

# Measurements of $K_{e4}$ and $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ decays

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On behalf of the NA48/2 collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz,  
Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

## 1 Introduction

## 2 $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu_e$

- Form factors and  $\pi\pi$  scattering lengths

## 3 $K^\pm \rightarrow \pi^0\pi^0 e^\pm \nu_e$

- Branching ratio and form factors

## 4 $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$

- The “cusp” effect
- Form factors and  $\pi\pi$  scattering lengths

## 5 Conclusions

# $\pi\pi$ scattering lengths: why and how measure them?

## WHY?

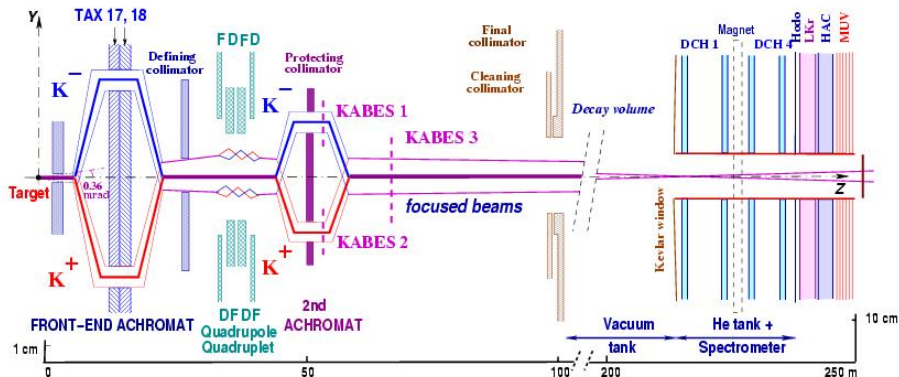
- **Very precise theoretical predictions** from  $\chi PT$  (2%) and generalised  $\chi PT$ , depending only on **one free parameter: the quark condensate  $\langle 0 | \bar{q}q | 0 \rangle$**
- $\langle 0 | \bar{q}q | 0 \rangle$  must be determined **experimentally**
- The size of  $\langle 0 | \bar{q}q | 0 \rangle$  determines the **order** at which mass terms appear in the perturbative expansion

## HOW?

- $K_{e4}$  ( $K \rightarrow \pi\pi e\nu$ ): no other hadrons, pions **close to threshold**  
No theoretical uncertainty on the form factors, only on  $a_0^2 = f(a_0^0)$
- $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ : “cusp” at  $M_{00}^2 = 4m_{\pi^+}^2$  due to **rescattering**  
Theoretical uncertainty of the Cabibbo-Isidori model: **5%**

# The NA48/2 beamline

Simultaneous  $K^+$  and  $K^-$  beams with  $p_K = (60 \pm 3) \text{ GeV}/c$   
to measure charge asymmetry in  $K \rightarrow 3\pi$  decays



# The NA48/2 detector

- **Magnetic spectrometer:**

4 drift chambers

+ 1 dipole magnet

$\sigma(p)/p \simeq 0.9\%$  @ 20 GeV/

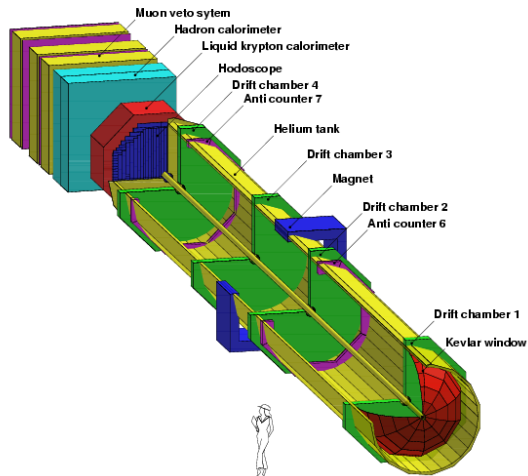
$(p_{\perp}^{kick} \simeq 120 \text{ MeV}/c)$

- **Electromagnetic calorimeter:**

Liquid krypton calorimeter

$\sigma(E)/E \simeq 1\%$  @ 20 GeV

$\sigma(t) \simeq 265 \text{ ps}$  for 50 GeV  $e^{-}$



# $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu_e$ : Signal and background

$K_{e4}^{+-}$  candidates (2003):  $\sim 370,000$  with 0.5% background

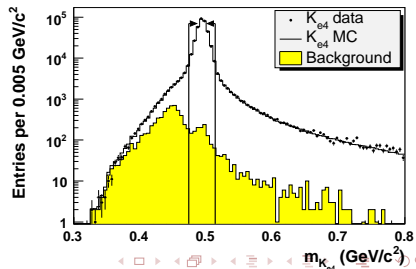
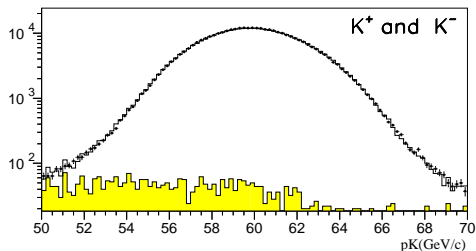
## Background estimate from data:

“Wrong sign” events ( $\pi^\pm \pi^\pm e^\mp \nu_e$ ) can only be background ( $\Delta S = \Delta Q$  rule)

Main contributions from  $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$  with  $\pi \rightarrow e\nu$  or  $\pi$  mis-ID

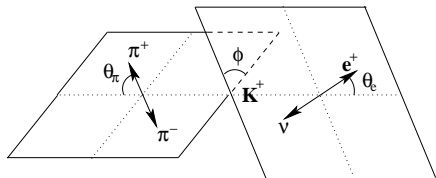
Factor 2 scaling applied to background from  $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$

Agreement between estimates from data and from MC



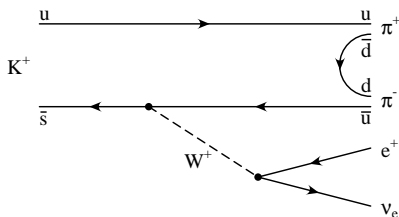
# $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu_e$ : Form factor parametrisation

## Kinematic variables



- $M_{\pi\pi}$
- $M_{e\nu}$
- $\cos \theta_\pi$  (in the  $\pi\pi$  cm system)
- $\cos \theta_e$  (in the  $e\nu$  cm system)
- $\phi$  (angle between the  $\pi\pi$  and  $e\nu$  planes in the  $K$  cm system)

## Matrix element



$$T = \frac{G_F}{\sqrt{2}} V_{us}^* \bar{u}(p_\nu) \gamma_\mu (1 - \gamma_5) v(p_e) (V^\mu - A^\mu)$$

## Form factors

$$F = F_s e^{i\delta_0^0} + F_p e^{i\delta_1^1} \cos \theta_\pi$$

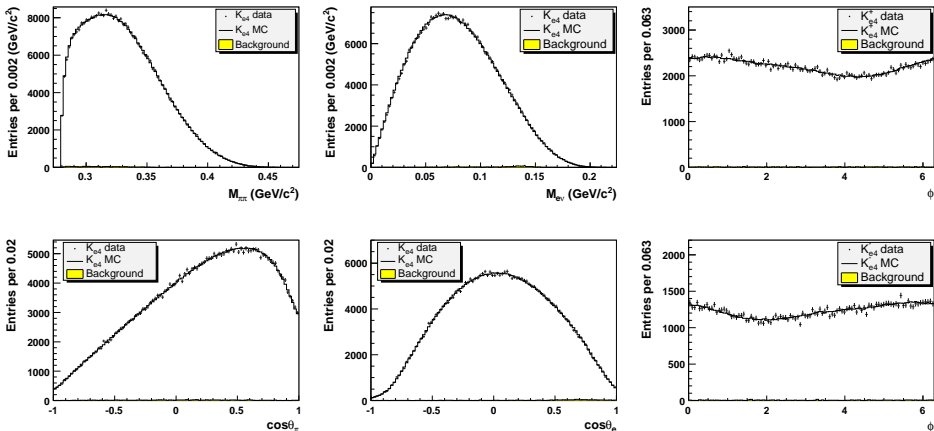
$$G = G_p e^{i\delta_1^1}$$

$$H = H_p e^{i\delta_1^1}$$

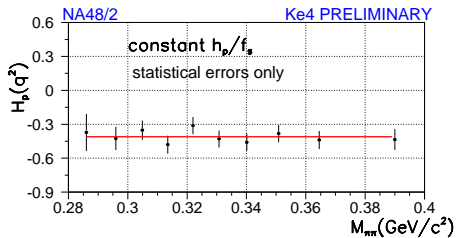
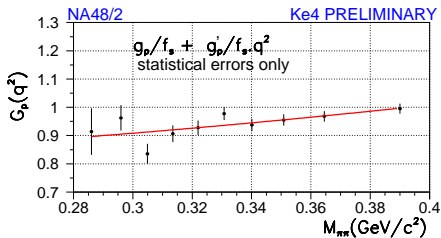
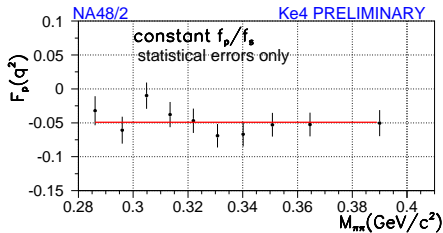
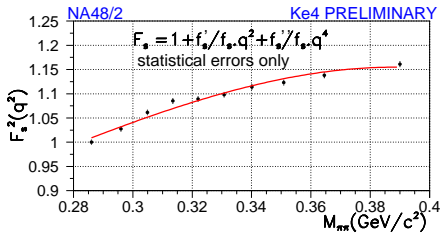
# $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu_e$ : Form factor fit

10 independent 5 parameter fits for each bin in  $M_{\pi\pi}$ .

For each fit 1500 equal population bins in the 4-dimensional space of the kinematic variables



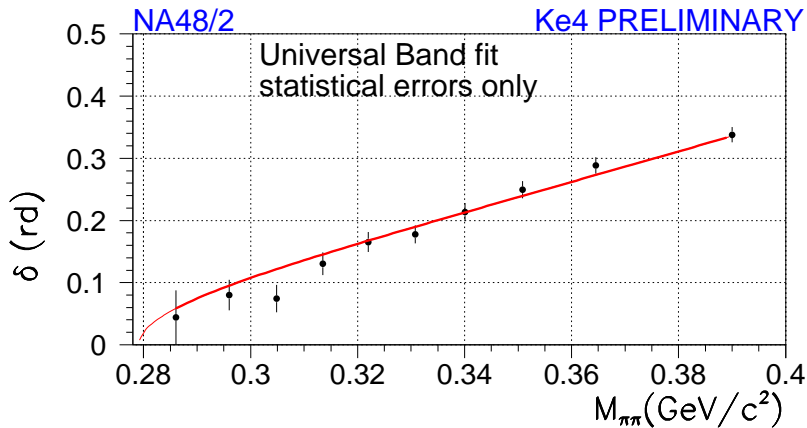
# $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu_e$ : F, G and H



$$K^\pm \rightarrow \pi^+\pi^-\nu_e: a_0^0 \text{ and } a_0^2$$

The  $\delta = \delta_0^0 - \delta_1^1$  distribution was fitted with a **1 parameter ( $a_0^0$ ) function** given by the numerical solution of the Roy equations in Phys. Rept. 353, 207

$a_0^0$  and  $a_0^2$  were constrained to lie on the **centre of the universal band**



# $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu_e$ : Systematics and preliminary result

## Systematic checks

- Two independent analyses with different **reconstruction methods**, **acceptance corrections**, **fit methods** and **MC parameters**
- Stability of **acceptance vs time** (variation of the simulated beam conditions)
- Uncertainty on **background estimate** (checked with data and MC)
- Uncertainty on **electron-ID efficiency** (variation of LDA cut)
- Uncertainty on **radiative corrections** (fraction of total effect with or w/o PHOTOS)
- Bias from **neglected  $s_e$  dependence** (MC tests)

## NA48/2 preliminary result (2003 data)

$$f'_s/f_s = 0.169 \pm 0.009_{stat} \pm 0.034_{syst}$$

$$f''_s/f_s = -0.091 \pm 0.009_{stat} \pm 0.031_{syst}$$

$$f_p/f_s = -0.047 \pm 0.006_{stat} \pm 0.008_{syst}$$

$$g_p/f_s = 0.891 \pm 0.019_{stat} \pm 0.020_{syst}$$

$$g'_p/f_s = 0.111 \pm 0.031_{stat} \pm 0.032_{syst}$$

$$h_p/f_s = -0.411 \pm 0.027_{stat} \pm 0.038_{syst}$$

$$a_0^0 = 0.256 \pm 0.008_{stat} \pm 0.007_{syst}$$

$$\pm 0.018_{theor} \text{ (universal band)}$$

$$\chi_{PT} \text{ prediction: } 0.220 \pm 0.005$$

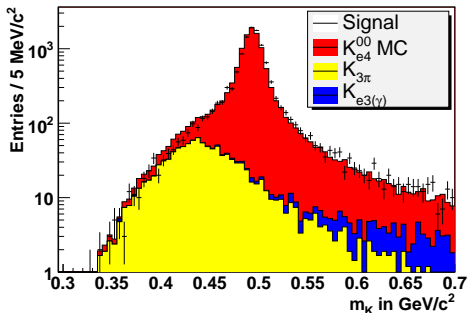
# $K^\pm \rightarrow \pi^0 \pi^0 e^\pm \nu_e$ : Branching ratio

$K_{e4}^{00}$  candidates (2003):  $\sim 10,000$   
with 3% background

$K_{e4}^{00}$  candidates (2004):  $\sim 30,000$   
with 2% background

**Background estimate from data,**  
reversing cuts

Main contribution from  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$   
with  $\pi$  mis-ID



## Systematic uncertainties

Acceptance, trigger efficiency and energy measurement of the calorimeter

## NA48/2 preliminary result (2003 data)

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5}$$

# $K^\pm \rightarrow \pi^0 \pi^0 e^\pm \nu_e$ : Form factors

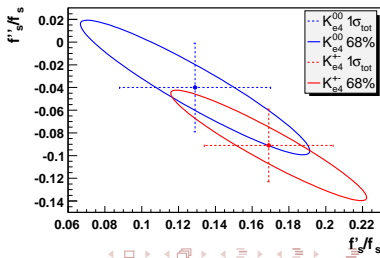
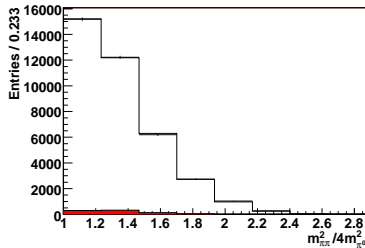
Same formalism as for  $K_{e4}^{+-}$ , but, for the symmetry of the  $\pi^0 \pi^0$  system,  
**no P-wave!**

Effect of cusp at  $M_{00}^2 = 4m_{\pi^+}^2$  taken into account

**NA48/2 preliminary result (2003+2004 data)**

$$f'_s/f_s = 0.129 \pm 0.036_{stat} \pm 0.020_{syst}$$

$$f''_s/f_s = -0.040 \pm 0.034_{stat} \pm 0.020_{syst}$$

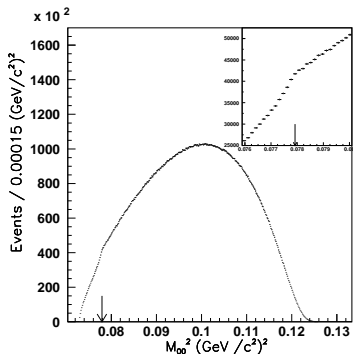
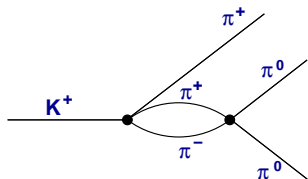


# $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ : Cusp in the $M_{00}^2$ distribution

**Selected events (2003):**  $23 \times 10^6$

$M_{00}^2$  computed imposing the mean vertex of the  $\pi^0$ 's  
(improved resolution close to threshold)

**Evidence for a cusp at  $M_{00}^2 = 4m_{\pi^+}^2$  due to  $\pi\pi$  rescattering**



Fit to the Cabibbo-Isidori model (JHEP03 021):

$$M_0 = A_0 \left( 1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 \right)$$

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$$g_0 = 0.645 \pm 0.004_{stat} \pm 0.009_{syst}$$

$$h' = -0.047 \pm 0.012_{stat} \pm 0.011_{syst}$$

$$a_2 = -0.041 \pm 0.022_{stat} \pm 0.014_{syst}$$

$$a_0 - a_2 = 0.268 \pm 0.010_{stat} \pm 0.004_{syst} \pm 0.013_{theor}$$

$K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ :  $k'v^2$  term

Modified matrix element:

$$M_0 = A_0 \left( 1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 + \frac{1}{2} k' v^2 \right)$$

**Systematic checks:**

Acceptance, trigger efficiency

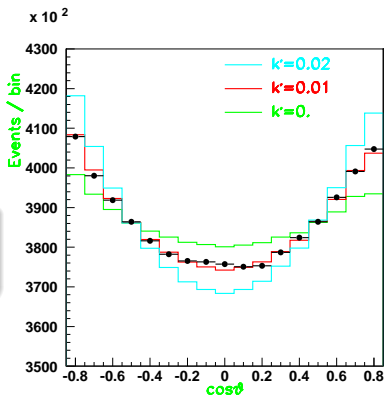
**NA48/2 preliminary result (2003 data)**

$$k' = 0.0097 \pm 0.0003_{stat} \pm 0.0008_{syst}$$

Meaning and value of  $g_0$  and  $h'$  change with the new matrix element

No change in  $a_0 - a_2$  and  $a_2$

**Note:**  $h'$  and  $k'$  are not the same as the PDG parameters  $h$  and  $k$



$\cos \theta =$  angle between  $\pi^\pm$   
and  $\pi^0$  in  $\pi^0 \pi^0$  cms

# Conclusions

- Form factor and  $\pi\pi$  scattering length measurement from  $K_{e4}^{+-}$

$$a_0^0 = 0.256 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{theor}$$

Compatible with  $\chi PT$  prediction and previous results, higher sensitivity

- Branching ratio and form factor measurements from  $K_{e4}^{00}$

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5}$$

BR: factor 8 better than latest measurement

Form factors: compatible with charged channel

- $\pi\pi$  scattering length and slope measurements from  $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$

$$a_0 - a_2 = 0.268 \pm 0.010_{stat} \pm 0.004_{syst} \pm 0.013_{theor}$$

- First evidence for a value of  $k \neq 0$  in the  $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$  Dalitz plot

$$k' = 0.0097 \pm 0.0003_{stat} \pm 0.0008_{syst}$$