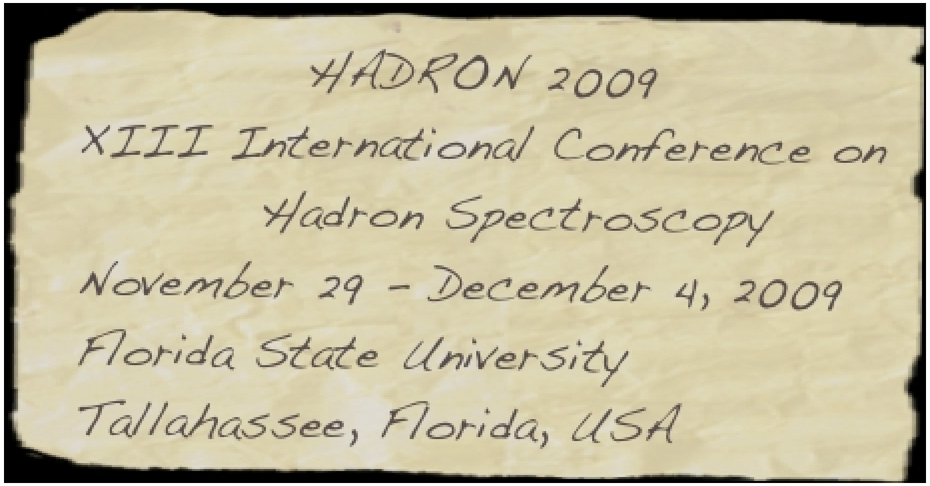
Pion-Pion scattering length measurement at NA48/2

Roberta Arcidiacono

University of Turin and INFN

On behalf of the NA48/2 collaboration

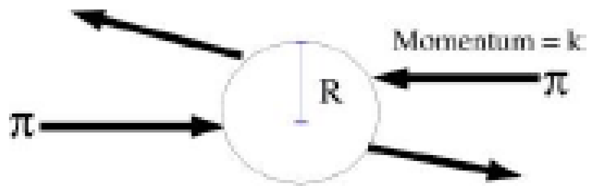
Cambridge, CERN, Chicago, Dubna, Edimburgh, Ferrara, Florence, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Turin, Vienna



Outline

- Why and How?
- The CERN NA48/2 experiment
- $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$ decays: the "cusp" effect
- Analysis: extraction of the $\pi\pi$ scattering length
- Results
- Conclusions

Why is ππ scattering length interesting?



At low energy $kr << 1$ S-wave dominates total cross section

Scattering matrix $S|\pi\pi> \doteq \exp(2i\delta)|\pi\pi>$ may be parametrized with 2 phases

$\delta_{0,2} = a_{0,2}k + O(k^2)$ (I = 0,2 allowed)

a_0 and a_2 are S-Wave scattering lengths in isospin I=0,2

At low energy the S-wave scattering lengths are essential parameters of Chiral Perturbation Theory (ChPT)

How to measure $\pi\pi$ scattering lengths

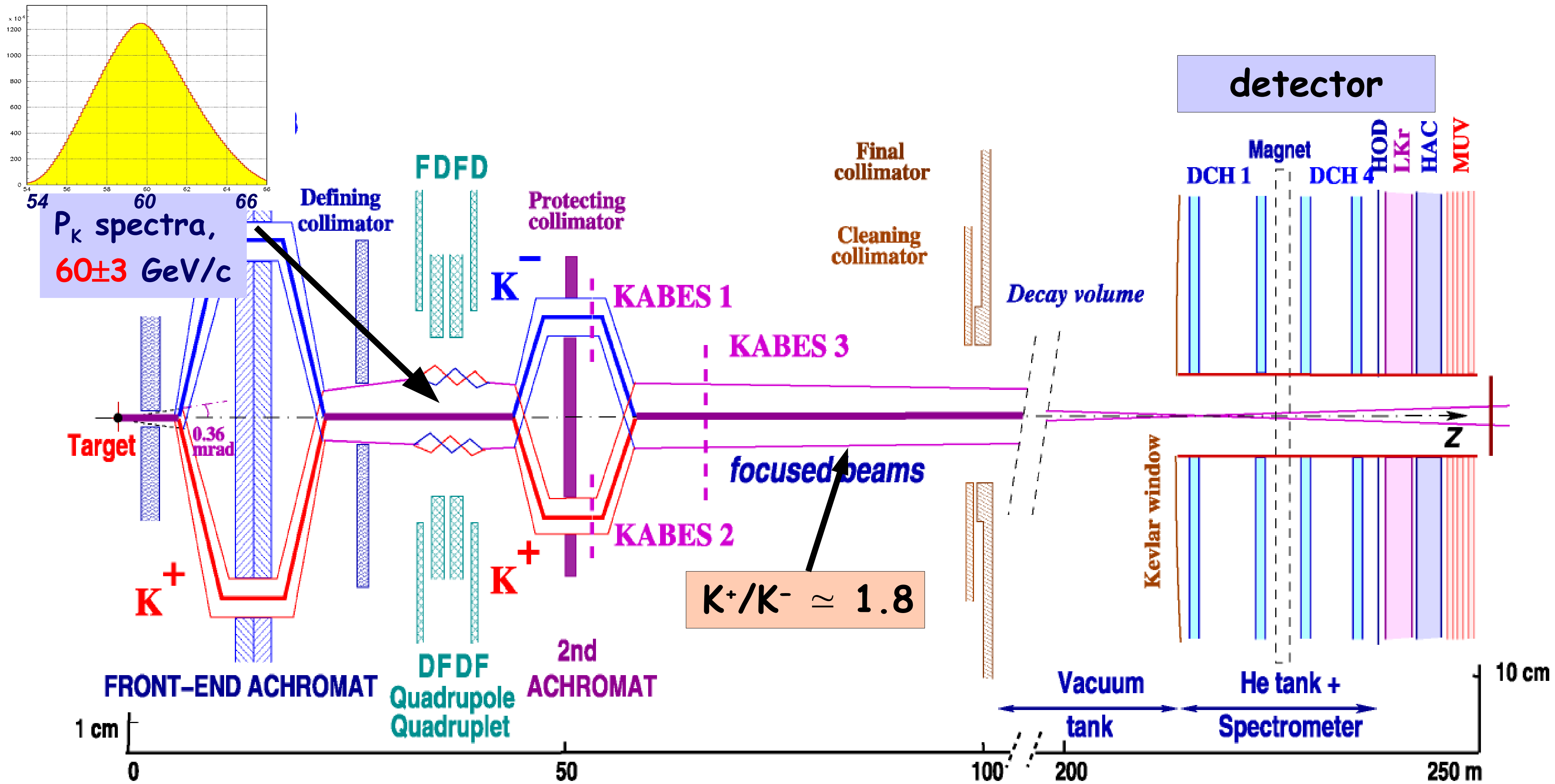
3 types of measurements performed so far:

- Ke4 decays $BR(K^{\pm}\to\pi^{+}\pi^{-}e^{\pm}\nu)= (4.09\pm0.09)\cdot 10^{-5}$
  - Geneva-Saclay CERN/PS S118: $3\cdot10^{4}$ (1977)
  - E685 @BNL: $4\cdot10^{5}$ (2003)
  - NA48/2 @CERN/SPS: $1.15\cdot10^{6}$ (2008)

See B.Peyaud talk [in P.5D]

- Pionium atoms   DIRAC CERN/SPS $\pi\pi$ lifetime
- K3π modes (cusp)
  - $BR(K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0})=(1.757\pm0.024)\cdot10^{-2}$
    - NA48/2 @CERN/SPS: $16\cdot10^{6}$ (2006), $60\cdot10^{6}$ (2008)
  - $BR(K_{L}\to\pi^{0}\pi^{0}\pi^{0})=(19.56\pm0.14)\cdot10^{-2}$
    - KTeV ($68\cdot10^{6}$) and NA48 ($100\cdot10^{6}$) work in progress

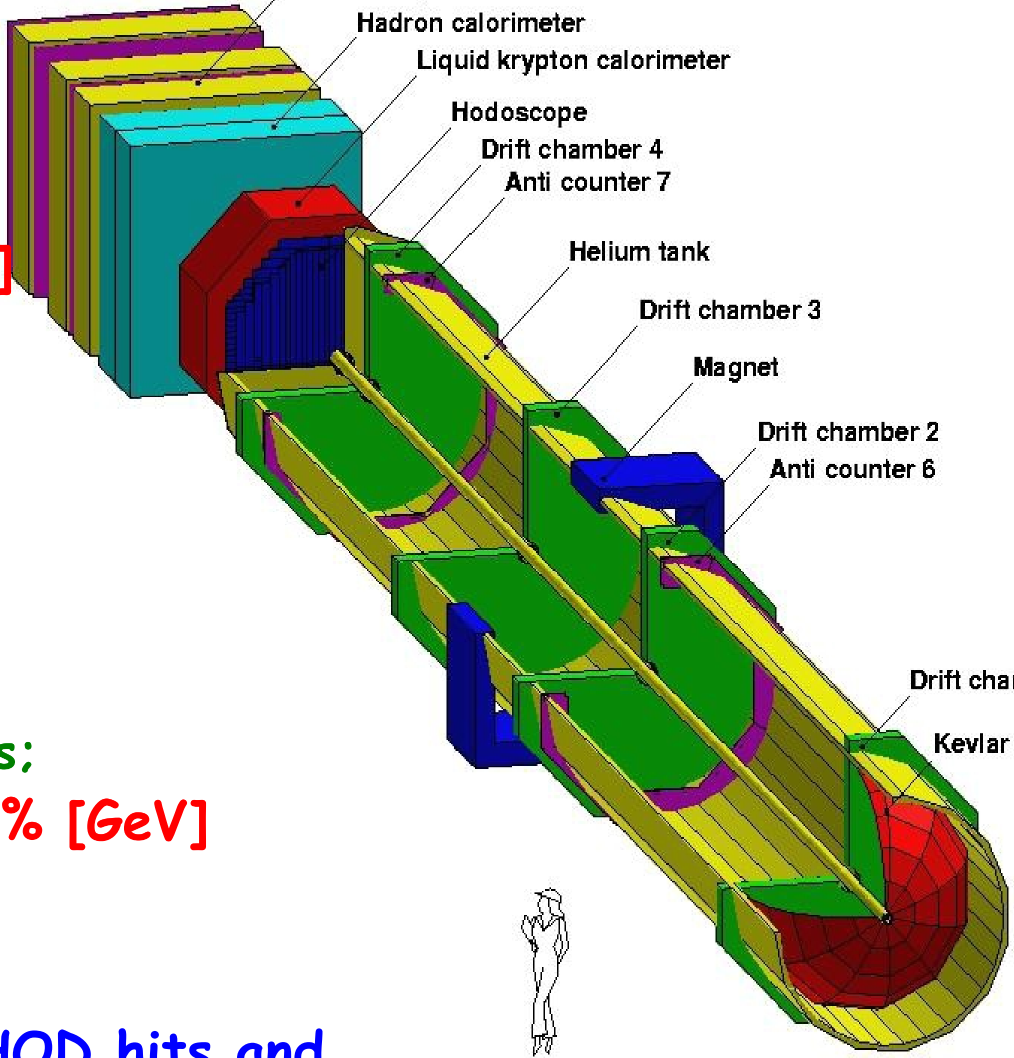
NA48/2: beam line



- Simultaneous K^+/K^- beams, very similar acceptance, ~7×10^{11} ppp, 6.3×10^7 particles per pulse in decay region

NA48/2: detector

- **Magnetic spectrometer (4 DCHs):**
  4 views/DCH → high efficiency;
  $\sigma_p/p = 1.0\% + 0.044\% * p$ [GeV/c]



- **Hodoscopes (neutral & charged)**
  fast trigger; precise time measurement (150ps).

- **Liquid Krypton EM calorimeter**
  High granularity, quasi-homogeneous;
  $\sigma_E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$ [GeV]
  $\sigma_x = \sigma_y \sim 1.5$mm for E=10 GeV

- **Triggers based on LKr peaks, CHOD hits and DCH multiplicity**

NA48/2 data taking

Main goal of NA48/2:
→ search for CP violation in $K^{\pm} \rightarrow 3\pi$

Detector and trigger optimized to collect 3 pions events. Unprecedented statistics in many channels

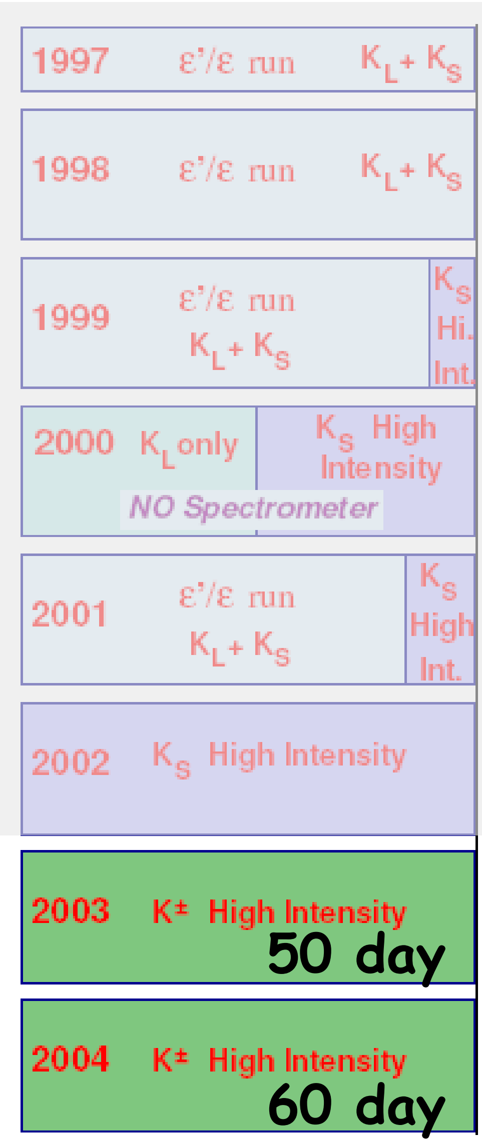
$K^{\pm} \rightarrow \pi^{+} \pi^{-} \pi^{\pm}$: **~ $4 \cdot 10^{9}$**

$K^{\pm} \rightarrow \pi^{0} \pi^{0} \pi^{\pm}$: **~ $1 \cdot 10^{8}$**

$K^{\pm} \rightarrow \pi^{+} \pi^{-} e^{\pm} \nu$: **~ $1.1 \cdot 10^{6}$**

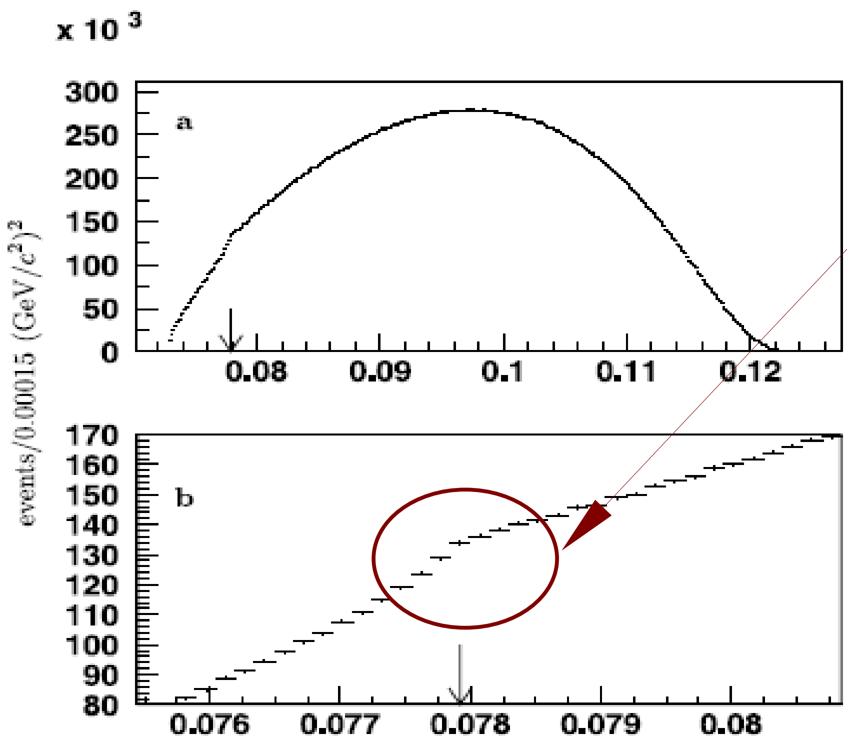
Ke4(2003) EPJ 54 (2008) 411 : ~ $6.7 \cdot 10^{5}$
Cusp(2003) Phys.Lett.B, 633(2006) 173 : ~ $1.6 \cdot 10^{7}$

Results on Cusp from the full sample presented here [Eur. Phys. J. C (2009) 64: 589–608]



"Cusp" in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$: evidence

Thanks to the big statistics collected by NA48/2 and the excellent calorimeter energy resolution, for the first time a structure has been observed in the M^{2}_{00} [already in the 2003 data], at the $\pi\pi$ threshold, not explainable by instrumental effects .



M^{2}_{00} in bin width of 0.00015 $(GeV/c^{2})^{2}$, with the 51st bin centered at $(2m+)^{2}$

A small pionium peak was expected … a CUSP was found!

First interpreted as due to the $\pi\pi$ charge exchange final state rescattering in the $K^{\pm} \rightarrow \pi+\pi-\pi^{\pm}$ [Cabibbo PRL 93,2004]

[Budini & Fonda – 1961] A general method which allows the determination of certain cross sections which are <u>out of reach of direct experimental measurement</u> is to observe a threshold effect of the kind of a cusp in the spectrum ...

"Cusp" interpretation - (CI)

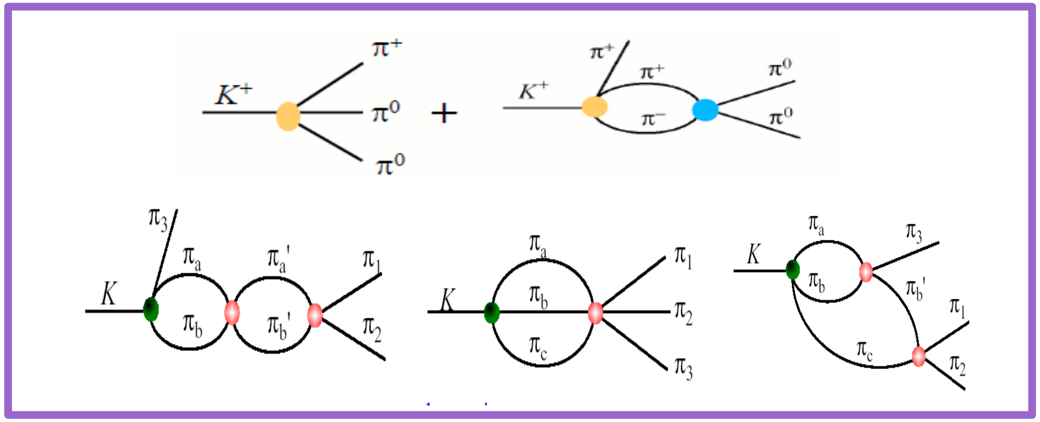
Cabibbo-Isidori approach

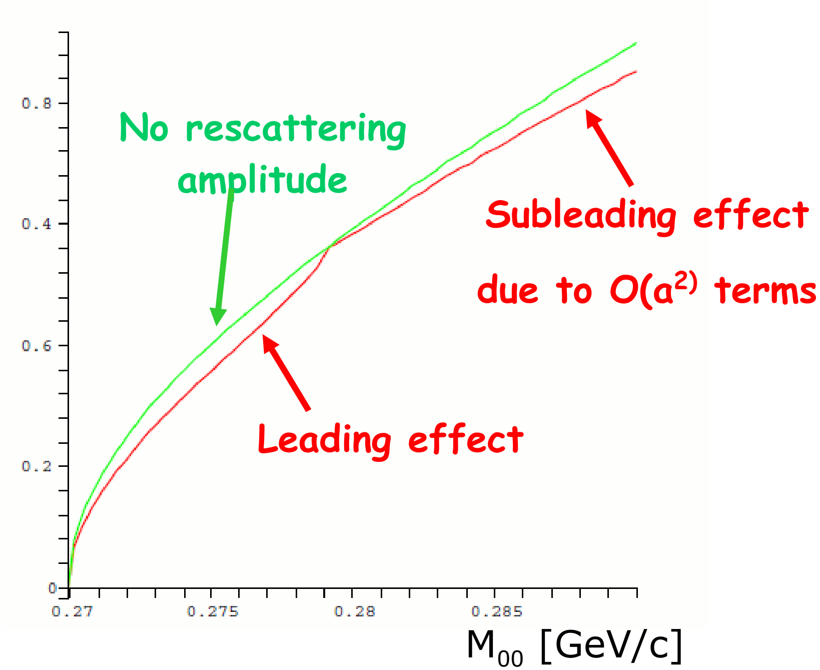
One loop calculation:

Cabibbo, PRL 93 (2004) 121801

Two loop calculation:

Cabibbo , Isidori, JHEP 503 (2005) 21





$$M\ (K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$$

M_0 → (no rescattering) standard expansion

$$M_0 = A_0(1 + g_0 u/2 + h_0 u^2/2 + k_0 v^2/2)$$

M_1 → real below threshold and imaginary above. Destructive interference below 2m+ threshold.

$$\mathcal{M}_1 = -2/3(a_0 - a_2) m_+ \mathcal{M}_+ \sqrt{1 - \frac{(M_{00})^2}{2m_+}}$$

with $M_+ = M\ (K^\pm \rightarrow \pi^\pm \pi^+ \pi^-)$

"Cusp" interpretation - (BB)

Bern-Bonn approach

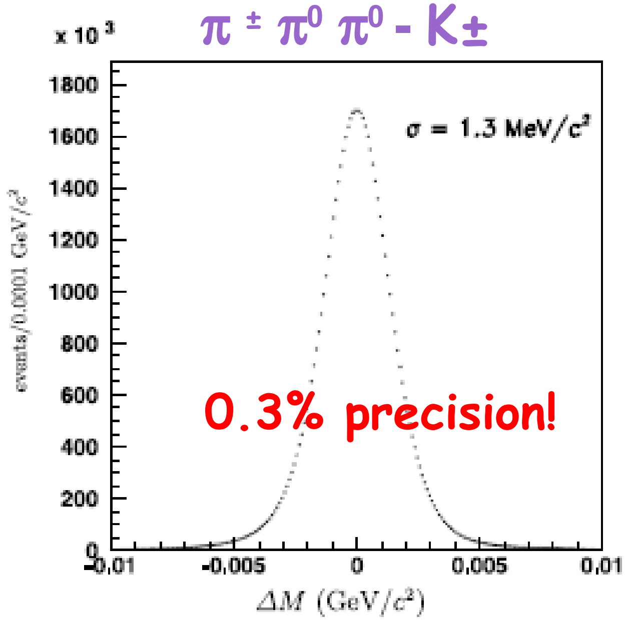
Colangelo, Gasser, Kubis, Rusetsky
Phys.Lett.B638:187-194,2006
Bissegger, Fuhrer, Gasser, Kubis, Rusetsky
NPH B806:178, 2009

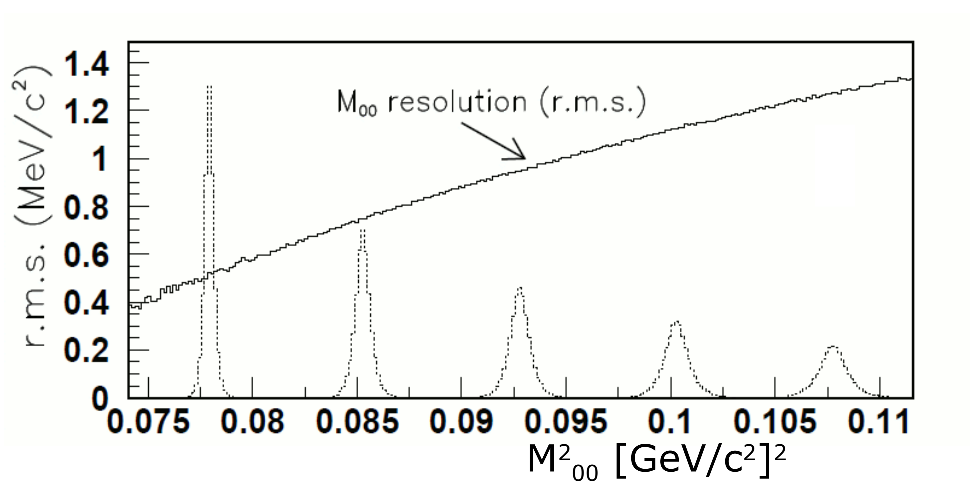
- Model based on a non-relativistic effective Lagrangian, using two expansion parameters :
  - **a** = generic $\pi\pi$ scattering length at threshold
  - **ε** = formal parameter such that pion momentum is of order $O(\varepsilon)$ and pion kinetic energy of order $O(\varepsilon^2)$

The formulation includes terms up to $O(\varepsilon^4, a\varepsilon^3, a^2\varepsilon^2)$ corresponding to 1-loop and 2-loops calculation. Valid over the full physical region.

- The electromagnetic effects are naturally included in this approach
- **Simultaneous fit of charged and neutral amplitude to extract M_+ slope parameters**
- In 2009, **radiative correction calculated** outside the cusp point

Analysis: events selection





Online selection:

•**L1** - one charged particle in CHOD and some minimum energy in the LKr (consistent with $2\pi^0$)

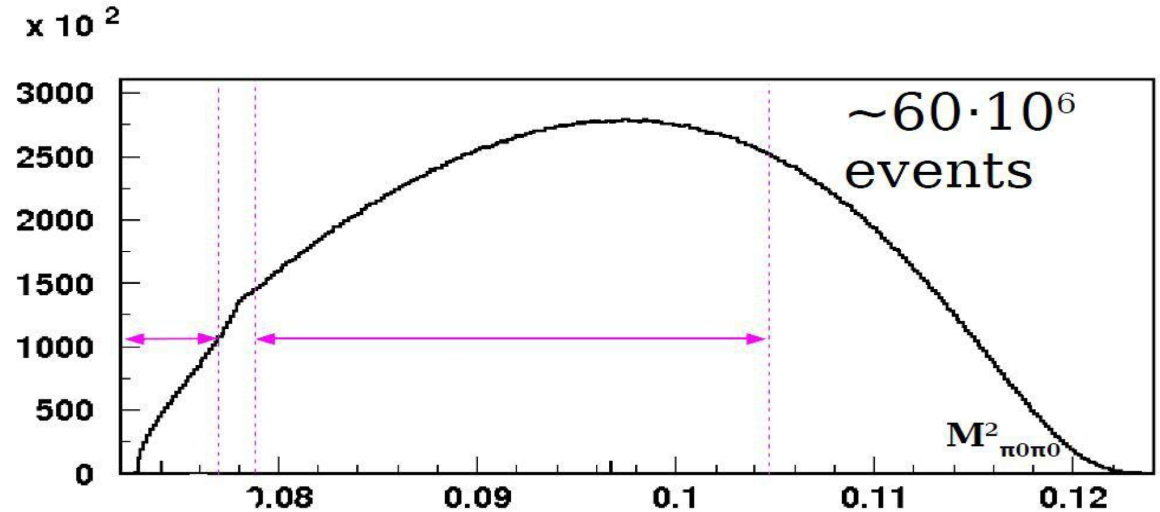
•**L2** -The missing mass seen by DCH (hyp. of $K^{\pm}$ decay) not consistent with one π^0

Offline selection:

- require 1 track + 4 e.m. clusters
- Cluster pairing ($\pi^0 \rightarrow \gamma\gamma$): consider all combination, reconstruct each $\pi^0 \rightarrow \gamma\gamma$ vertex using π^0 mass constraint $Z^2_{ik} = E_i E_k d^2_{ik} / m^2_{\pi 0}$
- choose 2 combinations with closest vtx
- K decay vertex = average of π^0 vertices
- Cuts on $K^{\pm}$ energy and fiducial cuts

Analysis: Fitting procedure

- fit the M^2_{00} distribution
- resolution and detector response matrix from accurate Geant3 simulation

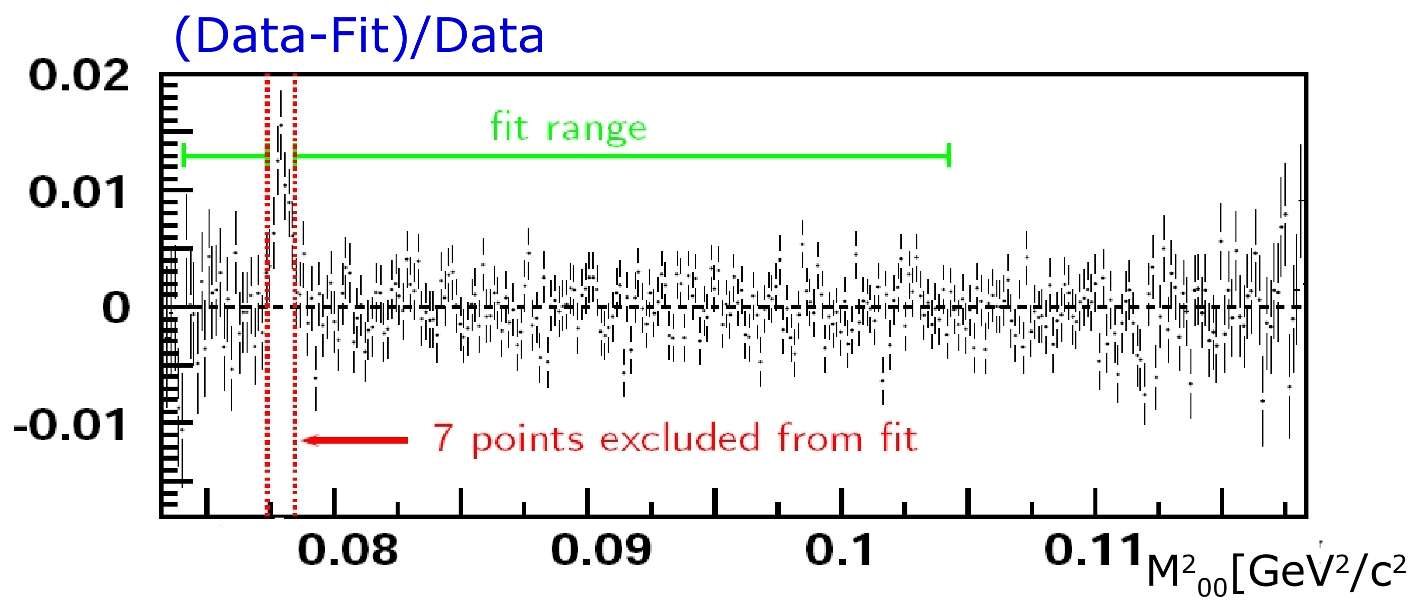


$M_0 = A_0(1+g_0u/2+h_0u^2/2+k_0v^2/2)$ u,v = Dalitz variables

- Fit parameters → (g, h) for M_0 (a0-a2, a2) for M_1, M_2, N
- The M_+ term in the CI theory is fixed by the recently measured $K^\pm\to\pi^\pm\pi^+\pi^-$ slope parameters [Phys.Lett.B649:349-358,2007]
- In the BB approach, the M_+ term is obtained simultaneously fitting M^2_{00} and $M^2_{\pm\pm}$ (from 470M $K^\pm\to\pi^\pm\pi^+\pi^-$ decays)

Analysis: Fitting procedure

The excess of events in the 2m+ position, $R=(1.8\pm0.3)\cdot10^{-5}$ can be explained by:



- **pionium (prediction $0.8\cdot10^{-5}$)** [Silagadze, JETP Lett. 60:689, 1994]
- **unbound state with resonant structure** [Gevorkian, Tarasov, Voskeresenskaya, Phys.Lett.B 649:159, 2007]

Two types of Fit :

- All bins, with f_{atoms} (relative excess in threshold bin) as free parameter

- 7 bins around cusp excluded and f_{atoms} fixed

Analysis: Results

2 param. Fit:

$(a0-a2)m_+ = 0.2571 \pm 0.0048_{stat} \pm 0.0025_{syst} \pm 0.0014_{ext}$

$a2m_+ = -0.024 \pm 0.013_{stat} \pm 0.009_{syst} \pm 0.002_{ext}$

Results based on BB model (better $\chi 2$, most complete theory)

Systematic uncertainties dominated by LKr non-linearity, trigger efficiency, Kaon P spectrum

External uncertainty due to $\Gamma+-/\Gamma 00=3.175\pm0.050$ taken from PDG

With ChPT constraint - 1 param Fit:

[Colangelo et al., PRL 86 (2001) 5008]:

$(a0-a2)m_+ = 0.2633 \pm 0.0024_{stat} \pm 0.0014_{syst} \pm 0.0019_{ext}$

Cusp and Ke4: comparison

Two independent measurements

- 60 M K3π
- 1.13 M Ke4

Different systematics

- Cusp: Calorimeter and trigger
- Ke4: electron misID and Background

Different theoretical inputs

- Cusp: rescattering in final state and ChPT expansion
- Ke4: Roy equation and isospin breaking correction

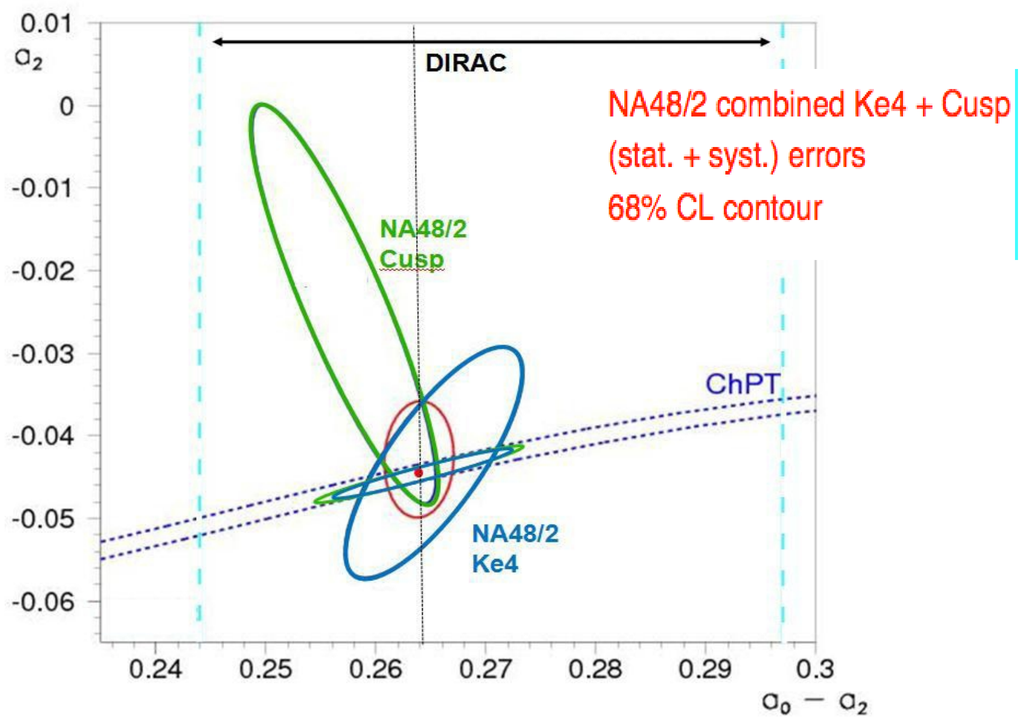
Very good agreement with ChPT prediction:
$(a0-a2)m_+ = 0.265 \pm 0.004$

Combined results

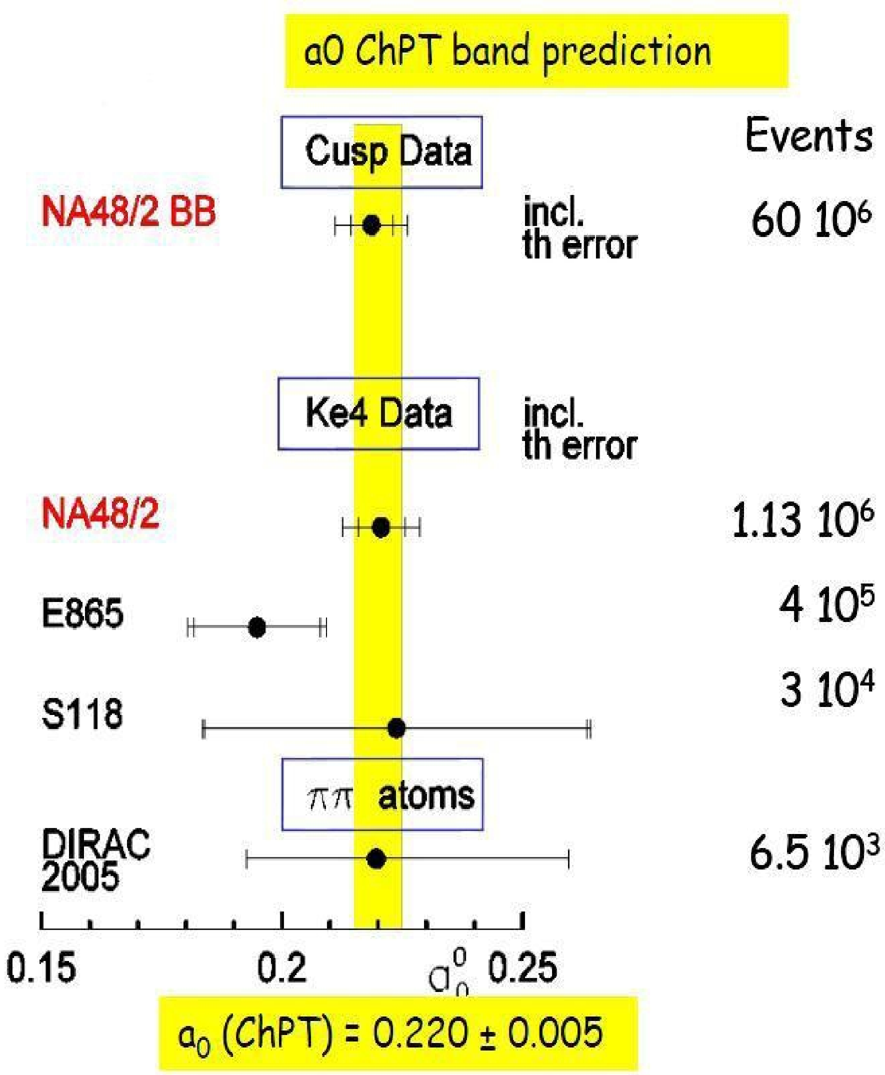
$(a0-a2)m_+ = 0.2639 \pm 0.0020 \pm 0.0004 \pm 0.0021$

Using ChPT constraints:

$(a0-a2)m_+ = 0.2640 \pm 0.0020 \pm 0.0017 \pm 0.0035$



Experimental results comparison



Yellow band is the ChPT prediction
a_0=0.220±0.005

The NA48/2 experimental precision now at the same level of theoretical prediction precision

Conclusions

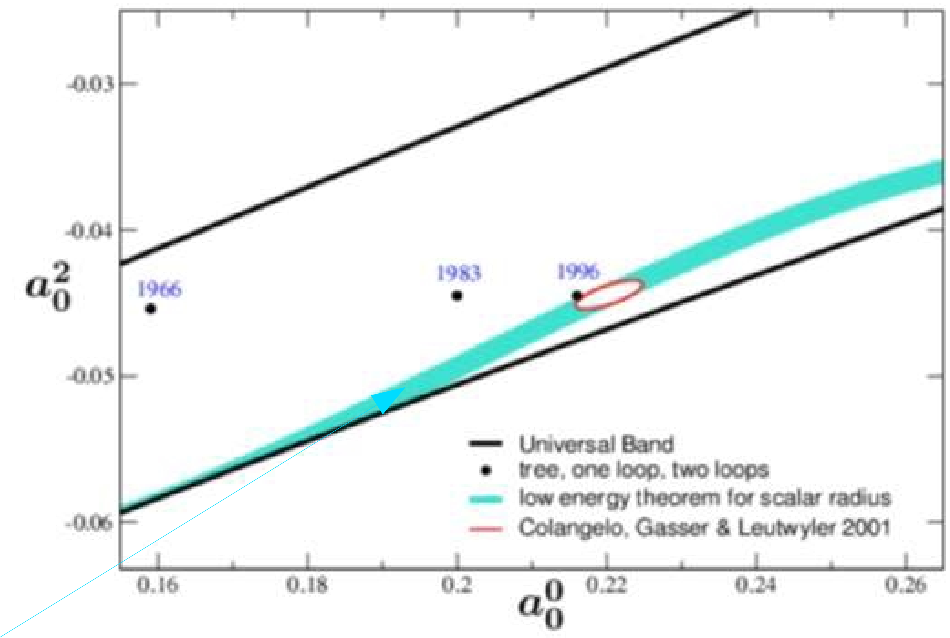
- Kaon decays give the possibility to study the low energy hadronic interaction with good precision
- NA48/2 has recorded high statistics and high quality data → can test ChPT predictions with high accuracy
- The achieved experimental precision on a_0 is now competitive with the theoretical precision (2.5%)
- $\pi\pi$ scattering lengths results shows an impressive agreement with the ChPT → very strong test of the theory

spares

a_0 and a_2 theoretical prediction

a0	= 0.220	+- 0.005
a2	= -0.0444	+- 0.0010
a0-a2	= 0.265	+- 0.004

Colangelo, Gasser, Leutwyler, PRL 86, 5008, (2001)



Analyticity and chiral symmetry predicts the constraint

$$a_2 = (-0.0444 \pm 0.0008) + 0.236(a_0 - 0.22) - 0.61(a_0 - 0.22)^2 - 9.9(a_0 - 0.22)^3$$

M^2_{00} fit results

Table 6 Fit results with electromagnetic corrections: $\pi\pi$ scattering parameters. Parameter values without errors have been kept fixed in the fit or calculated using the constraint between a_2 and a_0 given by (5)

Fit	χ^2/NDF	$a_0 m_+$	$a_2 m_+$	$(a_0 - a_2) m_+$	f_{atom}
CI	205.6/195	0.2391(56)	−0.0092(91)	0.2483(45)	0.0625(92)
CI_A	202.9/189	0.2400(59)	−0.0061(98)	0.2461(49)	0.0625
CI^{χ}	222.1/196	0.2203(28)	−0.0443	0.2646(21)	0.0420(77)
CI^{χ}_A	219.7/190	0.2202(28)	−0.0444	0.2645(22)	0.0420
BB	477.4/452	0.2330(92)	−0.0241(129)	0.2571(48)	0.0631(97)
BB_A	474.4/446	0.2350(97)	−0.0194(140)	0.2544(53)	0.0631
BB^{χ}	479.8/453	0.2186(32)	−0.0447	0.2633(24)	0.0538(77)
BB^{χ}_A	478.1/447	0.2178(33)	−0.0449	0.2627(25)	0.0538

K→3π Dalitz plot definition

- Three body decay can be analyzed using the Dalitz plot
- The most convenient variables are the Dalitz plot variables u and v defined as below:

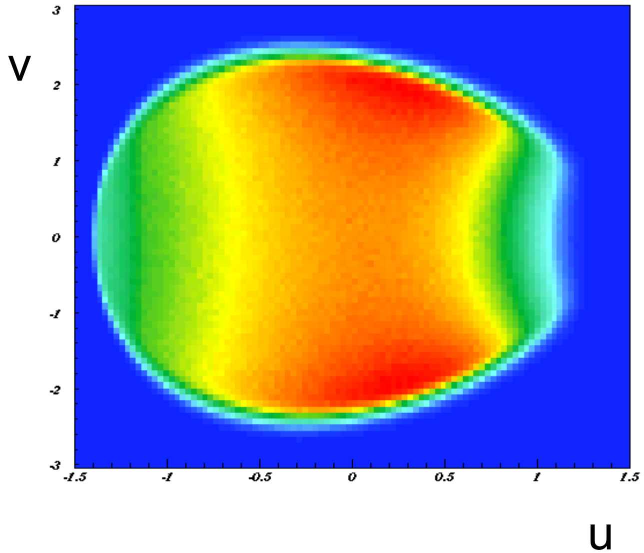
Dalitz variables:

$u=(s_3-s_0)/m_\pi^2$

$v=(s_2-s_1)/m_\pi^2$

$s_i=(p_K-p_i)^2 \; i=1,2,3$

$s_0=(s_1+s_2+s_3)/3$



- The Matrix element can be expanded as a function of u and v (a polynomial expansion)