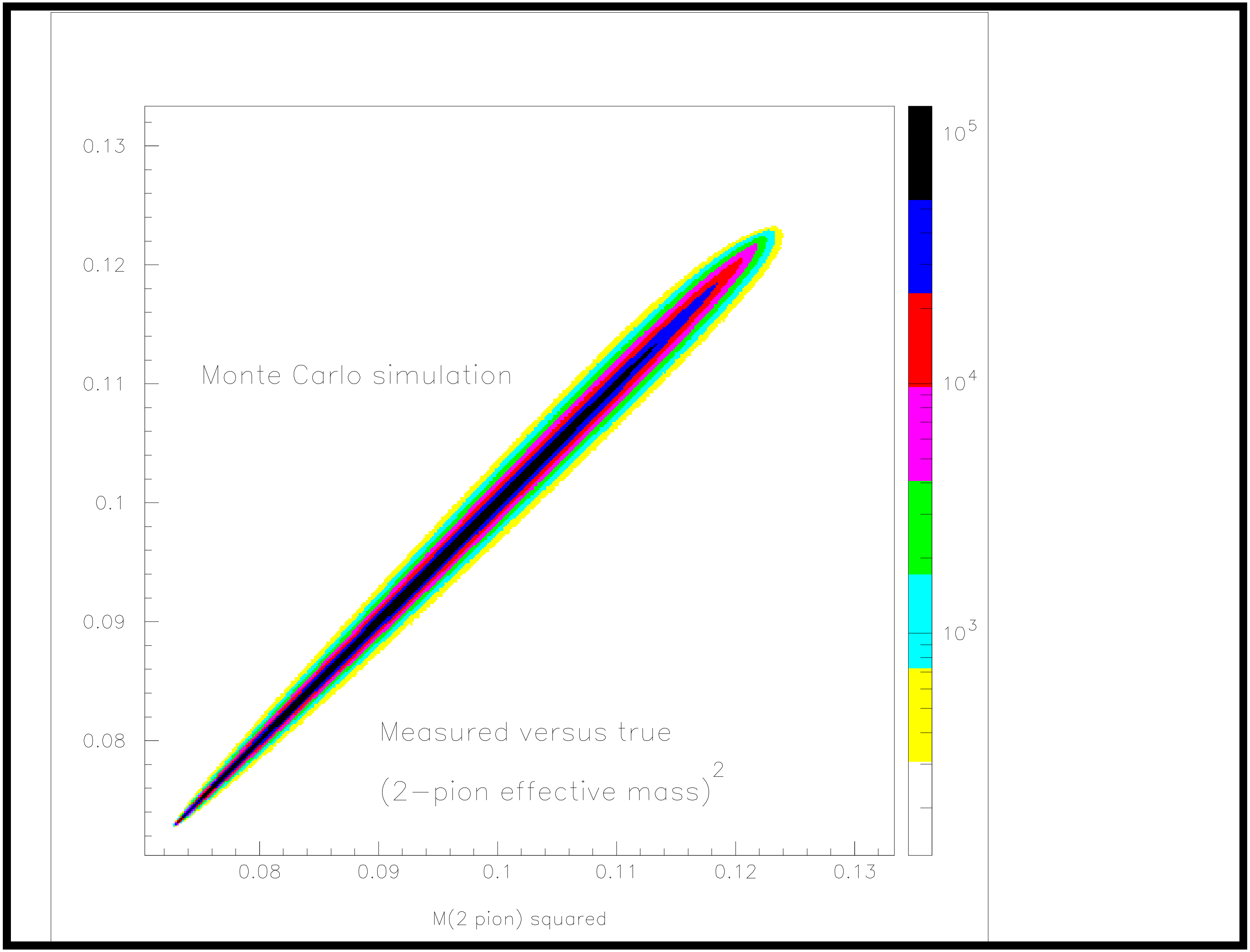
- Fit of data $M_{\pi^0\pi^0}$ distribution with the sum of rescattering and pionium contributions (integral is replaced with the sum over the bins):

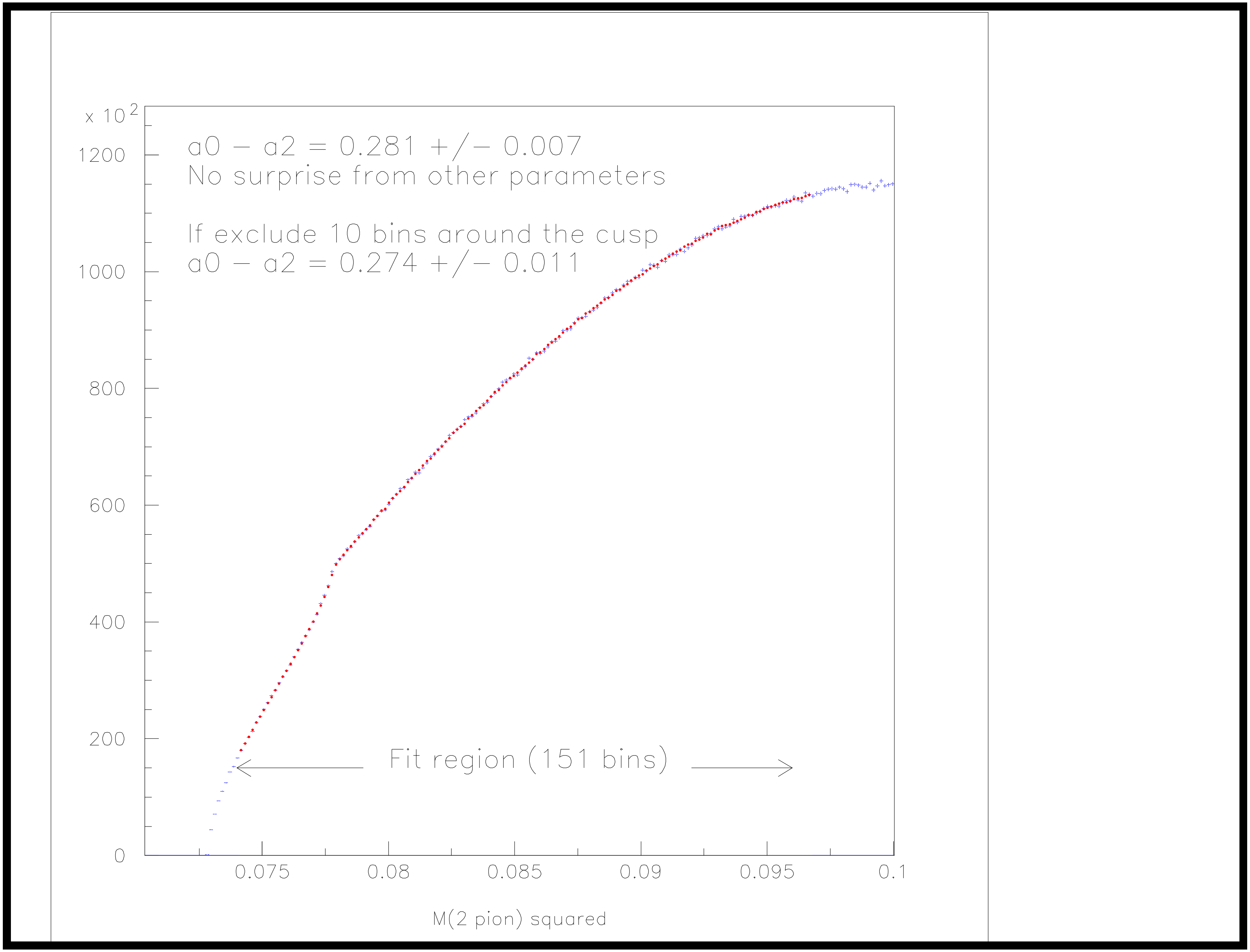
$$f1(S_3^r) = N_1 \Sigma H(S_3, S_3^r) \frac{|M_{fit}(par, < S_1 >, < S_2 >, S_3)|^2}{|M_{sim}(S_3)|^2} \quad (25)$$

$$f2(S_3^r) = N_2 H(4M_{\pi^\pm}^2, S_3^r) \quad (26)$$

Here $par = g0, h', a0 - a2, a2$, $< S_1 >=< S_2 >= \frac{3S_0^0 - S_3}{2}$,
$S_0^0 = (M_K^2 + 2M_{\pi^0}^2 + M_{\pi^\pm}^2)/3$.
S_3^r is a reconstructed S_3.

- A matrix H 420×420 bins of reconstructed $M_{\pi^0\pi^0}$ versus simulated one is used to calculate the convolution.


0.13
0.12
0.11
0.1
0.09
0.08
Monte Carlo simulation
Measured versus true
(2−pion effective mass)^2
0.08
0.09
0.1
0.11
0.12
0.13
M(2 pion) squared
10^5
10^4
10^3


x 10^2
1200
1000
800
600
400
200
0
a0 − a2 = 0.281 +/− 0.007
No surprise from other parameters
If exclude 10 bins around the cusp
a0 − a2 = 0.274 +/− 0.011
Fit region (151 bins)
0.075
0.08
0.085
0.09
0.095
0.1
M(2 pion) squared

6 Systematics checks summary

Sensitivity of the measured $a_0 - a_2$ to pionium production branching ratio $= 0.1\sigma(BR)/BR$.

3 independent analysis, two are based on the good toy Monte Carlo, one is GEANT - based simulation. Results are compatible withing 0.006.

Excluding of pionium region from the fit	0.008
Cuts variation	0.004
Z - dependence	0.009
MC-related difference	0.006
TOTAL (adding in quadrature)	0.014

7 Preliminary result

$$a_0 - a_2 = 0.281 \pm 0.007(stat) \pm 0.014(syst) \qquad (27)$$

Doesn't include theoretical uncertainties (ChiPT next order, radiative corrections, isospin breaking effects).

8 Conclusions

- Full statistics 2003-2004 of the same quality is 4 times more.
- The systematics study is well under way.
- The data quality calls for new theoretical efforts for extracting pion-pion scattering parameters.
- A PDG page describing K dalitz plot will be deeply modified.