

**$|V_{td}|, |V_{ts}|$  and rare kaon decays**F. BUCCI<sup>(1)</sup>

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**Summary.** — We present a review of results on the CKM matrix elements  $V_{td}$  and  $V_{ts}$  from  $B\bar{B}$  mixing and radiative penguin  $B$  decays. We discuss the prospects for measuring  $|V_{td}|$  from the rare kaon decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and we describe the NA62 proposal for an experiment which measures the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching fraction with a statistical precision better than 10%.

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**1. – Introduction**

Second order charged weak processes involving box or penguin diagrams containing virtual quarks are sensitive to  $V_{td}$  and  $V_{ts}$ . The absolute value of the CKM matrix element  $V_{tq}$  ( $q = d, s$ ) can be determined by measuring the oscillation frequency  $\Delta M_q$  in the  $B_q^0 \bar{B}_q^0$  mixing:

$$(1) \quad \Delta M_q = \frac{G_F^2}{6\pi^2} m_{B_q} \left( \hat{B}_{B_q} F_{B_q}^2 \right) \eta_B m_W^2 S_0(x_t) |V_{tq}|^2$$

Here,  $\hat{B}_{B_q}$  is the bag parameter of the  $B$  meson,  $F_{B_q}$  is the weak  $B$  decay constant,  $\eta_B$  is a QCD correction and  $S_0(x_t)$  is a slowly varying function of the top quark and  $W$  boson mass. To reduce the theoretical uncertainties coming from  $\hat{B}_{B_q}$  and  $F_{B_q}$ ,  $|V_{td}/V_{ts}|$  is extracted from the  $\Delta M_d$  over  $\Delta M_s$  ratio.  $|V_{td}/V_{ts}|$  can also be extracted by measuring  $R = \mathcal{B}(B \rightarrow \rho/\omega \gamma) / \mathcal{B}(B \rightarrow K^* \gamma)$ :

$$(2) \quad R \propto \left| \frac{V_{td}}{V_{ts}} \right|^2 \left( \frac{m_B^2 - m_{\rho/\omega}^2}{m_B^2 - m_{K^*}^2} \right)^3 \frac{1}{\xi^2} (1 + \Delta R)$$

where  $\xi$  is a ratio of form factors and  $\Delta R$  accounts for explicit  $O(\alpha_s)$  corrections. Physics beyond the SM could create inconsistencies between the  $|V_{td}/V_{ts}|$  measurement from radiative decays and  $B\bar{B}$  mixing.

TABLE I. –  $B_s^0\bar{B}_s^0$  oscillation frequency  $\Delta M_s$  measured by CDF ( $1fb^{-1}$ ) and D0 ( $2.4fb^{-1}$ )

CDF: World First Observation ( $5\sigma$ )	D0: Evidence ( $3\sigma$ )
$\Delta M_s = 17.77 \pm 0.10(\text{stat.}) \pm 0.07(\text{syst.}) \text{ ps}^{-1}$	$\Delta M_s = 18.53 \pm 0.93(\text{stat.}) \pm 0.30(\text{syst.}) \text{ ps}^{-1}$

## 2. – $B^0\bar{B}^0$ mixing

The experimental status of  $\Delta M_d$  is dominated by the results from the  $B$  factories where two  $B_d^0$  mesons with opposite flavor are produced in a coherent state from the  $\Upsilon(4S)$  decay. The oscillation frequency is determined with an unbinned maximum likelihood fit that uses for each event the measured difference in decay times of the two  $B$  mesons.

Combining all published measurements and accounting for all identified correlations yields  $\Delta M_d = 0.507 \pm 0.005 \text{ ps}^{-1}$  [1]. The  $\Delta M_d$  result can be used to extract the magnitude of the CKM matrix element  $V_{td}$  within the SM:  $|V_{td}| = (7.4 \pm 0.8)^{-3}$ . The extraction is at present completely dominated by the uncertainty on the hadronic matrix element  $f_{B_d}\sqrt{\hat{B}_{B_d}} = 244 \pm 11 \pm 24 \text{ MeV}$  [2] obtained from the lattice QCD calculations.

Unlike  $B$  factories, Tevatron can produce all  $B$  species and  $b\bar{b}$  pairs do not evolve coherently. The proper time  $t = \frac{m_B L}{p}$  is measured from the distance  $L$  between the production vertex and the  $B$  decay vertex, and from an estimate of the  $B$  momentum  $p$ . To extract information useful for the interpretation of  $B_s^0$  oscillation searches a  $B_s^0$  oscillation amplitude  $\mathcal{A}$  is measured as a function of a fixed test value of  $\Delta m_s$ , using a maximum likelihood fit based on  $\Gamma_s e^{-\Gamma_s t} (1 \pm \mathcal{A} \cos(\Delta m_s t))/2$ . If  $\Delta m_s$  is equal to its true value, one expects  $\mathcal{A} = 1$  within the total uncertainty  $\sigma_{\mathcal{A}}$ . However, if  $\Delta m_s$  is (far) below its true value, a measurement consistent with  $\mathcal{A} = 0$  is expected. Table I summarizes the CDF and D0 results [3].

The information on  $|V_{ts}|$  obtained, in the framework of the SM, from the CDF amplitude spectrum is hampered by the hadronic uncertainty, as in the  $B_d^0$  case. However, several uncertainties cancel in the frequency ratio  $\Delta m_s/\Delta m_d$ :

$$(3) \quad \left| \frac{V_{ts}}{V_{td}} \right| = 0.2060 \pm 0.0007(\Delta m_s)_{-0.0060}^{+0.0081}(\Delta m_d + \text{theor})$$

## 3. – $B \rightarrow V\gamma$ decays

The  $B^+ \rightarrow \rho^+ \gamma$ ,  $B^0 \rightarrow \rho^0 \gamma$  and  $B^0 \rightarrow \omega \gamma$  decays have been reconstructed by both BABAR and Belle [4]. Two almost uncorrelated kinematical variables are computed to separate the signal from backgrounds: the difference between the  $B$  candidate center of mass (CM) energy and the CM beam energy  $\Delta E$ , and the ‘‘beam energy-substituted mass’’  $m_{ES}$  obtained substituting the CM beam energy for the  $B$  candidate CM energy in the invariant mass expression. The signal content of the data is determined by a multidimensional unbinned maximum likelihood fit which is constructed individually for each of the three signal decay modes. The significance is computed as  $\sqrt{2\Delta\log\mathcal{L}}$ , where  $\Delta\log\mathcal{L}$  is the log-likelihood difference between the best fit and the null-signal hypothesis.

By using the world average value of  $\mathcal{B}(B \rightarrow K^*\gamma) = (41.8 \pm 1.7) \cdot 10^{-6}$ , both BABAR and Belle have computed the  $|V_{td}/V_{ts}|$  ratio from eq.(2). The results are shown in

TABLE II. – Significance ( $\Sigma$ ) in standard deviations including systematic errors, the isospin-averaged branching fraction ( $\mathcal{B}$ ) and the  $|V_{td}/V_{ts}|$  measured by BABAR and Belle.

	$\Sigma$	$\mathcal{B}(B \rightarrow \rho/\omega \gamma) \cdot 10^6$	$ V_{td}/V_{ts} $
BABAR	6.4	$1.25^{+0.25}_{-0.24} \pm 0.09$	$0.200^{+0.021}_{-0.020} \pm 0.015$
Belle	6.2	$1.14 \pm 0.20^{+0.10}_{-0.12}$	$0.195^{+0.020}_{-0.019} \pm 0.015$

Table II. The measurement of  $|V_{td}/V_{ts}|$  is consistent with the one from  $B\bar{B}$  mixing and gives a competitive constraint on the unitarity triangle (UT) [5].

#### 4. – Rare kaon decays

The rare kaon decays  $K \rightarrow \pi\nu\bar{\nu}$  are sensitive probes of the physics at high energy scales and allow in particular to access the CKM couplings of the top quark in a very clean way. We limit the discussion to  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  transitions. The SM prediction can be parameterized as follows [6]:

$$(4) \quad \mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) \approx 1.6 \times 10^{-5} \cdot |V_{cb}|^4 \cdot (\sigma\bar{\eta}^2 + (\rho_c - \bar{\rho})^2) \approx (8.0 \pm 1.1) \times 10^{-11}$$

where  $\sigma = 1/(1 - \lambda^2/2)^2$  and  $\rho_c \approx 1.4$ .

The determination of  $|V_{td}|$  is subject to various uncertainties:

$$(5) \quad \frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 0.39 \frac{\sigma(P_c)}{P_c} \pm 0.70 \frac{\sigma(\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}))}{\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})} \pm \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

where the parameter  $P_c$  results from Z-penguin and electroweak box diagrams involving internal charm quark exchange and has been calculated at NNLO [7].

On the experimental side, the decay was studied with stopped kaons by E787 and its upgraded version, E949, leading to the publication of three candidates and to a first determination of the branching ratio:  $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$  [8]. The mean value is about twice larger than the SM prediction but, given the large errors, completely in agreement with it.

The proposal to measure the  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  decay at the CERN SPS with a 10% accuracy has been put forward by the NA62 collaboration [9]. The aim is to observe  $\sim 100$  signal events in 2 years with a 10/1 signal to background ratio. This can be achieved by exploiting a decay in flight technique which allows a 10% signal acceptance and by using a beam line able to provide of the order of  $10^{13}$  kaon decays. The signature of the  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  event is one reconstructed positive track in the downstream detector. Background rejection relies on the event kinematics, photon veto, and particle identification. The squared missing mass,  $m_{miss}^2$ , defined as the squared difference between the kaon and the pion candidate four-momentum allows a kinematical separation between the signal and almost 92% of the background. To reject the main sources of background,  $K^+ \rightarrow \mu^+\nu_\mu$  and  $K^+ \rightarrow \pi^+\pi^0$ , we need a  $\mu/\pi$  separation at  $10^{-8}$  level and a photon veto at  $10^{-5}$ , respectively. To further suppress the  $K^+ \rightarrow \pi^+\pi^0$  background, the kaon beam is chosen to have 75GeV/c. Requiring the  $\pi^+$  to have less than 35 GeV of energy, leaves at least

40 GeV of energy associated to the  $\pi^0$ . The drawback of a high energy kaon beam is that pions and protons cannot be efficiently separated from kaons. Although only about 6% of particles are kaons and only  $\approx 10\%$  of the kaons decay in the useful decay volume, all particles (800MHz) have to be tracked and precisely timed. The main elements of the proposed detector are:

- A differential Cherenkov counter (CEDAR) placed on the incoming beam for positive kaon identification.
- Thin silicon micro-pixel detectors (GIGATRACKER) able to work at 1 GHz to measure the momentum of the incoming particle.
- A magnetic spectrometer (STRAW) measuring the direction and the momentum of the out-going pion.
- A gas ring imaging Cherenkov counter (RICH) providing pion/muon separation with a muon suppression factor of at least  $10^{-2}$ . The RICH must also work as a timing detector for the downstream track with a requested time resolution of 100 ps.
- The NA48 hodoscope for fast timing and triggering (CHOD).
- A photon veto system composed by a set of ring-shaped anti-counters (LAV) for photons originating from kaon decays in the fiducial region from 10 mrad to 50 mrad, a high-performance liquid krypton calorimeter (LKr) for photons in the region between 1 and 10 mrad, and two calorimeters (IRC and SAC) to cover the region below 1 mrad.
- A hadron calorimeter (HAC) and a muon detector (MUD) able to identify muons with  $10^{-5}$  inefficiency.

A preliminary analysis has been done based on simulation. We expect  $\sim 50$  signal events per year of data taking with a background to signal ratio between 14% and 17%.

## 5. – Conclusions

We reviewed the status of the  $|V_{td}|$  and  $|V_{ts}|$  measurements and we discussed the prospects of a theoretically clean determination of  $|V_{td}|$  from  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decays. However, in the future, the role of the  $K \rightarrow \pi \nu \bar{\nu}$  will shift towards the search for new physics.

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