

PRECISION MEASUREMENT OF $\pi\pi$ SCATTERING LENGTHS AT THE NA48/2 EXPERIMENT

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On behalf of the NA48/2 collaboration

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Valencia



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OUTLINE

- NA48/2 experiment: detectors and collected data;

$K^\pm \rightarrow e^\pm \nu \pi^+ \pi^-$ (Ke4) decay:

- Form factor and phase shift estimation;
- Extraction of pion scattering lengths (a_0 and a_2);

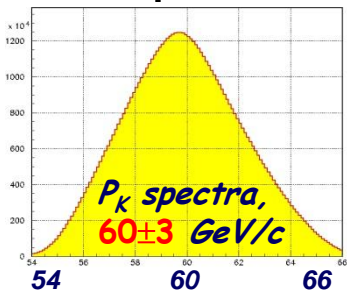
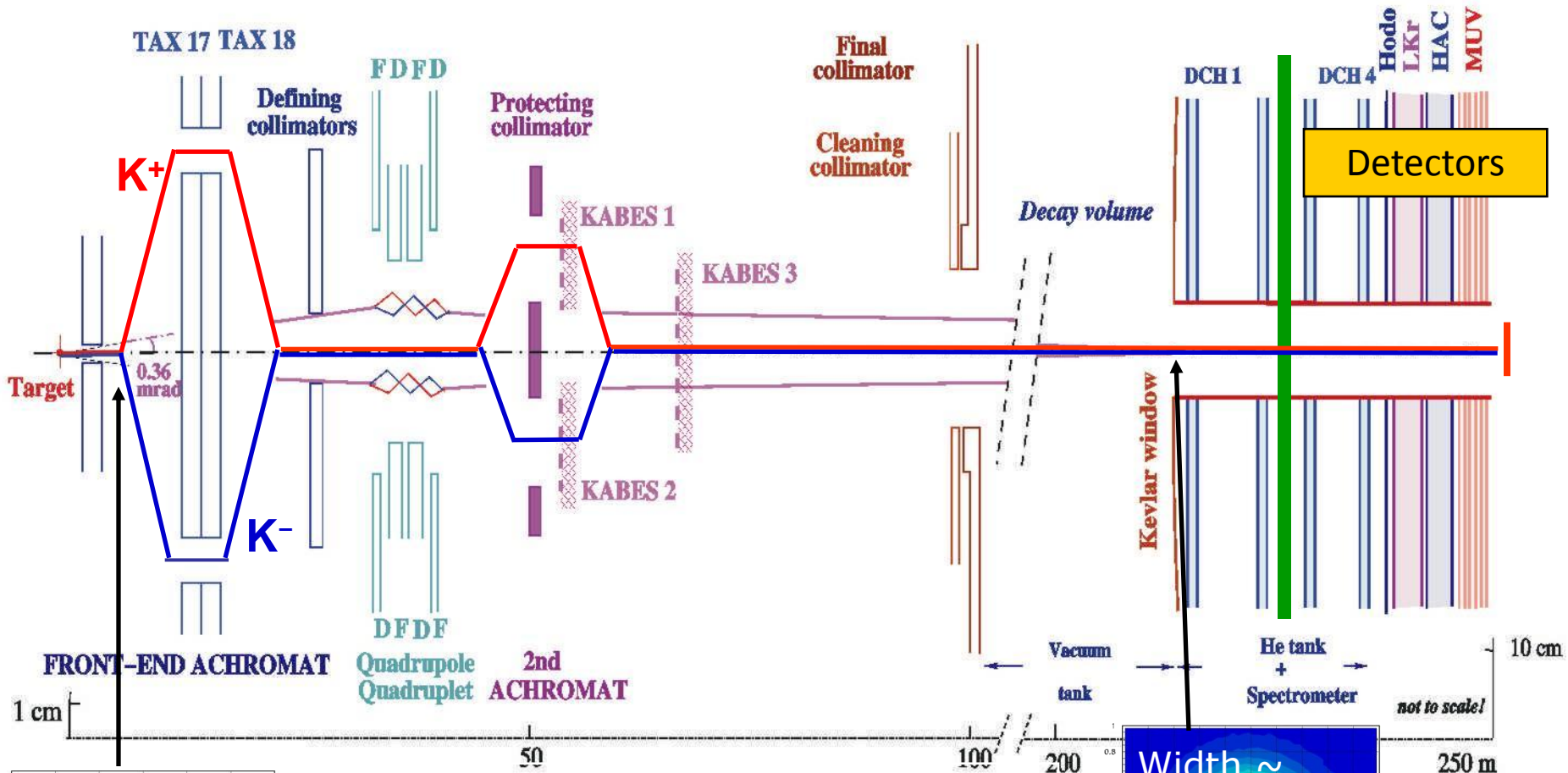
Cusp in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ (Kpi3n) decay:

- Evidence and theoretical models
- Fitting procedure and extraction of scattering lengths;
- Conclusions.

NA48	1997	ϵ'/ϵ run	$K_L + K_S$
	1998	ϵ'/ϵ run	$K_L + K_S$
	1999	ϵ'/ϵ run $K_L + K_S$	K_S Hi-Int.
	2000	K_L only NO Spectrometer	K_S High Intensity
	2001	ϵ'/ϵ run $K_L + K_S$	K_S High Int.
NA48/1	2002	K_S High Intensity	
NA48/2	2003	K^\pm High Intensity	
	2004	K^\pm High Intensity	
NA62	2007	Ke2/K μ 2 measurement	
	2012/13	$K^+ \rightarrow \pi^+ \nu \nu$ measurement	

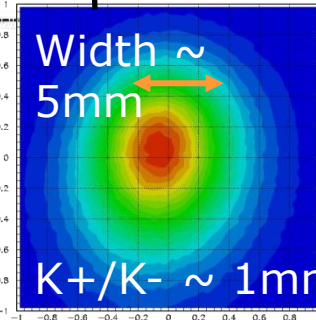


NA48/2 BEAM LINE



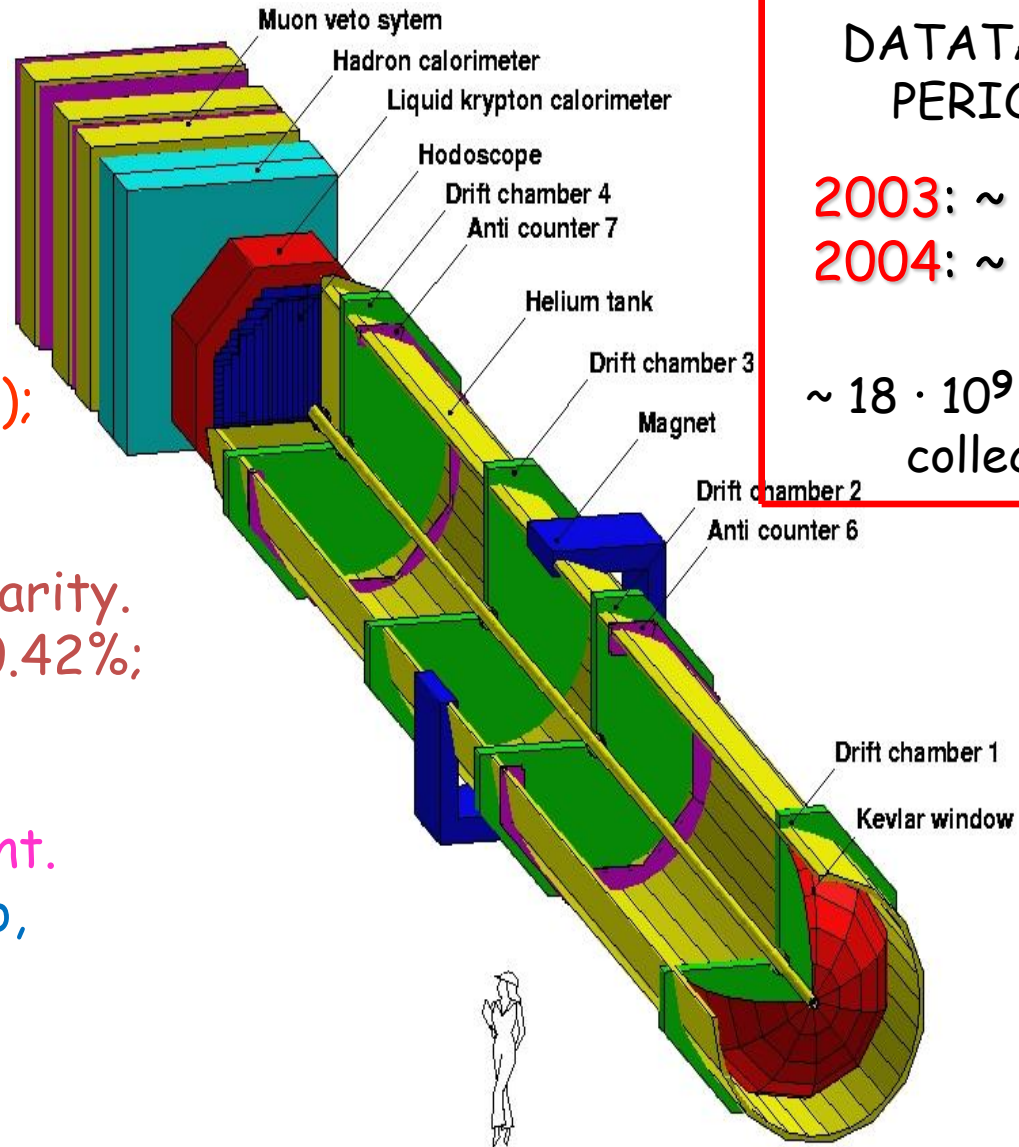
- 400 GeV protons from SPS
- K^+/K^- flux ~ 1.8
- 6.3×10^7 p.p.p.

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NA48/2 DETECTORS

- Spectrometer:
4 DCHs -> redundancy.
 $\sigma_p/p = 1.0\% + 0.044\% \times p(\text{GeV})$;
- Liquid Krypton EM calorimeter:
13000 cells -> high granularity.
 $\sigma_E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$;
- Hodoscopes (charged, neutral):
Trigger, time measurement.
- Muon veto, Hadronic calo, Kabes, photon vetoes.



DATA TAKING PERIODS:

2003: ~ 50 days

2004: ~ 60 days

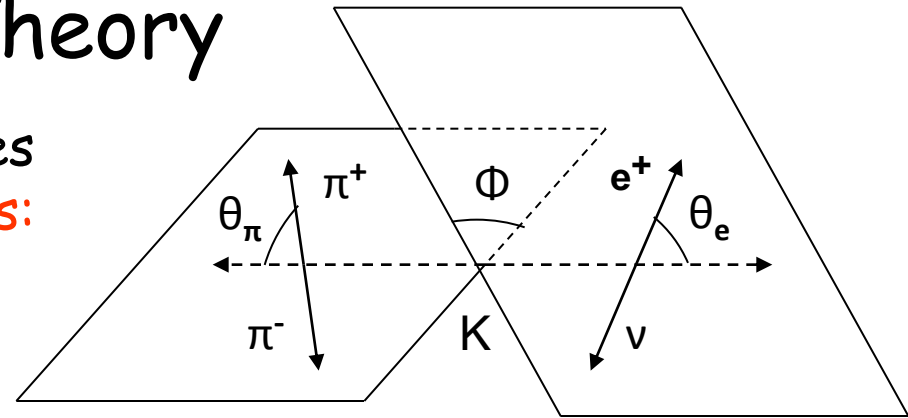
~ $18 \cdot 10^9$ triggers collected

Ke4: Theory

4 body decay \rightarrow 5 independent variables

Using Cabibbo-Maksymowicz variables:

$$S_\pi(M_{\pi\pi}^2), S_e(M_{e\nu}^2), \cos\theta_\pi, \cos\theta_e, \Phi$$



$$\text{Hadronic ME: } \langle \pi^+ \pi^- | A_\lambda + V_\lambda | K \rangle = (1/M_K) [F P_\lambda + G Q_\lambda + R(K-P)_\lambda + (H/M^2) \epsilon_{\lambda\mu\nu\sigma} K_\mu P_\nu Q_\sigma]$$

(P_λ =dipion ; Q_λ =dilepton; K_λ =kaon 4 vectors)

F, G, R : axial form factors

H : vector form factor

R negligible since proportional to m_e^2 (relevant in $K\mu 4$)

Partial wave expansion of amplitude:

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi + \text{d-wave terms}$$

$$G = G_p e^{i\delta_g} + \text{d-wave terms}$$

$$H = H_p e^{i\delta_h} + \text{d-wave terms}$$

f.f. expansion wrt S_e, q^2 :

$$F_s = f_s + f'_s q^2 + f''_s q^4 + f_e (S_e/4m_\pi^2) + \dots$$

$$F_p = f_p + f'_p q^2 + \dots$$

$$G_p = g_p + g'_p q^2 + \dots$$

$$H_p = h_p + h'_p q^2 + \dots$$

$$q^2 = (S_\pi/4m_\pi^2) - 1$$

Fit parameters: $F_s, F_p, G_p, H_p, \delta = \delta_s - \delta_p$

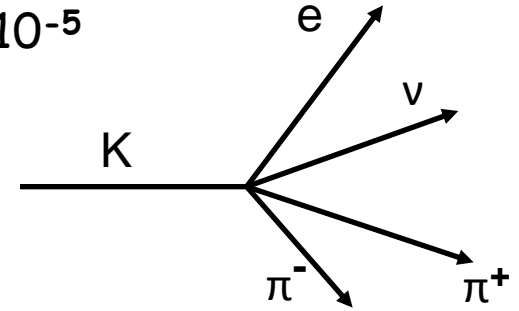
Ke4 Analysis: selection and BG rejection

Ke4 → 3 charged tracks (2 opposite sign pions), BR ~ $4 \cdot 10^{-5}$

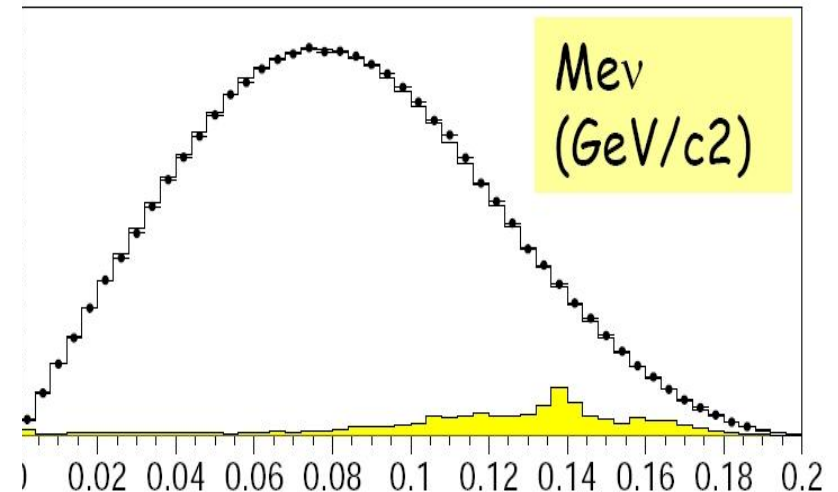
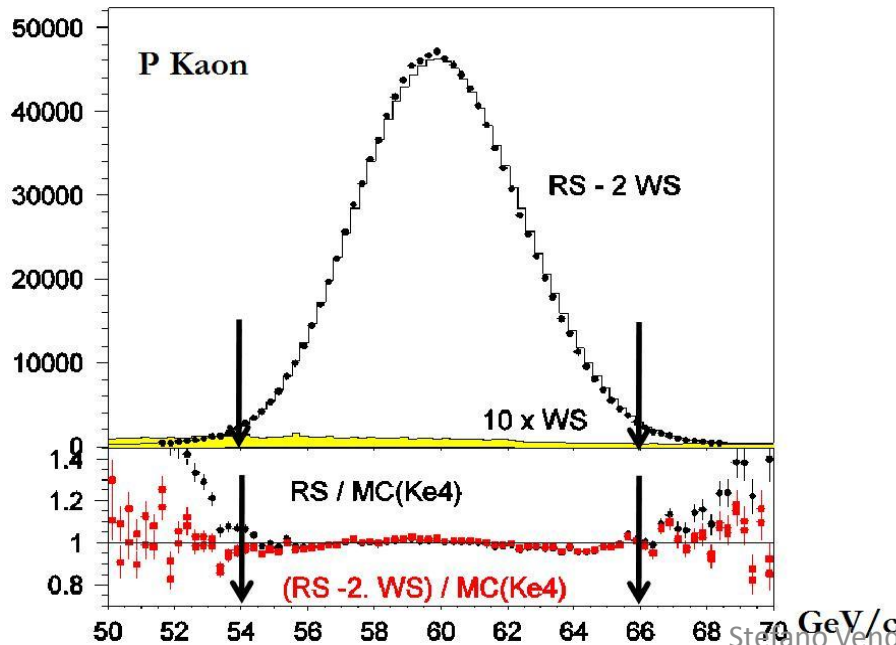
- Spectrometer for momenta measurement;
- LK_R info used to tag electron and pions (E/p);
- Missing energy and Pt cuts.

Main BG sources:

- $\pi^\pm\pi^+\pi^-$, with $\pi \rightarrow e\nu$ in-flight decay or π misidentified as e;
- $\pi^\pm\pi^0, \pi^\pm\pi^0\pi^0$, with $\pi^0 \rightarrow e^+e^-\gamma$ Dalitz decay, π^\pm/e^\pm misID and $\gamma(s)$ undetected.



Elliptic cut on $(M_{3\pi}, p_T)$ plane to reject BG, residual BG at the level of 0.5%, checked using wrong sign events (es: $K^- \rightarrow e^+\pi^-\pi^-\nu, K^+ \rightarrow e^-\pi^+\pi^+\nu$)



Ke4: Fitting procedure

Iso-populated $10(M_{\pi\pi}) \times 5(\text{Mev}) \times 5(\cos\theta_e) \times 5(\cos\theta_\pi) \times 12(\Phi) = 15000$ bins used. Form factor values minimize a DATA-MC estimator well-suited for small numbers.

Ten independent fits in $M_{\pi\pi}$ bins, assuming constant f.f. over single bins.

K^+ : **Data**: 0.72M events, 48 evts/bin • No BR measurement → only relative f.f.

MC: 17.7 M events, ~1160 evts/bin (e.g. F_p/F_s) and their variation wrt

K^- : **Data**: 0.40 M events, 27 evts/bin kinematical variables are measurable;

MC: 9.8 M events, ~373 evts/bin

- Fits in 4-dim space for K^+/K^- separately;
- Residual variation (linear slope) observed wrt S_e for F_s ;

$K^+/K^- \sim 1.8$ (data & MC)

MC/Data ~ 23 (K^+ & K^-)

- $\langle \text{Relative normalization} \rangle = 1$ by F_s rescaling;
- f.f. values deconvoluted of F_s variation.

$$f'_s/f_s = 0.152 \pm 0.007 \pm 0.005$$

$$f''_s/f_s = -0.073 \pm 0.007 \pm 0.006$$

$$f'_e/f_s = 0.068 \pm 0.006 \pm 0.007$$

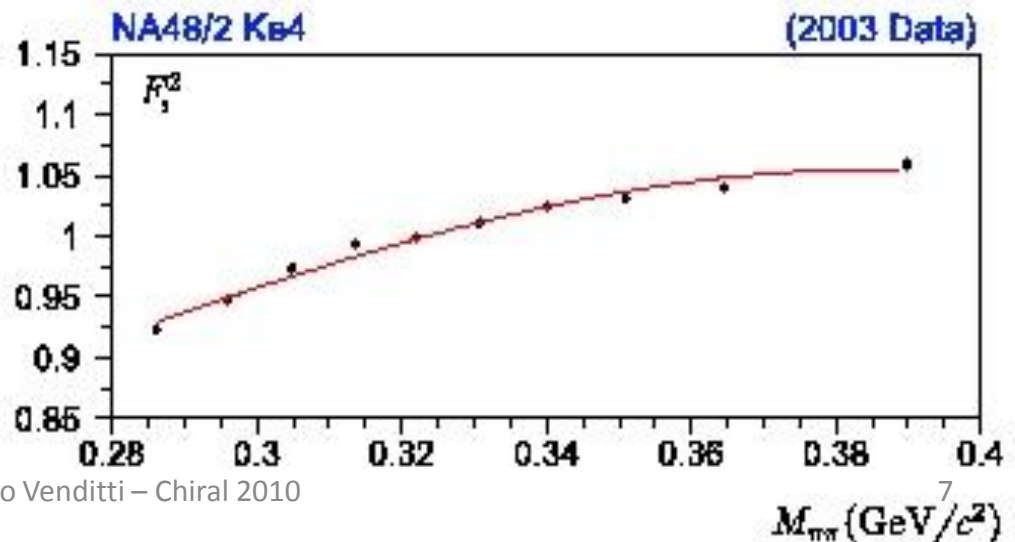
$$f_p/f_s = -0.048 \pm 0.003 \pm 0.004$$

$$g_p/f_s = 0.868 \pm 0.010 \pm 0.010$$

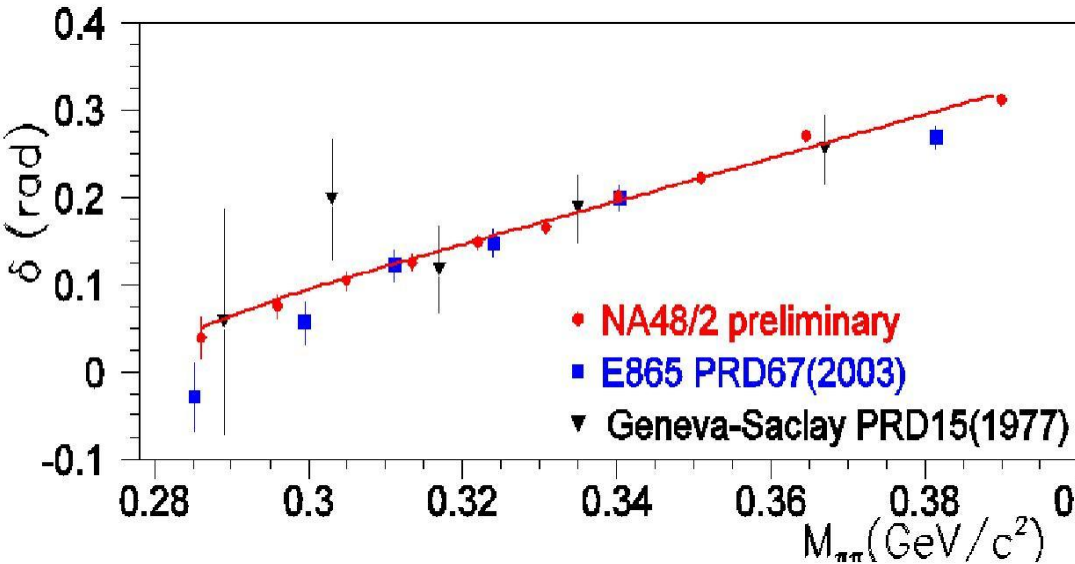
$$g'_p/f_s = 0.089 \pm 0.017 \pm 0.013$$

$$h_p/f_s = -0.398 \pm 0.015 \pm 0.008$$

$$F_s^2 \sim f_s^2 (1 + f'_s q^2 + f''_s q^4 + f'_e S_e / 4m_\pi^2) / (1 + f'_e S_e / 4m_\pi^2), \quad q^2 = (S_\pi / 4m_\pi^2) - 1$$



Ke4: δ phase shift and a_0, a_2 extraction



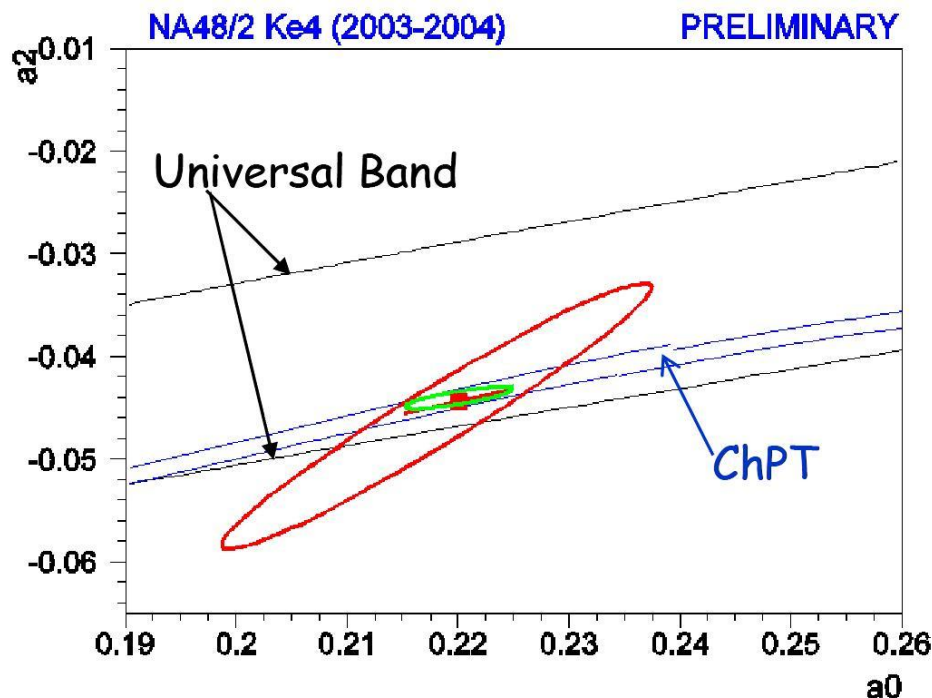
- Roy equations relate $\pi\pi$ phase difference $\delta = (\delta_0^0 - \delta_1^1)$ variation at threshold to $\pi\pi$ s-wave scattering lengths at $I=0$ (a_0) and 2 (a_2) using experimental data at $M_{\pi\pi} > 0.8$ GeV (1,2,3);
- The Universal Band parameterization is a constraint in the (a_0, a_2) plane, i.e. a 1-dim δ fit with a fixed relation between a_0 and a_2 ;
- An additional constraint in the (a_0, a_2) plane comes from ChPT:

$$a_2 = -0.0444 + 0.236(a_0 - 0.22) - 0.61(a_0 - 0.22)^2 - 9.9(a_0 - 0.22)^3 \pm 0.0008$$

- Effect of Coulomb correction (Gamow factor) and real photons included (PHOTOS);
- Isospin correction prescription given by Gasser (4) results in 10 to 15 mrad shift in fitted $M_{\pi\pi}$ range;
- 2 par. fit translates a 68% CL contour in the (a_0, a_2) plane;
- Isospin corrections \sim error in each point

- 1)[Ananthanarayan, Colangelo, Gasser, Leutwyler Phys.Rept.353:207-279 (2001)]
- 2)[Descotes-Genon, Fuchs, Girlanda, Stern Eur.Phys.J.C24:469-483,2002]
- 3)[Kaminski, Pelaez, Yndurain Phys.Rev.D77 (2008)]
- 4)[Gasser et al. Eur.Phys.J. C59:777,2009]

Ke4: a_0 and a_2 results



- Theory prediction from ChPT:
[Colangelo, Gasser, Leutwyler Nucl.Phys.B603,2001]
[Colangelo, Gasser, Leutwyler Phys.Rev.Lett.86,2001]:

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0008$$

ChPT constraint (1 par. fit): $a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{th}}$
 2 par. fit: $a_0 = 0.2220 \pm 0.0128_{\text{stat}} \pm 0.0050_{\text{syst}} \pm 0.0037_{\text{th}}$
 $a_2 = -0.0432 \pm 0.0086_{\text{stat}} \pm 0.0034_{\text{syst}} \pm 0.0028_{\text{th}}$ (97% CORRELATION)

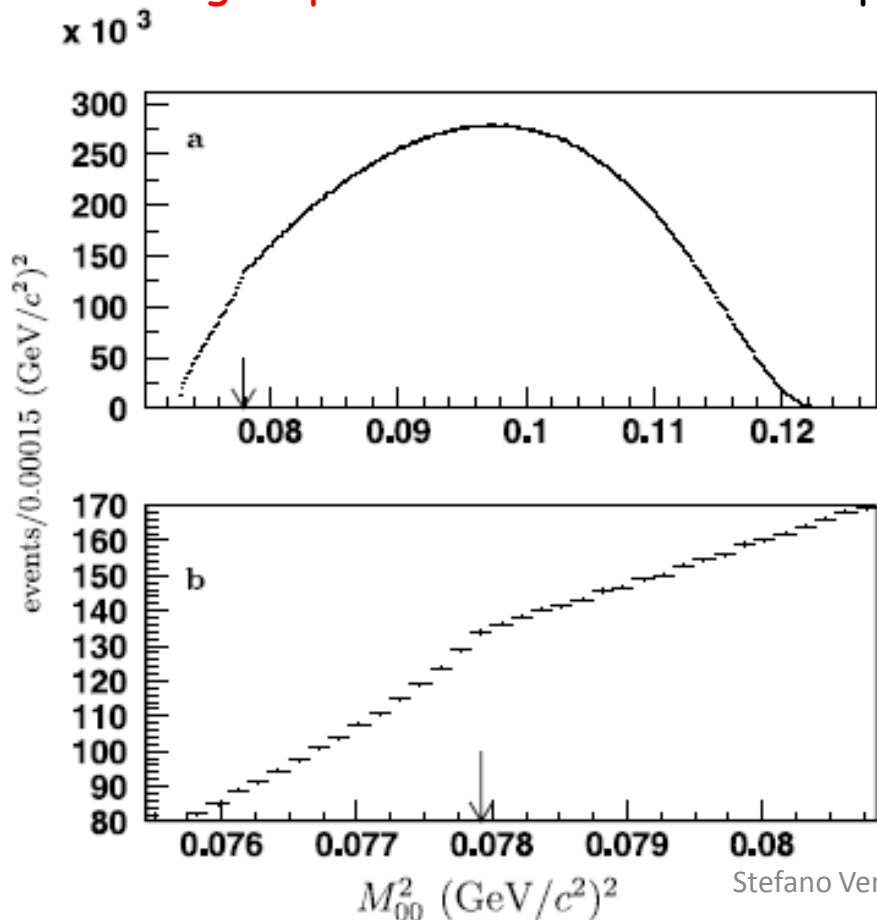
Theoretical error computed from isospin corrections and Roy equation inputs
 [Gasser et al. Eur.Phys.J. C59:777,209]

Model independent procedure allows to use data from NA48/2 and past experiments:

$$a_0 = 0.2199 \pm 0.0125_{\text{exp}} \pm 0.0037_{\text{th}} \quad a_2 = -0.0430 \pm 0.0083_{\text{exp}} \pm 0.0028_{\text{th}}$$

Cusp effect in Kpi3n decay

- Main goal of Kpi3n analysis: measure of CPV asymmetry [Eur.Phys.J.C52(2007)];
- "Bonus" goal: cusp-like structure in $\pi^0\pi^0$ invariant mass (M_{00}) distribution, due to $\pi^+\pi^-\rightarrow\pi^0\pi^0$ rescattering;
- Effect of $\pi\pi$ rescattering expected to show only at $M_{00}\sim 2M_{\pi^+}$; evidence for an **interfering amplitude** over the whole spectrum was found.



THEORETICAL MODELS

- Budini & Fonda (1961): first prediction of a possible effect due to $\pi^+\pi^-\rightarrow\pi^0\pi^0$ rescattering in Kpi3n decay [PRL 6, 419(1961)];
- Cabibbo Isidori (CI) approach, using decay amplitude expansion in powers of $\pi\pi$ scattering lengths;
- Bern-Bonn approach, based on a non-relativistic Lagrangian framework

Both methods require data from $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ (Kpi3c) decay, also measured by NA48/2 (4.7×10^8 events sample used)

Theoretical models: CI approach

Kpi3n and Kpi3c amplitudes at tree level can be parameterized as:

$$M_0 = 1 + \frac{1}{2} g_0 u + \frac{1}{2} h_0 u^2 + \frac{1}{2} k_0 v^2 \quad M_+ = 1 + \frac{1}{2} g_+ u + \frac{1}{2} h_+ u^2 + \frac{1}{2} k_+ v^2 \quad u, v = \text{phase space variables}$$

- One-Loop calculation:

Cabibbo, PRL 93 (2004) 121801

$$M(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$$

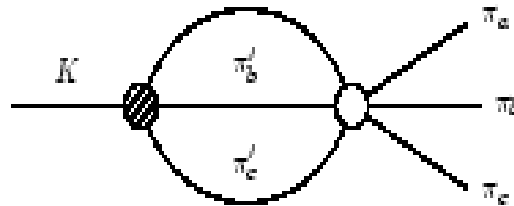
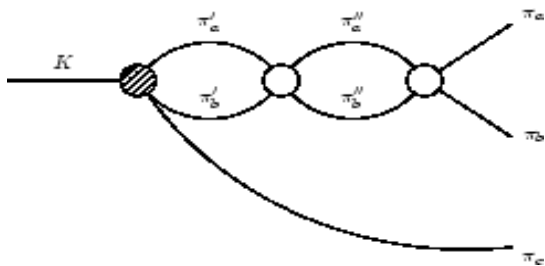
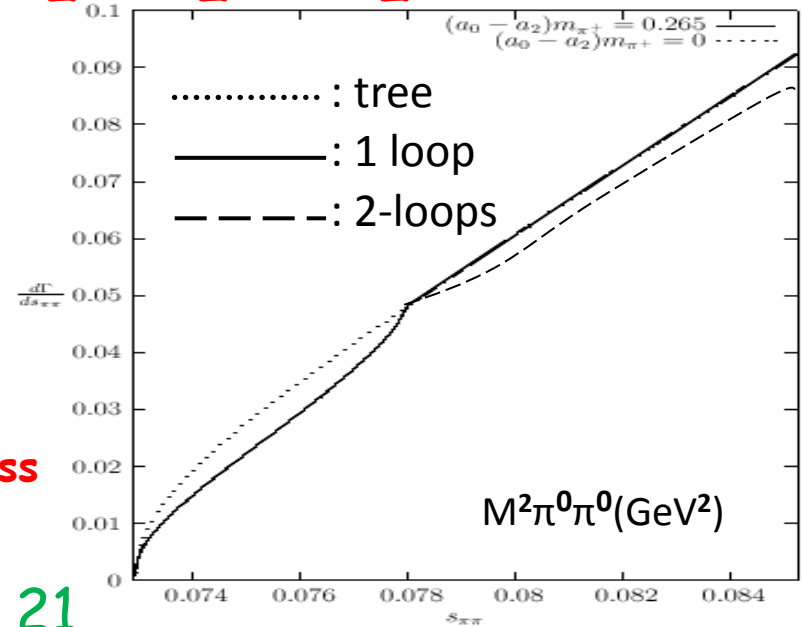
$$M_1 = -2a_x m_+ M_+ \sqrt{\left(\frac{2m_+}{M_{00}}\right)^2 - 1}$$

$a_x = (a_0 - a_2)/3$ if Isospin sym. holds, $m_+ = \pi^+$ mass

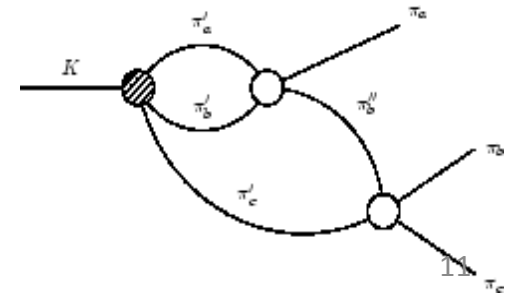
- Two-Loop calculation:

Cabibbo, Isidori, JHEP 503 (2005) 21

- Pion rescattering effects are computed in terms of 5 S-wave scattering lengths, which are function of only a_0 and a_2 if isospin symmetry holds;
- Radiative corrections not included



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Theoretical models: BB approach

Colangelo, Gasser, Kubis, Rusetsky *Phys.Lett.B638:187-194,2006* Bissegger, Fuhrer, Gasser, Kubis, Rusetsky *NPH B806:178, 2009*

The model describes the decay amplitude in terms of 2 parameters:

a = generic $\pi\pi$ amplitude at threshold

ε = formal parameter such that $P_{\pi} \sim \varepsilon$ and $T_{\pi} \sim \varepsilon^2$

- Simultaneous fit of M^2_{00} and $M^2_{\pm\pm}$ needed ($M^2_{\pm\pm}$ fixed in CI approach);
- EM effects are naturally included in this approach;
- Calculation include terms up to 2 loops;
- Radiative corrections outside the cusp point computed in 2009.

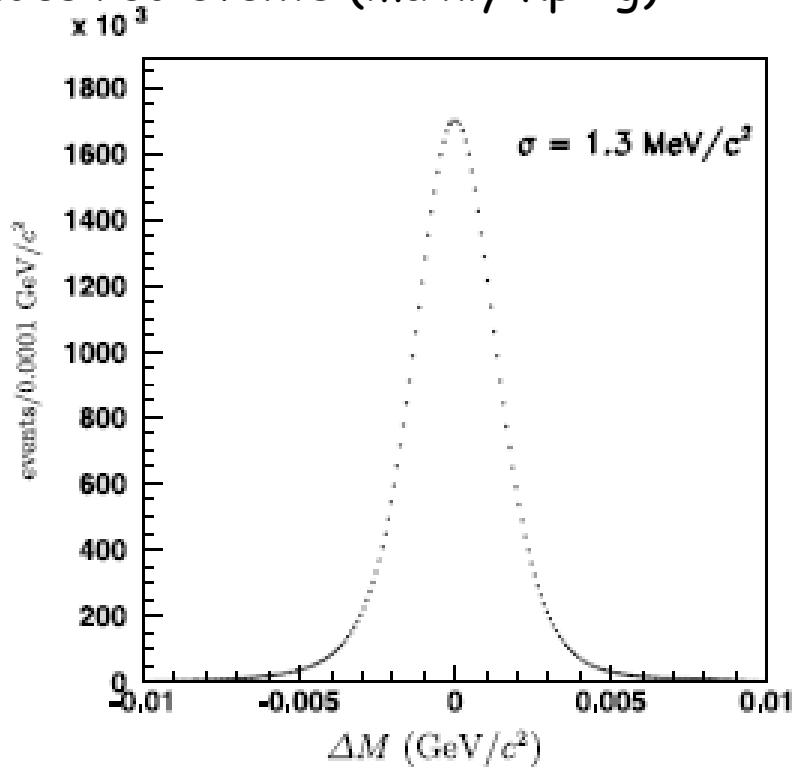
Analysis

A dedicated trigger was used for NA48/2 to collect $O(10^9)$ $K\pi 3n$ events in order to measure the neutral CPV asymmetry.

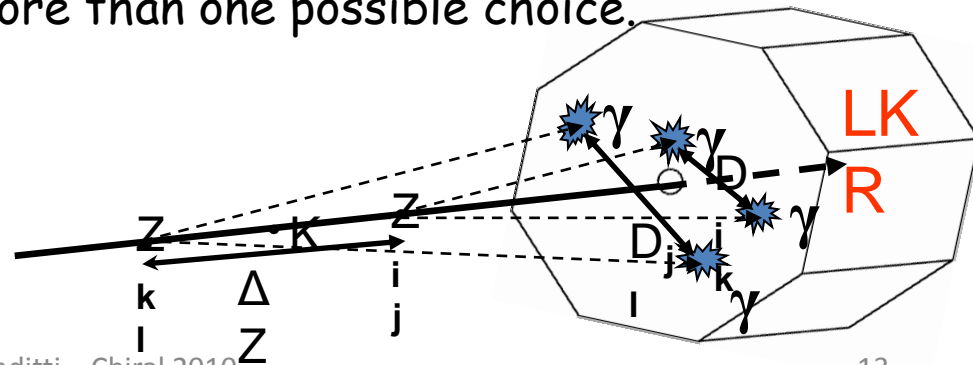
L0 Trigger: Signal in the hodoscope and at least 2 energy deposits in X or Y views of the LKr required;

L1 Trigger: Online cut on charged pion missing mass to reduce collection of undesired events (mainly $K\pi 2g$)

OFFLINE ANALYSIS



- 1 Charged track ($P > 5 \text{ GeV}/c$), at least 4 LKr clusters ($E > 3 \text{ GeV}$);
- The 2 neutral vertices (from imposing M_{π^0}) must differ less than 500 cm, decay vertex from their average;
- Cuts on acceptance and total energy;
- χ^2 minimization technique used in case of more than one possible choice.



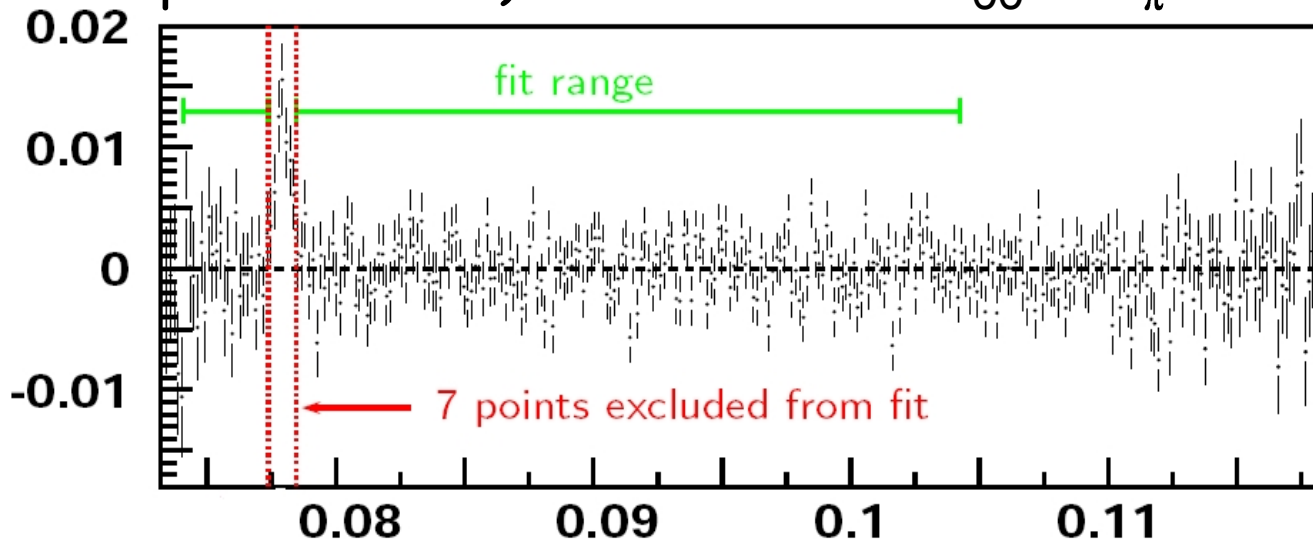
Difference between reconstructed and nominal K mass. Small tails due to $\pi \rightarrow \mu \nu$ decay and γ mispairing ($\sim 0.19\%$)

Fitting procedure

- Detector response computed with a full GEANT3 MC. Resolution matrix applied on MC-generated data → **theoretical distribution can be turned into expected distribution** and compared with data;
- 5 free parameters ($g_0, h_0, m_+(a_0-a_2), m_+a_2, N_1$) for CI, 9 (+ g, h, k and N_2 from $K\pi 3c$ decay) for BB;
- Minimization of χ^2 based on Data-MC difference:

$$\chi^2(g_0, h_0, \dots) = \sum_{\text{bins}} \frac{(F_{\text{data}} - N F_{\text{MC}})^2}{\delta F_{\text{data}}^2 + \delta^2 N^2 F_{\text{MC}}^2}$$

- Two fits: **1)** all bins, with normalization of the bin at $M_{00} = 2M_\pi$ as free parameter **2)** 7 bins around $M_{00} = 2M_\pi$ excluded from the fit.



EXCESS OF EVENTS AT $M_{00} = 2M_\pi$ ($R = 1.8 \cdot 10^{-3}$):

- PIONIUM ($\sim 0.8 \cdot 10^{-3}$)
Silagadze, JETP Lett. 60:689, 1994
- UNBOUND STATES
Gevorkian et al., Phys.Lett.B 649:159, 2007

FINAL RESULTS

$$(a_0 - a_2)m_+ = 0.2571 \pm 0.0048_{\text{stat.}} \pm 0.0025_{\text{syst.}} \pm 0.014_{\text{ext.}}$$
$$a_2 m_+ = -0.024 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.002_{\text{ext.}}$$

RESULTS BASED ON BB APPROACH

Using the chiral symmetry constraint [Colangelo et al., PRL 86 (2001) 5008]:

$$a_2 = -0.0444 + 0.236(a_0 - 0.22) - 0.61(a_0 - 0.22)^2 - 9.9(a_0 - 0.22)^3$$

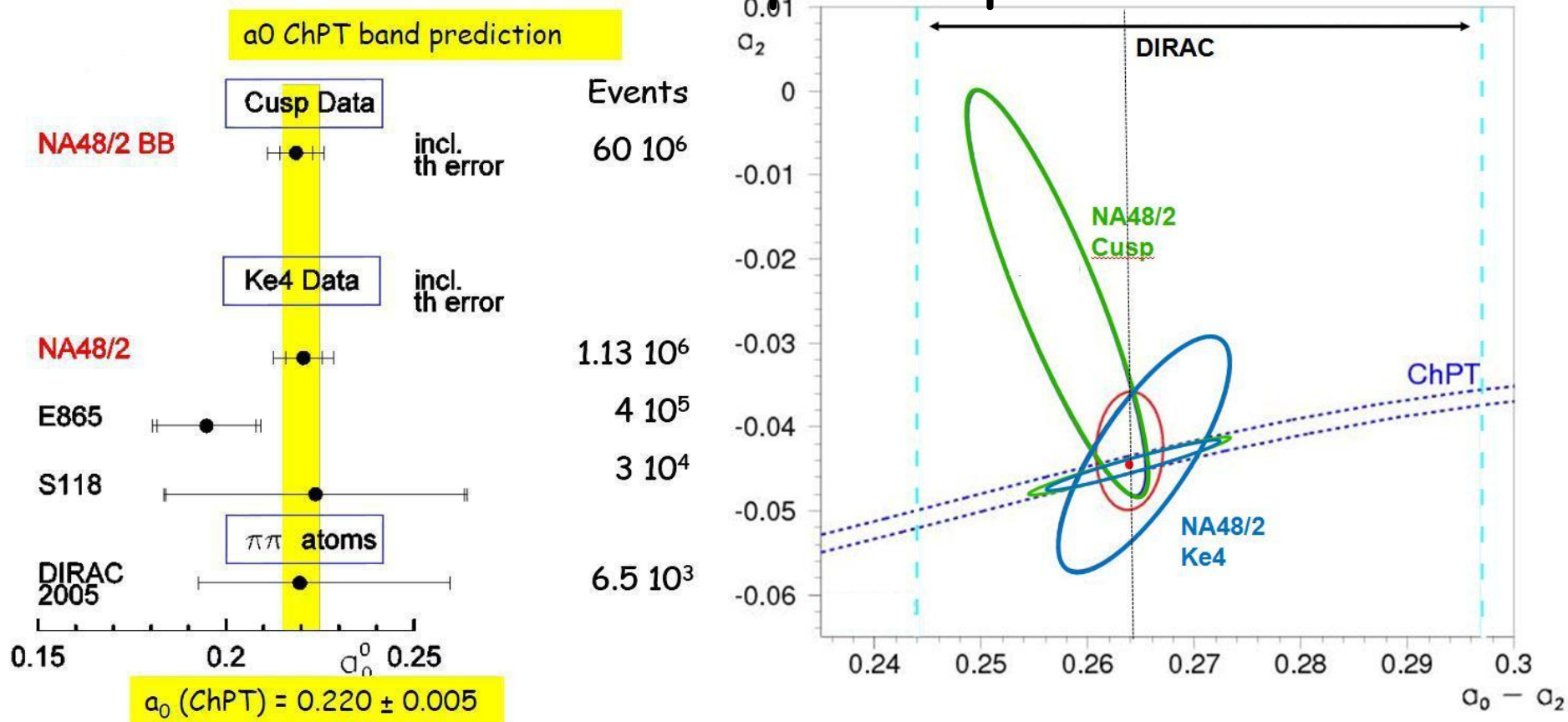
$$(a_0 - a_2)m_+ = 0.2633 \pm 0.0024_{\text{stat.}} \pm 0.0014_{\text{syst.}} \pm 0.019_{\text{ext.}}$$

Eur. Phys. J. C Vol. 64, 4 (2009)

Main systematics: Acceptance, trigger efficiency, LKr non-linearity

Main external uncertainty from $\Gamma(K\pi^3c)/\Gamma(K\pi^3n)$ ratio = 3.175 ± 0.050

Ke4 and cusp: Comparison



COMBINED Ke4 AND CUSP RESULT:

$$(a_0 - a_2)m_+ = 0.2639 \pm 0.0020 \pm 0.0004 \pm 0.0021$$

$$(a_0 - a_2)m_+ = 0.2640 \pm 0.0020 \pm 0.0017 \pm 0.0035 \quad (\text{ChPT constraint})$$

DIRAC RESULT (pionium lifetime) in good agreement:

$$(a_0 - a_2)m_+ = 0.264^{+0.033}_{-0.020}$$

Conclusions

- Accurate study of low-energy hadronic interactions thanks to the unprecedented statistics available in NA48/2;
- Two independent estimations of pion scattering lengths using $K^\pm \rightarrow e^\pm \nu \pi^+ \pi^-$ decay and M_{00} fit in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decay;
- Good agreement between the two results;