

# **New results on kaon decays from NA48/2**

**Cristina Morales**  
*(University of Mainz)*

on behalf of the **NA48/2** Collaboration:

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara,  
Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay,  
Siegen, Torino, Vienna

***XLIIIrd Rencontres de Moriond***  
***La Thuile, March 13<sup>th</sup>, 2008***

# Outline

## New Measurements of ChPT Parameters

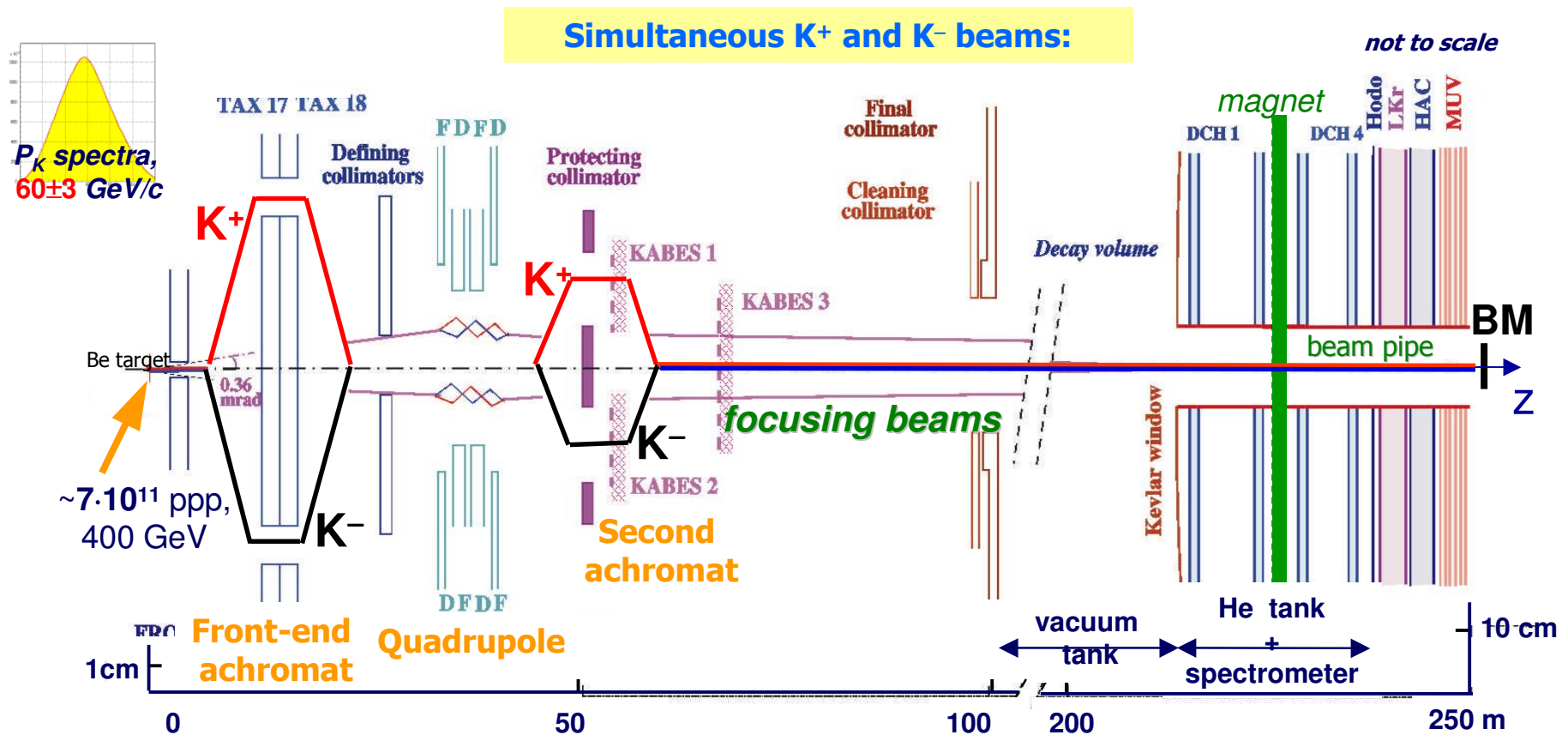
- ★ The **NA48/2** Experiment.
- ★  $\pi\pi$ -Scattering lengths in  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  and  $K^\pm \rightarrow \pi^\pm \pi e^\pm \nu$ .
- ★ Precise study of  $K^\pm \rightarrow \pi^\pm \gamma$  and  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ .
- ★ BR and form factors from  $K^\pm \rightarrow \pi^\pm e^+ e^-$ .
- ★ Summary.

NA48	1997	$\epsilon'/\epsilon$ run	$K_L + K_S$	
	1998	$\epsilon'/\epsilon$ run	$K_L + K_S$	
	1999	$\epsilon'/\epsilon$ run	$K_L + K_S$	$K_S$
				Hi. Int.
	2000	$K_L$ only	$K_S$ High Intensity	NO Spectrometer
NA48/1	2001	$\epsilon'/\epsilon$ run	$K_S$ High Int.	
		$K_L + K_S$		
NA48/2	2002	$K_S$ High Intensity		
NA48/2	2003	$K^\pm$ High Intensity		
	2004	$K^\pm$ High Intensity		
NA62	2007	$K_{e2}^+ / K_{\mu 2}^+$ run		

# The NA48/2 Experiment

Fixed target kaon experiment at CERN's SPS.

- Designed for precision measurement of charge asymmetries in  $K_{3\pi}$  ;
- Other goals:  $\pi\pi$  scattering, ChPT, lepton universality, CKM unitarity tests, etc.



# The NA48/2 Experiment

## Main detector components:

- Magnet spectrometer (4 DCHs):  
1% resolution for  $p=20$  GeV/c.
- Liquid Krypton EM calorimeter (LKr)  
1.6% energy resolution for  $E_\gamma=20$  GeV/c.

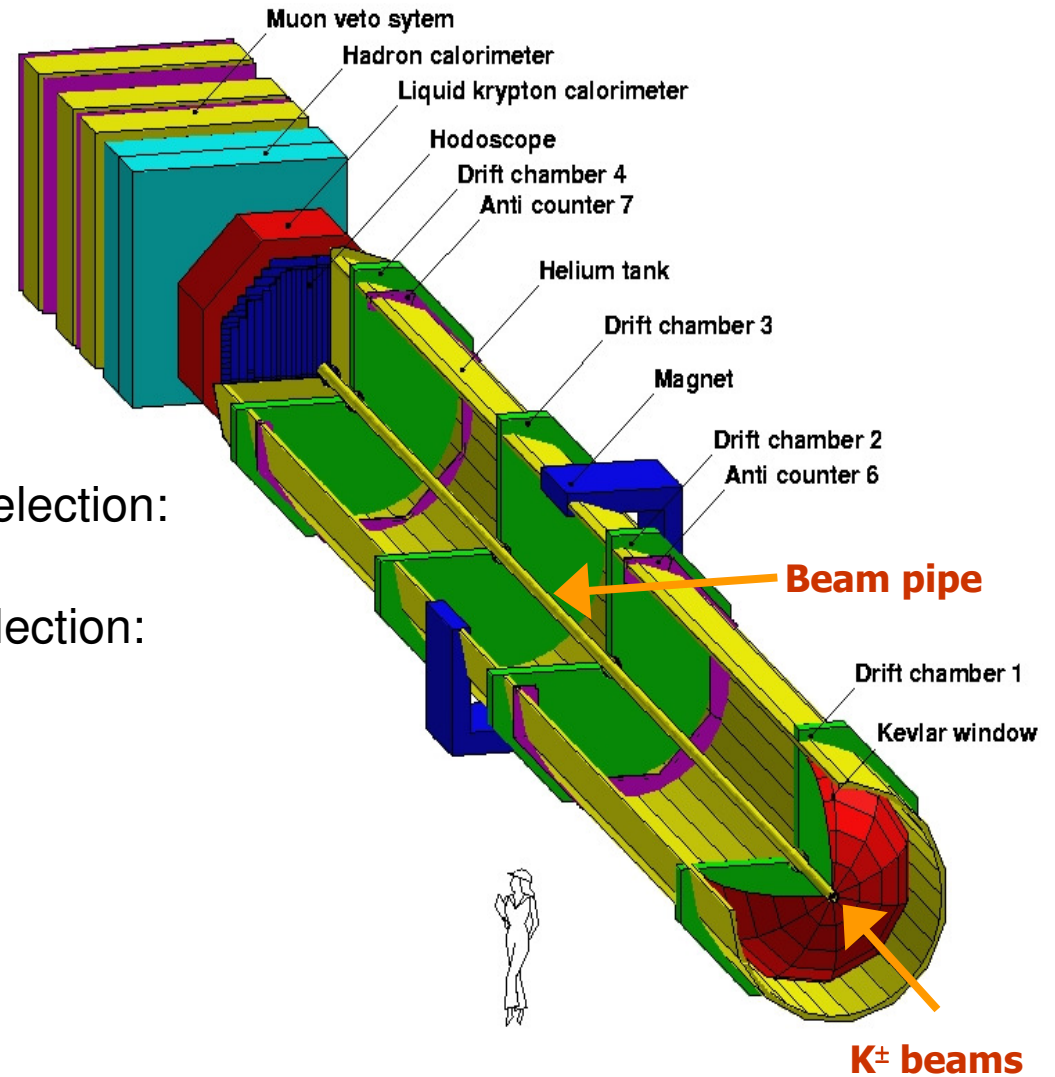
## Two main trigger modes:

- Charged. Devoted to  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  selection:  
3 charged tracks.
- Neutral. Devoted to  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  selection:  
> 2 em clusters in LKr x or y projection

## Total statistics (2003-2004):

$$K^\pm \rightarrow \pi^- \pi^+ \pi^\pm: \sim 4 \cdot 10^9$$

$$K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm: \sim 1 \cdot 10^8$$



**$\pi^+\pi^-$  Scattering in  $\mathbf{K}^\pm \rightarrow \pi^\pm \pi^0 \pi^0$   
&  
 $\mathbf{K}^\pm \rightarrow \pi^+\pi^- e^\pm \nu$  ( $K_{e4}$ )**

# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ & $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$

## Motivation:

- At low energies, s-wave  $\pi\pi$ -scattering lengths for isospin 0 and 2,  $a_{0,2}$ , are **fundamental parameters of ChPT**.
- $\pi\pi$ -Scattering lengths directly connected to  $M_\pi$ :

$$a_0, a_2 \propto M_\pi^2 + O(m^2) \quad [\text{Weinberg, 1966}]$$

and to quark condensate:

$$M_\pi \sim (m_u + m_d) \cdot |\langle 0 | \bar{q}q | 0 \rangle| / F_\pi^2 \quad [\text{Colangelo, Gasser, Leutwyler (2000)}]$$

**➡ Measurement of  $a_0, a_2$  crucial test for ChPT**

**Two clean measurements** of scattering lengths  $a_{0,2}$  in NA48/2:

- Wigner-cusp effect in  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  decays;
- $\delta$  in  $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$  decays.

# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

~  $60 \cdot 10^6$   $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  in 2003 + 80% of 2004. Background negligible.

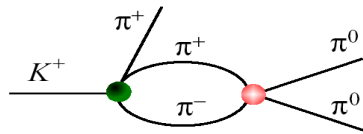
Original motivation: search for **pionium** decay

signature in  $M_{00}$ :  $K^\pm \rightarrow \pi^\pm (\pi^+ \pi^-)_{\text{atom}} \rightarrow \pi^\pm (\pi^0 \pi^0)$

⇒ Unexpected observation of cusp at  $M_{00} = 2m_{\pi^+}$ .

## Interpretation :

- [N. Cabibbo PRL 93 (2004) 12181]:



$\mathcal{M}_{\text{rescattering}} \propto - (a_0 - a_2)$  real and negative for  $M_{00} < 2m_{\pi^+}$ .

$\mathcal{M}_{\text{rescattering}} \propto (a_0 - a_2)i$  imaginary for  $M_{00} > 2m_{\pi^+}$ .

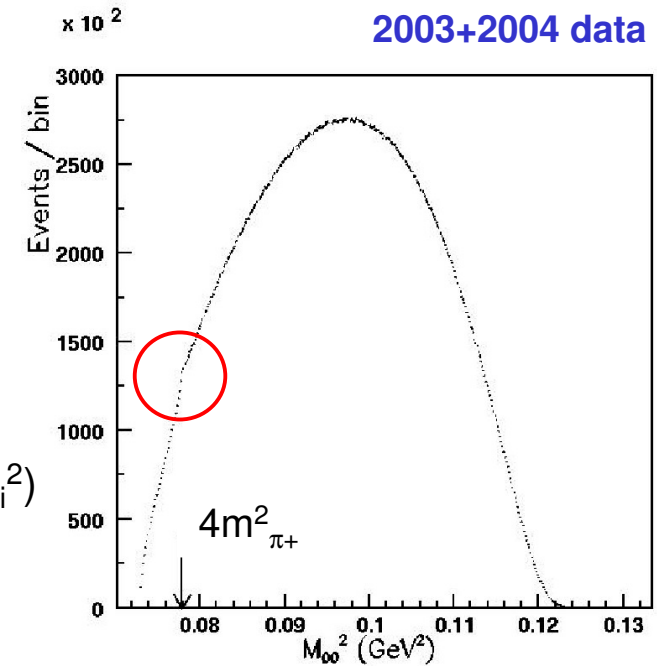
- 2nd order rescattering corrections + Isospin breaking +  $O(a_i^2)$   
[N. Cabibbo, G. Isidori, JHEP03 (2005) 021].

## Method of measurement :

Fit to:

$$\frac{\partial \Gamma}{\partial M_{00}} = N \cdot F(a_0 - a_2, a_2, g, h, M_{00})$$

with **5 free parameters**, and **g** and **h** slope parameters from PDG matrix element ( $|\mathcal{M}|^2 = 1 + gu + hu^2$ ).



# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$

$\sim 677 \cdot 10^3 K_{e4}$  in 2003; 0.5% Background.

Decay amplitude described by **form factors partial wave expanded**.

$\delta_0 = \pi^+\pi^-$  scattering phase shift for  $l=0, L=0$  (s-wave),

$\delta_1 = \pi^+\pi^-$  scattering phase shift for  $l=1, L=1$  (p-wave).

Decay rate depends on  $\delta = \delta_0 - \delta_1$  with  $\delta = \delta(M_{\pi\pi})$ .

## Five independent kinematic variables

(Cabibbo-Maksymowicz variables):

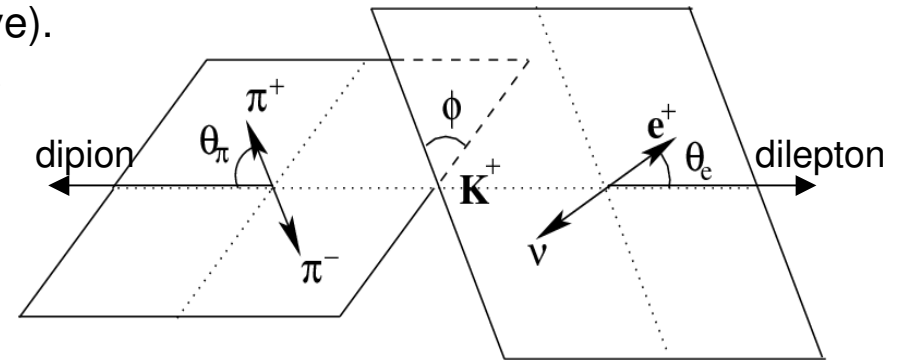
$M_{\pi\pi}$ : invariant **di-pion mass** squared;

$M_{e\nu}$ : invariant **di-lepton mass** squared;

$\theta_\pi$ : **angle of  $\pi^+$**  in  $\pi\pi$  rest frame wrt  $\pi\pi$  direction of flight in  $K$  rest frame;

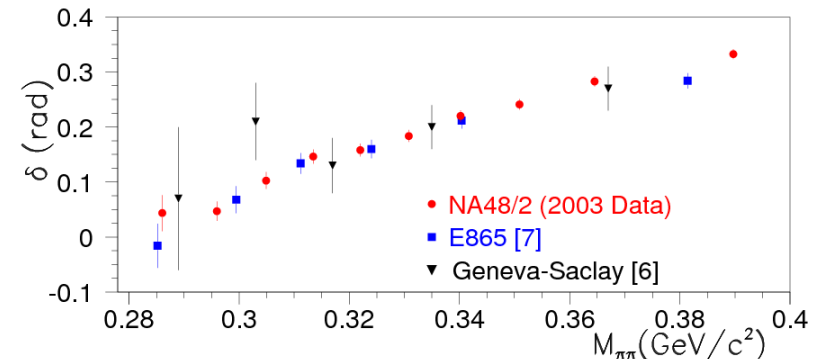
$\theta_e$ : **angle of  $e^+$**  in  $e\nu$  rest frame wrt  $e\nu$  direction of flight in  $K$  rest frame;

$\Phi$ : **angle of  $\pi\pi$  decay plane wrt  $e\nu$  decay plane.**



**Method of Measurement**: In each  $M_{\pi\pi}$  bin, simultaneous fit to  $K_{e4}$  **form factors** and  $\delta$  in the 4 other dimensions.

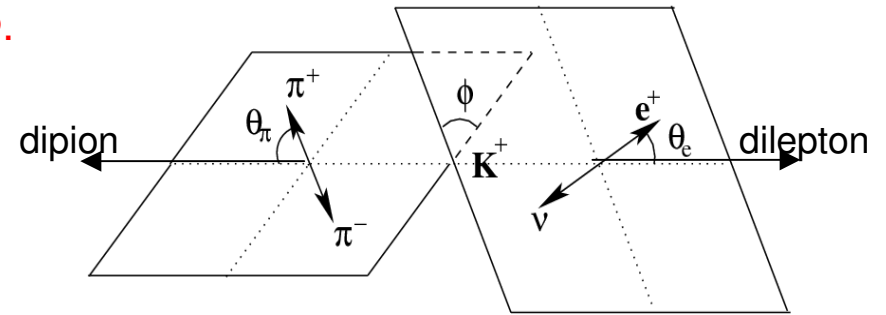
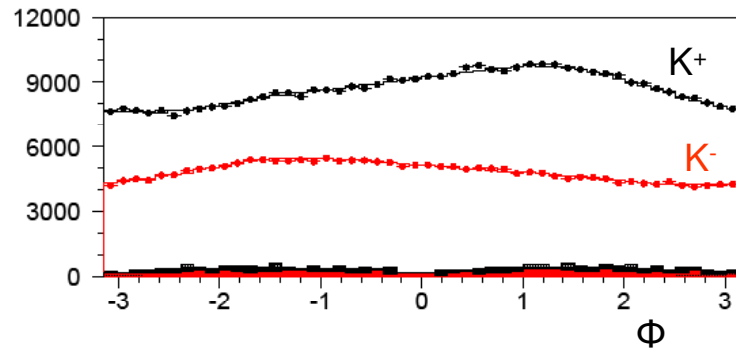
Roy equations relate  $\delta$  with  $\pi\pi$  scattering lengths.



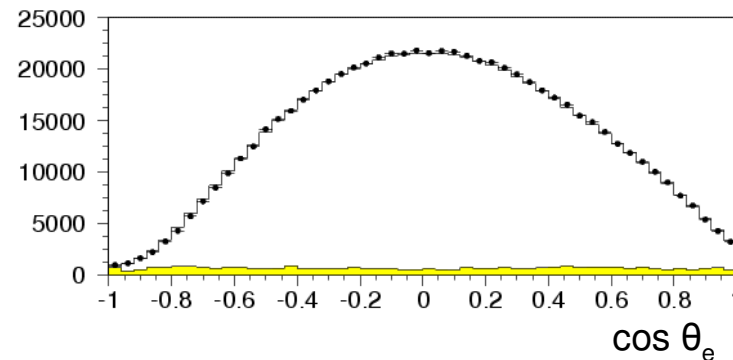
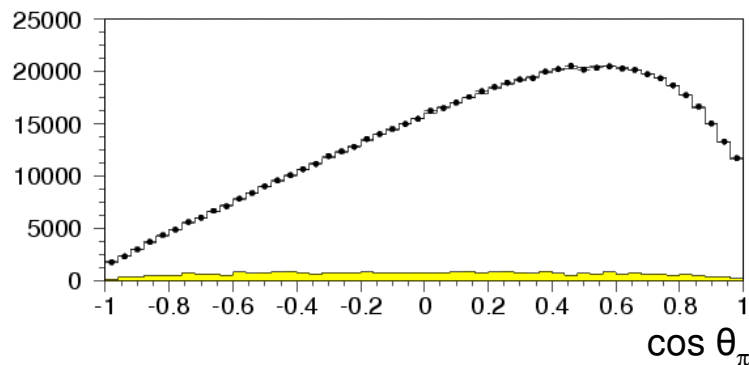
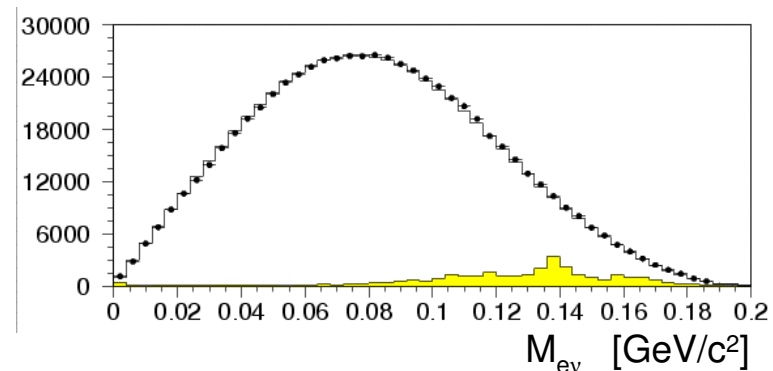
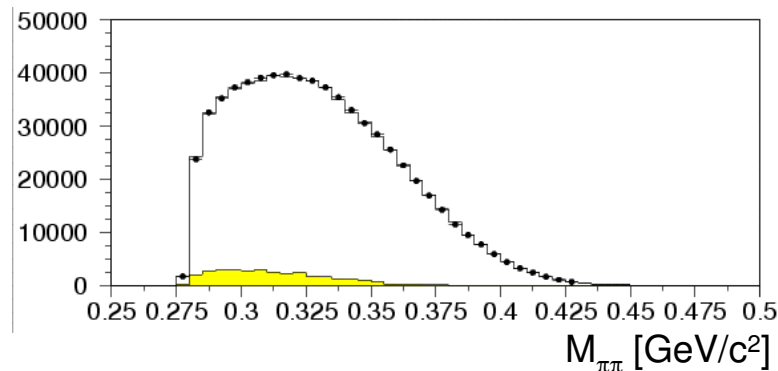
# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$

Five independent kinematic variables

(Cabibbo-Maksymowicz variables):  $M_{\pi\pi}, M_{e\nu}, \theta_\pi, \theta_e, \Phi$ .



Background 10 times scaled in all plots to make it visible!



# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ & $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$

- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  results:

$$(a_0 - a_2) m_{\pi^+} = 0.261 \pm 0.006_{\text{stat}} \pm 0.003_{\text{syst}} \pm 0.001_{\text{ext}} \pm 0.013_{\text{theory}}$$

$$a_2 m_{\pi^+} = -0.037 \pm 0.013_{\text{stat}} \pm 0.009_{\text{syst}} \pm 0.002_{\text{ext}}$$

(preliminary)

with  $\rho = -0.92$ :

$$a_0 m_{\pi^+} = 0.224 \pm 0.008_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.003_{\text{ext}} \pm 0.013_{\text{theory}}$$

Theoretical uncertainty: 5% due to neglected higher order terms and radiative corrections.

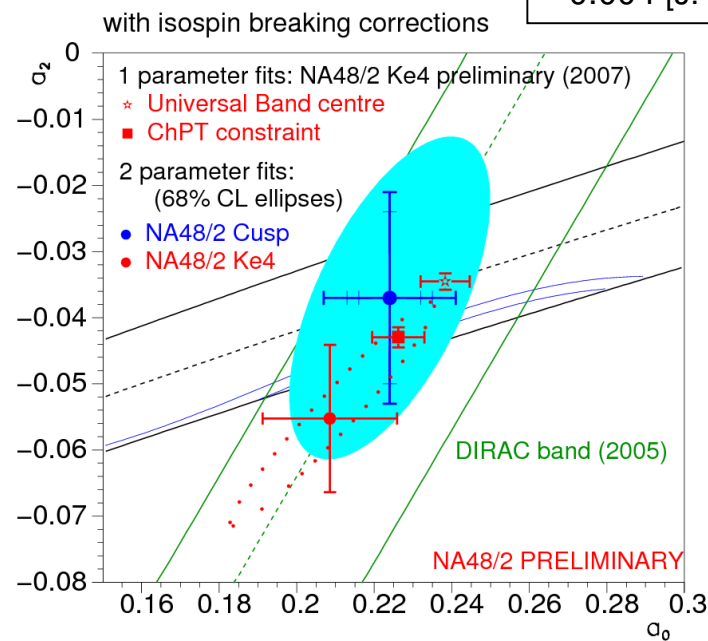
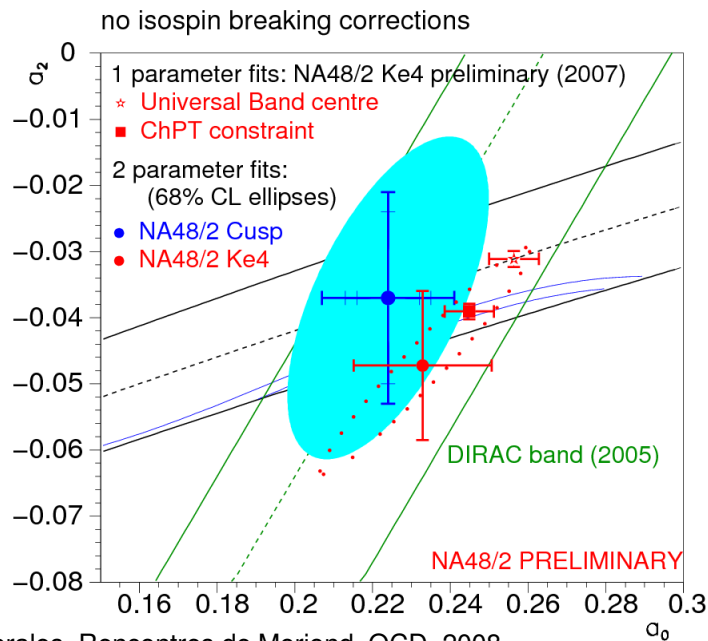
- Ke4 results:

[<http://dx.doi.org/10.1140/epjc/s10052-008-0547-0>]

(1-par fit + univ band)  $a_0 = 0.256 \pm 0.006_{\text{stat}} \pm 0.002_{\text{syst}} + 0.018_{\text{theo}} - 0.017_{\text{theo}}$

(2-par fit)  $a_0 = 0.233 \pm 0.016_{\text{stat}} \pm 0.007_{\text{syst}}$   
 with  $\rho = 0.967$ :  $a_2 = -0.047 \pm 0.011_{\text{stat}} \pm 0.004_{\text{syst}}$

Note: corrections for isospin breaking shift  $a_0$  by -0.02 and  $a_2$  by -0.004 [J. Gasser].



$$\mathbf{K}^{\pm} \rightarrow \pi^{\pm} \gamma \gamma \quad \& \quad \mathbf{K}^{\pm} \rightarrow \pi^{\pm} e^{+} e^{-} \gamma$$

# $K^\pm \rightarrow \pi^\pm \gamma\gamma$ in ChPT

•  $K^\pm \rightarrow \pi^\pm \gamma\gamma$ : 
$$\frac{\partial^2 \Gamma}{\partial y \partial z} = \frac{m_K}{2^9 \pi^3} \left[ z^2 (|A + B|^2 + |C|^2) + \underbrace{\left( y^2 - \frac{1}{4} \lambda(1, r_\pi^2, z) \right)^2}_{\text{Important at low } m_\gamma} (|B|^2 + |D|^2) \right]$$

## At order O(4):

[G. Ecker, A. Pich, E. de Rafael, Nucl.Phys.B303(1988) 665]

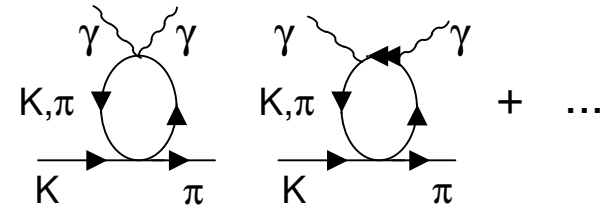
- $A(z, \hat{c})$  contains: **Loops** +  $\hat{c}$  ( $O(1)$ ).
- $A$  changes slope ( $\Rightarrow$  cusp) at  $m_{\gamma\gamma} = 2m_{\pi^+}$

- $C(z)$  contains **Wess-Zumino poles and tadpoles**.

[M. Gerard, C. Smith, S. Trine, hep-ph/0508189]

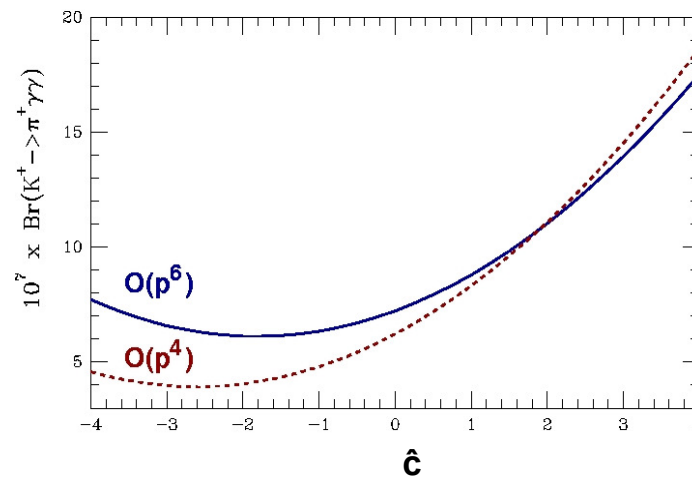
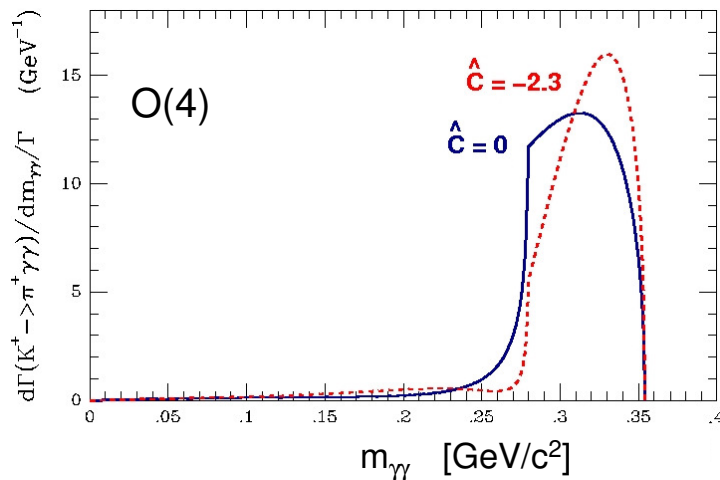
$$y = \frac{p \cdot (q_1 - q_2)}{m_K^2}, \quad z = \frac{(q_1 + q_2)^2}{m_K^2}$$

(only z dynamically relevant)



## At order O(6): Unitarity correction effects. Could increase BR in 30%- 40%.

[G. D'Ambrosio, J. Portolés. Nucl.Phys. B386 (1996), 403].



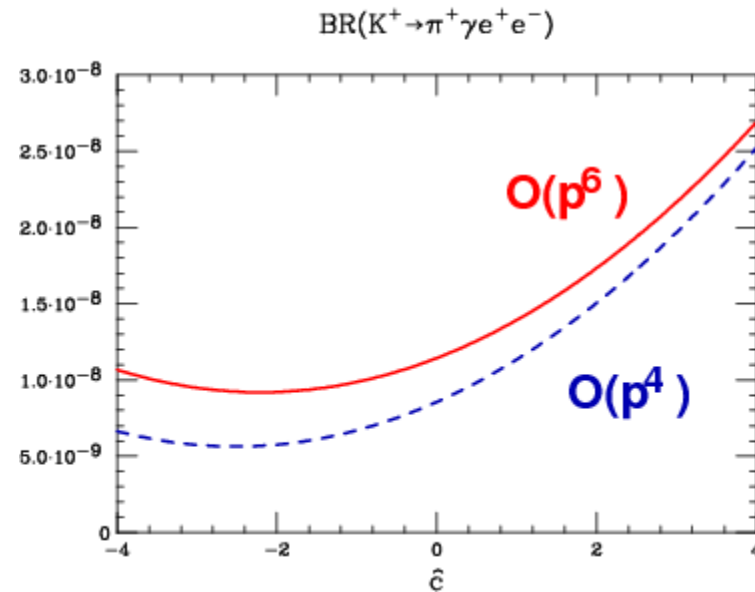
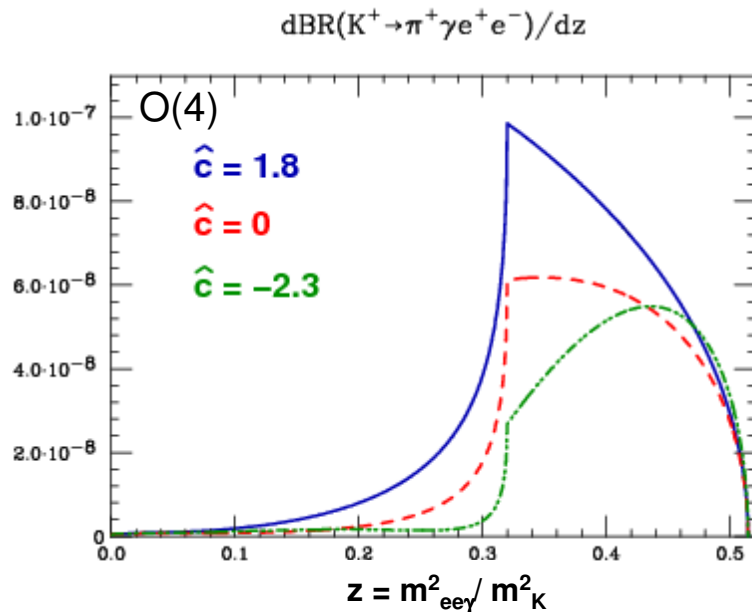
# $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ in ChPT

•  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$  similar to  $K^\pm \rightarrow \pi^\pm \gamma \gamma$  with  $\gamma$ -conversion.

- Like for  $K^\pm \rightarrow \pi^\pm \gamma \gamma$ ,  $\hat{c}$  determines the value of BR and  $m_{ee\gamma}$  spectrum shape.

$$\Gamma(O(4)) = \underbrace{(2.80 + 0.87\hat{c} + 0.14\hat{c}^2)}_{\text{Loops}} \underbrace{+ 0.23}_{\text{Wess-Zumino}} \cdot 10^{-23} \text{ GeV}$$

- **O(6)** unitarity corrections could increase BR in 30%-40% [F. Gabbiani, Phys.Rev.Lett. D59 (1999), 094022].



# Br ( $K^\pm \rightarrow \pi^\pm \gamma\gamma$ )

$$BR(\pi^\pm \gamma\gamma) = \frac{1}{\Phi_{\text{flux}}} \times \frac{(\#\pi^\pm \gamma\gamma) - (\#\text{bkg})}{\text{Acc}(\pi^\pm \gamma\gamma) \times \text{Eff}}$$

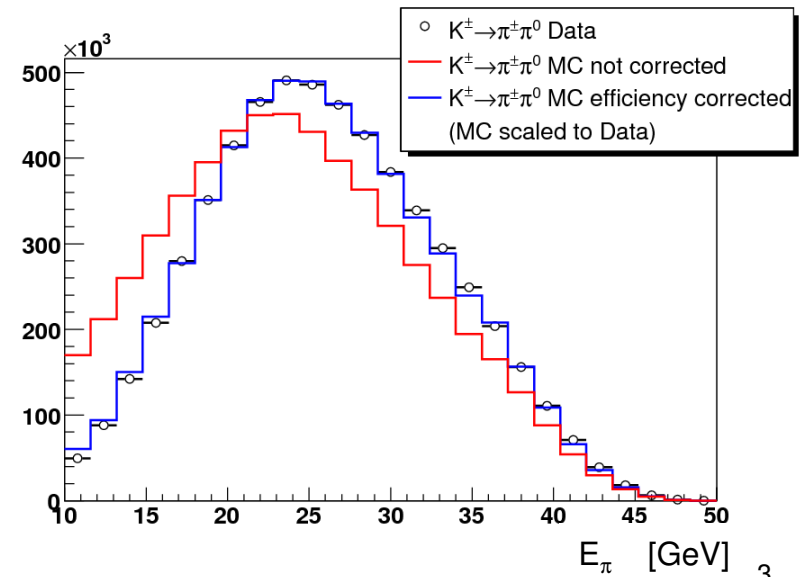
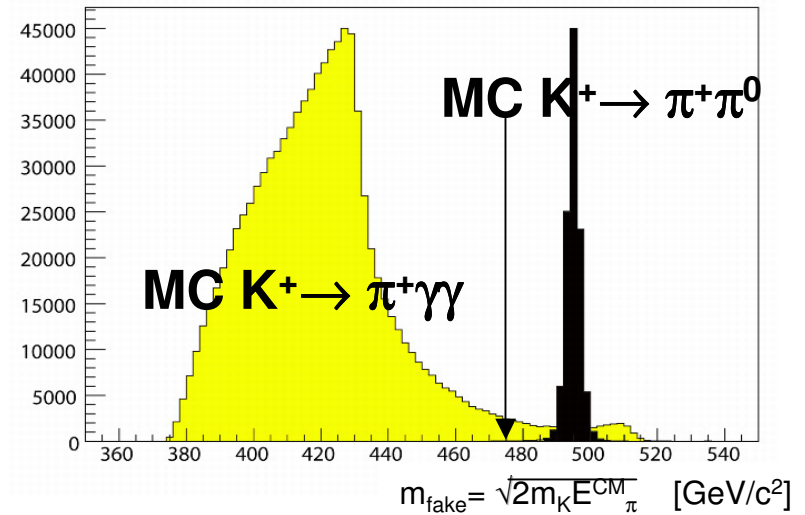
- **Bad trigger conditions:**
  - **Selected through neutral trigger:**  
L1 required more than 2 clusters in e.m. calorimeter  $\Rightarrow$  **50% efficiency.**
  - L2 rejected  $K^\pm \rightarrow \pi^\pm \pi^0$  decays (BR=20.92%!!)  
cutting on  $E_\pi^{\text{CM}}$ : **80% efficient.**

- Due to **low amount of statistics**, trigger efficiencies not measured with  $K^\pm \rightarrow \pi^\pm \gamma\gamma$  data.
  - Solution: **use background events**
- & study dependencies on **kinematic variables:**

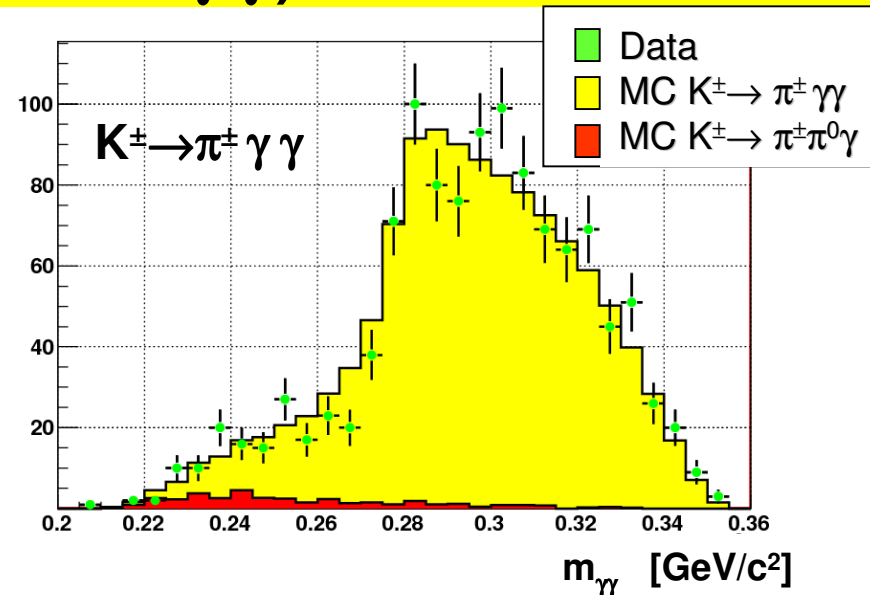
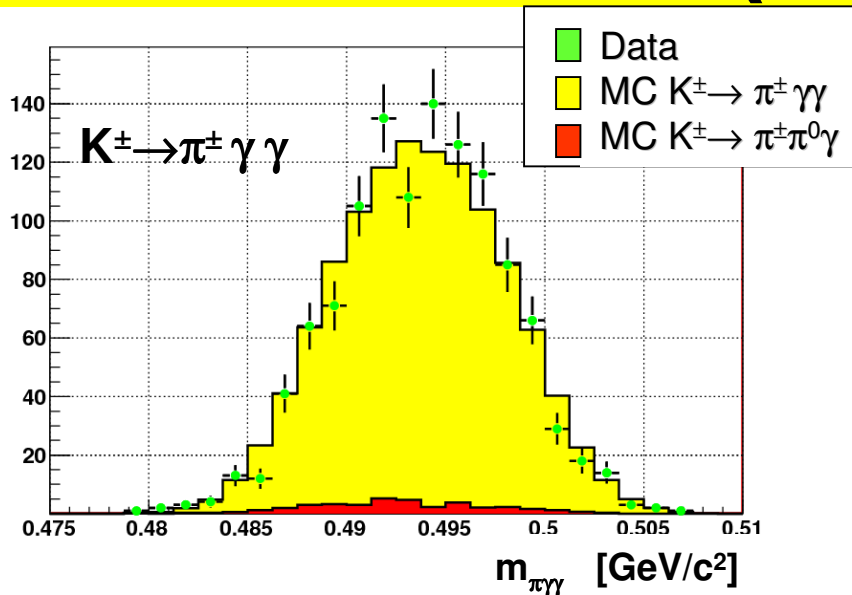
$$\text{Eff}^{\text{L1}}(p_{\text{track}}, d_{c1c2}), \text{Eff}^{\text{L2}}(p_{\text{track}}, z_{\text{vertex}}, r_{\text{beampipe}})$$

& use **MC  $K^\pm \rightarrow \pi^\pm \gamma\gamma$**  to “reshape” variables and integrate

$$\text{Eff}^{\text{L1,L2}} = \frac{\sum_n (\#\text{MC}(v_1, \dots, v_n) \times \text{Eff}^{\text{L1,L2}}(v_1, \dots, v_n))}{\sum_n (\#\text{MC}(v_1, \dots, v_n))}$$



# Br ( $K^\pm \rightarrow \pi^\pm \gamma \gamma$ )



**1164  $K^\pm \rightarrow \pi^\pm \gamma \gamma$  candidates in 40% of NA48/2. 3.3% Background mainly from  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  (IB).  
Sample ~ 40 times larger than the previous world sample!**

## Result :

$$\text{BR}(K^\pm \rightarrow \pi^\pm \gamma \gamma, \hat{c}=2, \text{O}(6)) = (1.07 \pm 0.04_{\text{stat}} \pm 0.08_{\text{syst}}) \cdot 10^{-6}$$

(preliminary)

**Systematics:** Main contributions arising from efficiency measurement method.

- **Model independent** measurement and  $\hat{c}$  extraction in preparation.

**Previous measurement:** BNL E787 (1997). 31 candidates with 5 background events [PDG2006].

$$\text{Br}(K^\pm \rightarrow \pi^\pm \gamma \gamma) = (1.10 \pm 0.32) \cdot 10^{-6} ; \quad \hat{c} = 1.8 \pm 0.6$$

# Br and $\hat{c}$ extraction in $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$

- Unlike  $K^\pm \rightarrow \pi^\pm \gamma \gamma$ , decay selected through **charged trigger** (3 charged tracks).

• All NA48/2 data sample used .

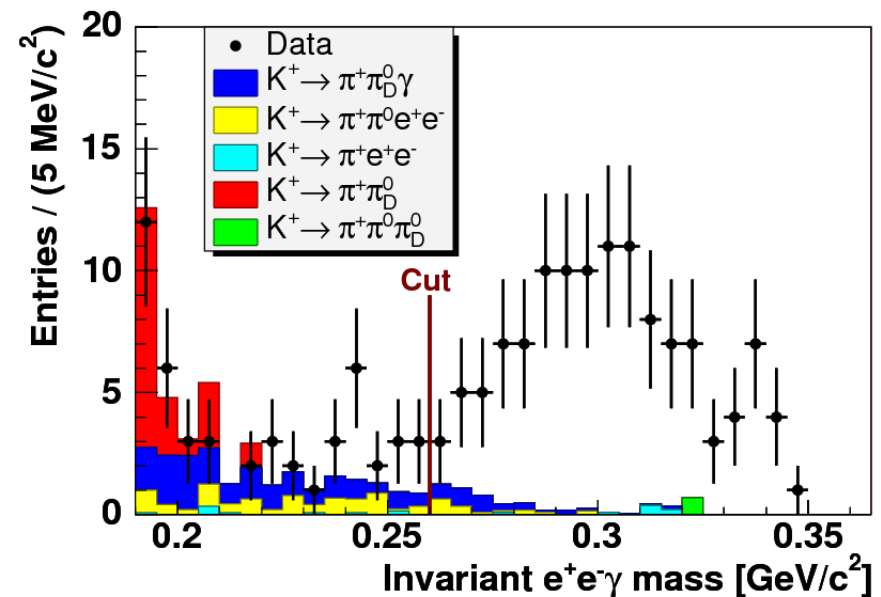
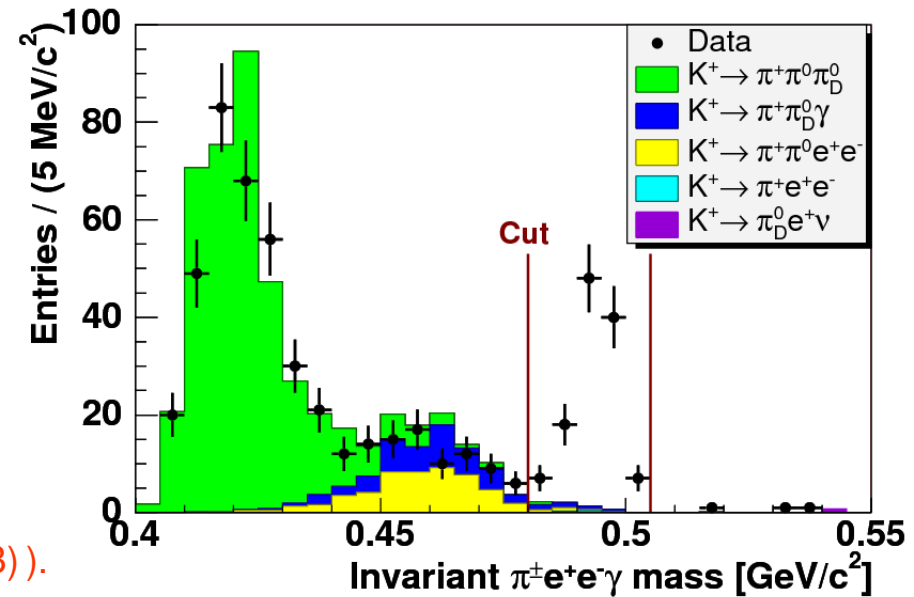
**120**  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$  candidates.

$7.3 \pm 1.7$  estimated background ( $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  (IB)).

**FIRST OBSERVATION OF THIS DECAY!**

• Normalization channel:

$K^\pm \rightarrow \pi^\pm \pi^0 \rightarrow \pi^\pm e^+ e^- \gamma$  14M events.



# Br and $\hat{c}$ extraction in $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$

**Results:** [NA48/2 Collaboration, PLB 659 (2008) 493]

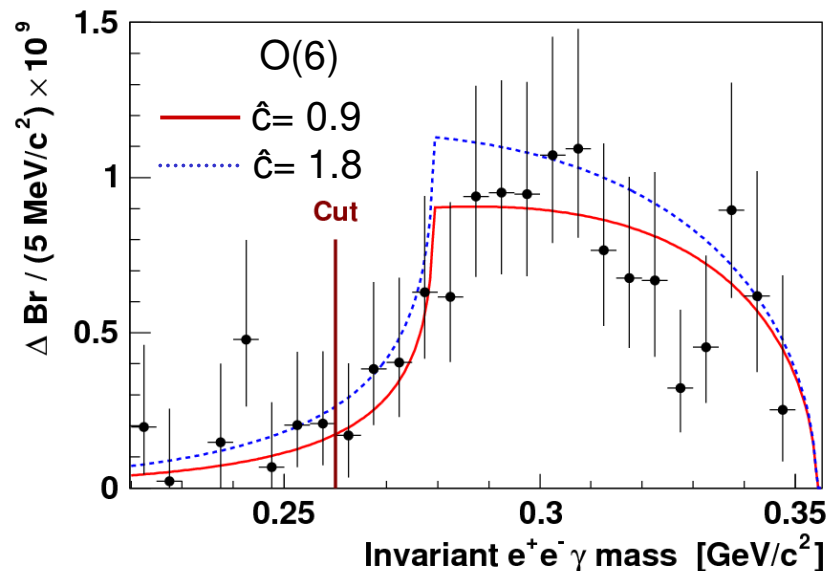
- **Model independent** measurement performed evaluating  $d\Gamma_{\pi\gamma}/dz$  and integrating over whole  $z$  region ( $z = m_{ee\gamma}^2 / m_K^2$ ).

$$\text{BR}(K^\pm \rightarrow \pi^\pm e^+ e^- \gamma, m_{ee\gamma} > 260 \text{ MeV}/c^2) = (1.19 \pm 0.12_{\text{stat}} \pm 0.04_{\text{syst}}) \times 10^{-8}$$
$$\text{BR}(K^\pm \rightarrow \pi^\pm e^+ e^- \gamma) = (1.29 \pm 0.13_{\text{stat}} \pm 0.03_{\hat{c}}) \times 10^{-8}$$

- **Model dependent** measurement: use  $\text{BR}(m_{ee\gamma})$  to extract  $\hat{c}$  by fitting to the absolute O(6) ChPT prediction [F. Gabbiani, Phys.Rev.Lett. D59 (1999), 094022].

$$\hat{c} = 0.90 \pm 0.45$$

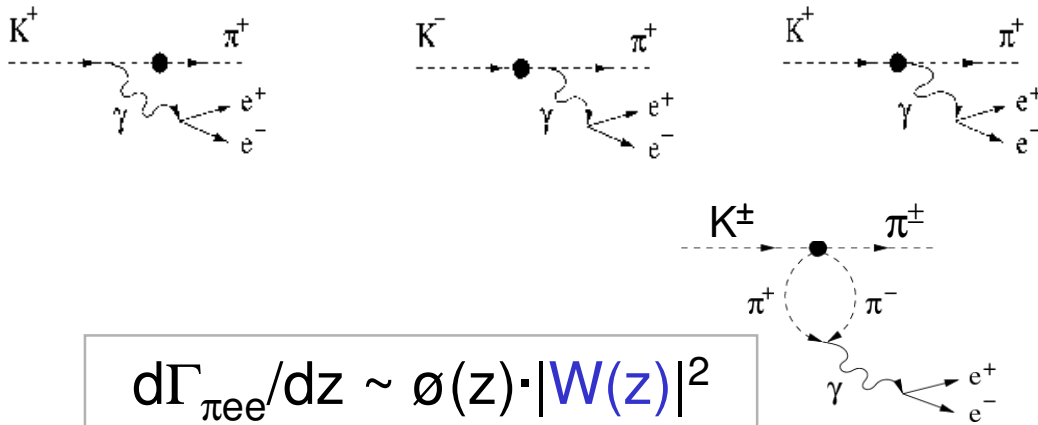
$\chi^2/\text{ndof} = 8.1/17$   
96.4% probability



$$K^{\pm} \rightarrow \pi^{\pm} e^{+} e^{-}$$

# $K^\pm \rightarrow \pi^\pm e^+ e^-$

- **FCNC suppressed** process  $K^\pm \rightarrow \pi^\pm \gamma^* \rightarrow \pi^\pm l^+ l^-$  proceeding through **one-photon exchange**.



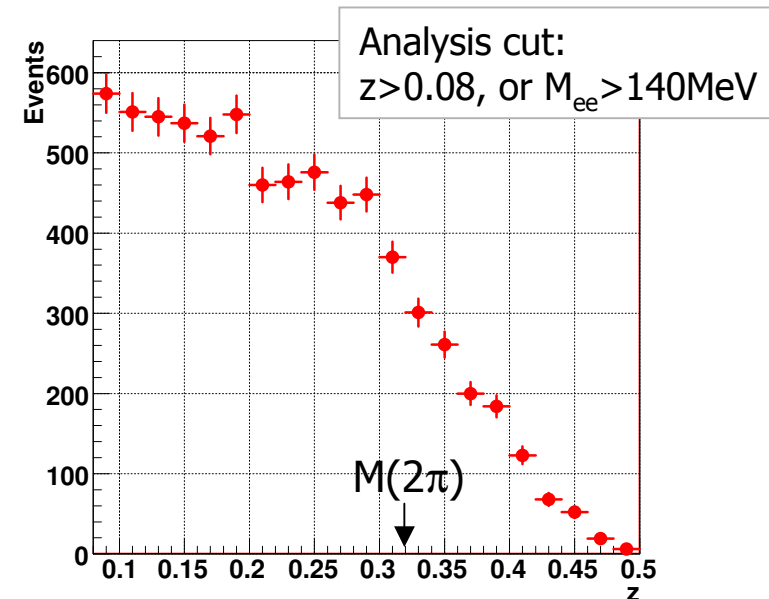
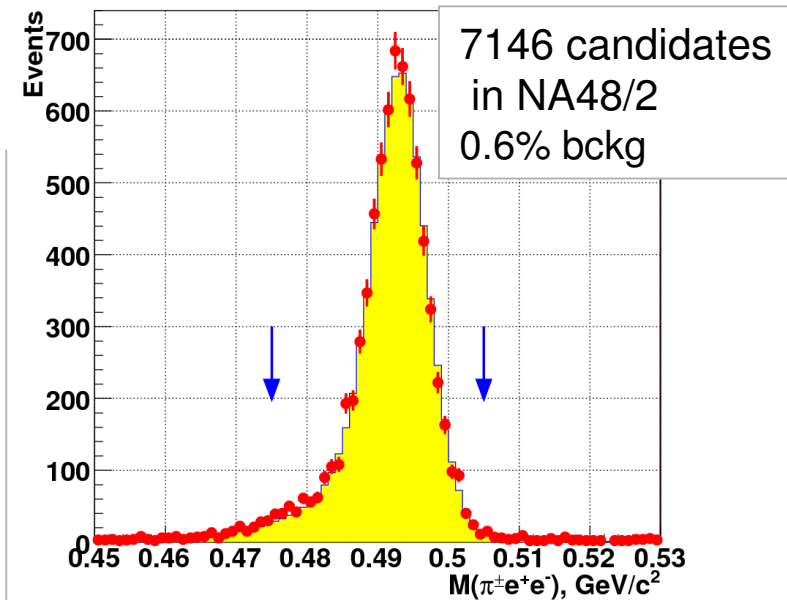
$$d\Gamma_{\pi ee}/dz \sim \varnothing(z) \cdot |W(z)|^2$$

$z = (M_{ee}/M_K)^2$ ,  $\varnothing(z)$  phase space factor.

W(z) Form-factor:

- (1) **polynomial:**  $W(z) = G_F M_K^2 \cdot f_0 \cdot (1 + \delta z)$
- (2) **ChPT  $O(p^6)$ :**  $W(z) = G_F M_K^2 \cdot (a_+ + b_+ z) + W^{\pi\pi}(z)$   
[JHEP 8 (1998) 4]
- (3) **Dubna ChPT:**  $W(z) = W(M_a, M_p, z)$   
[hep-ph/0611175]

⇒ **Form factors** determine model-dependent BR.



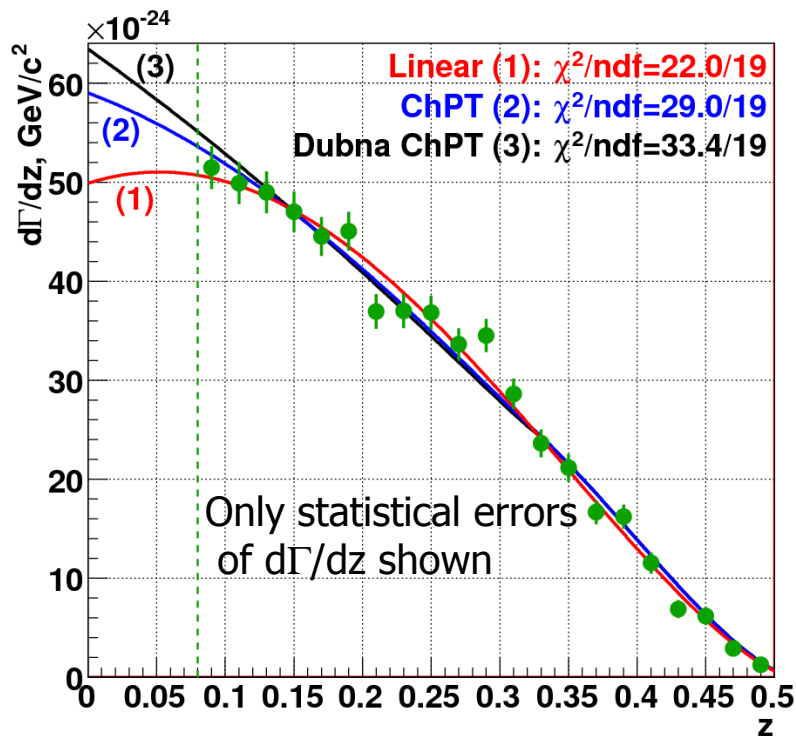
# Br and form factors in $K^\pm \rightarrow \pi^\pm e^+ e^-$

(preliminary)

- **Model-independent Br** computed integrating  $d\Gamma/dz$  :

$$\text{BR}(K^\pm \rightarrow \pi^\pm e^+ e^-, z > 0.08) = (2.26 \pm 0.03_{\text{stat}} \pm 0.03_{\text{syst}} \pm 0.06_{\text{ext}}) \cdot 10^{-7}$$

- **Model-dependent** method: fit  $z$ , obtain form factors and plug them into theoretical prediction of Br for numerical integration.



$$\begin{aligned} \delta &= 2.35 \pm 0.15_{\text{stat}} \pm 0.09_{\text{syst}} \\ (1) \quad f_0 &= 0.532 \pm 0.012_{\text{stat}} \pm 0.008_{\text{syst}} \pm 0.008_{\text{ext}} \\ \text{BR} &= (3.02 \pm 0.04_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.08_{\text{ext}}) \cdot 10^{-7} \end{aligned}$$

$$\begin{aligned} (2) \quad a_+ &= -0.579 \pm 0.012_{\text{stat}} \pm 0.008_{\text{syst}} \pm 0.007_{\text{ext}} \\ b_+ &= -0.798 \pm 0.053_{\text{stat}} \pm 0.037_{\text{syst}} \pm 0.017_{\text{ext}} \\ \text{BR} &= (3.11 \pm 0.04_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.08_{\text{ext}}) \cdot 10^{-7} \end{aligned}$$

$$\begin{aligned} (3) \quad M_a &= 0.965 \pm 0.028_{\text{stat}} \pm 0.018_{\text{syst}} \pm 0.002_{\text{ext}} \\ M_p &= 0.711 \pm 0.010_{\text{stat}} \pm 0.007_{\text{syst}} \pm 0.002_{\text{ext}} \\ \text{BR} &= (3.15 \pm 0.04_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.08_{\text{ext}}) \cdot 10^{-7} \end{aligned}$$

$$\text{BR}(K^\pm \rightarrow \pi^\pm e^+ e^-) = (3.08 \pm 0.12) \times 10^{-7}$$

$$\Delta(K^\pm_{\pi ee}) = \frac{(\text{BR}^+ - \text{BR}^-)}{(\text{BR}^+ + \text{BR}^-)} = (-2.1 \pm 1.5_{\text{stat}} \pm 0.3_{\text{syst}}) \%$$

# Summary

## New Measurements by NA48/2 of ChPT Parameters

★  $\pi\pi$  - **Scattering lengths from  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  and  $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ .**

- Full 2004 being worked on and fitting different theoretical expressions (Bern).
- Isospin breaking symmetry corrections for Ke4 need to be evaluated and included.

★ **Precise study of the  $K^\pm \rightarrow \pi^\pm \gamma \gamma$  decay.**

- **BR** measurement in agreement with ChPT.
- shape analysis and a larger sample (2004 data) coming soon.

★ **First observation of the  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$  decay.**

- **BR** and  $\hat{c}$  measurement in agreement with ChPT.

★ **Precise study of  $K^\pm \rightarrow \pi^\pm e^+ e^-$ .**

- Precision comparable with world's best.
- **BR** and **form factor** measurements in agreement with ChPT and other measurements.
- First limit on **CPV** asymmetry.

# **Backup Slides**

# Other NA48/2 Results

★ Measurement of direct **CP violating charge asymmetries** of the Dalitz plot linear slopes  $A_g = (g^+ - g^-)/(g^+ + g^-)$  in  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  and  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  :

$$A_g^c = (-1.5 \pm 2.2) \cdot 10^{-4} \text{ from } 3.11 \cdot 10^9 \text{ } K^\pm \rightarrow \pi^\pm \pi^+ \pi^- \text{ decays,}$$

$$A_g^n = (1.8 \pm 1.8) \cdot 10^{-4} \text{ from } 9.13 \cdot 10^7 \text{ } K^\pm \rightarrow \pi^\pm \pi^0 \pi^0 \text{ decays.}$$

In agreement with SM. Measurements statistically limited [Eur. Phys. J. C52, 875-891 (2007)].

★ First measurement of Direct Emission and Interference terms in  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  has been performed in the region  $0 \text{ MeV} < T_\pi^* < 80 \text{ MeV}$  :

-  $\text{Frac(DE)} = (3.35 \pm 0.35_{\text{stat}} \pm 0.25_{\text{syst}}) \%$

-  $\text{Frac(INT)} = (-2.67 \pm 0.81_{\text{stat}} \pm 0.73_{\text{syst}}) \%$

- **First evidence** of a non vanishing **interference term**.

★ **Semileptonic** channels [Eur. Phys. J. C50, 329--340 (2007)] :

-  $R_{K_{e3}/K_{2\pi}} = (0.2496 \pm 0.0009_{\text{stat}} \pm 0.0004_{\text{syst}})$

-  $R_{K_{\mu 3}/K_{2\pi}} = (0.1637 \pm 0.0006_{\text{stat}} \pm 0.0003_{\text{syst}})$

-  $R_{K_{\mu 3}/K_{e3}} = (0.656 \pm 0.003_{\text{stat}} \pm 0.001_{\text{syst}})$

★ **Leptonic** channels:

-  $R_{K_{e2}/K_{\mu 2}} = (2.416 \pm 0.043_{\text{stat}} \pm 0.024_{\text{syst}})$  [4670 evts, 2003 preliminary]

-  $R_{K_{e2}/K_{\mu 2}} = (2.455 \pm 0.045_{\text{stat}} \pm 0.041_{\text{syst}})$  [3407 evts, 2004 preliminary]

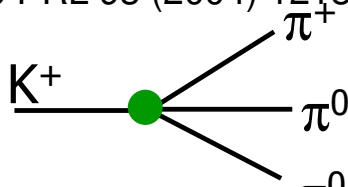
# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$

## Theoretical interpretation :

$$\mathcal{M}(K^\pm \rightarrow \pi^\pm\pi^0\pi^0) = \mathcal{M}_0 + \mathcal{M}_1$$

### At first order:

(N. Cabibbo PRL 93 (2004) 12181)



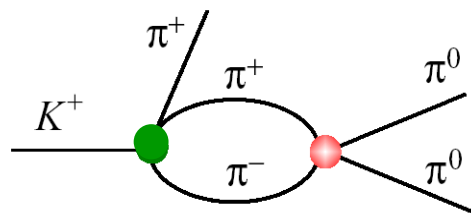
$$\mathcal{M}_0 \propto 1 + g_0 u/2 + h' u^2/2 + k' v^2/2$$

### Kaon rest frame:

$$u = 2m_K \cdot (m_K/3 - E_{\text{odd}})/m_{\pi^2}$$

$$v = 2m_K \cdot (E_1 - E_2)/m_{\pi^2}$$

$$[g_0 \approx g_{\text{PDG}}, h' \approx h_{\text{PDG}} - g_{\text{PDG}}^2/4, k'_{\text{PDG}} \approx k]$$



$\mathcal{M}_1$  real and negative for  $M_{00}^2 < 4m_{\pi^+}^2$  :

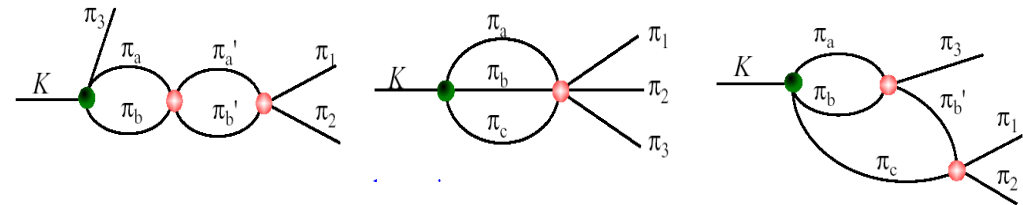
$$\mathcal{M}_1 \propto -(a_0 - a_2)m_{\pi^+} \mathcal{M}_{3\pi, \text{th}} \cdot \pi \sqrt{(4m_{\pi^+}^2/M_{00}^2 - 1)}$$

$\mathcal{M}_1$  imaginary for  $M_{00}^2 > 4m_{\pi^+}^2$  :

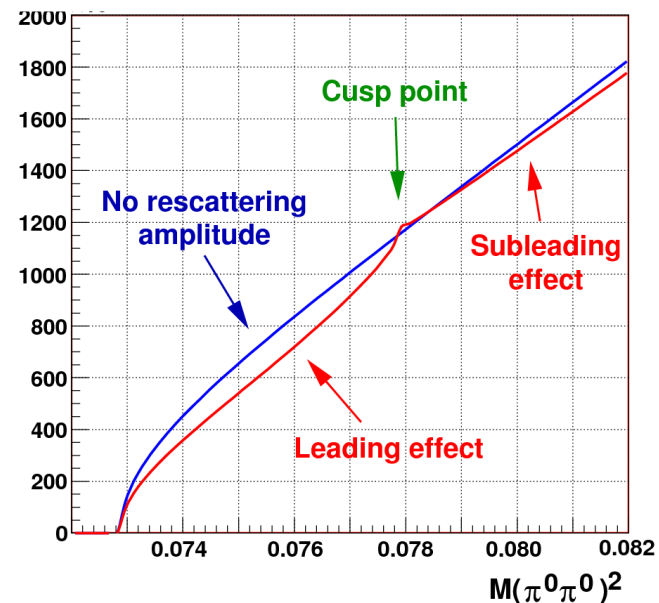
$$\mathcal{M}_1 \propto -(a_0 - a_2)m_{\pi^+} \mathcal{M}_{3\pi, \text{th}} \cdot (-i)\pi \sqrt{(1 - 4m_{\pi^+}^2/M_{00}^2)}$$

### 2<sup>nd</sup> order: (N. Cabibbo, G. Isidori, JHEP03 (2005) 021)

- All other rescattering corrections:  $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ ,  $\pi^+\pi^0 \rightarrow \pi^+\pi^0$ , ...
- 5 S-wave scattering lengths dependent on  $a_0$  and  $a_2$
- Isospin symmetry breaking in
- Two-loop level  $O(a_i^2)$  corrections:



$\Rightarrow$  separate **determination of  $a_2$**  possible



# $\pi^+\pi^-$ Scattering in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ & $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$

## Previous determinations:

### • $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu(\nu)$ decays:

· Geneva-Saclay ('76): ~30000 events.

· BNL E865('03): 400000 events, Isospin symmetry breaking corrections not applied. Measured  $a_0$  and  $a_2$  and obtained:

$$(a_0 - a_2)m_{\pi^+} = 0.258 \pm 0.013$$

### • Pionium ( $\pi^+\pi^-$ atom) lifetime: $\Gamma_{\text{pionium}} \propto (a_0 - a_2)^2$

· DIRAC ('05):  $|a_0 - a_2| m_{\pi^+} = 0.264^{+0.020}_{-0.011}$

• Precise prediction within **ChPT** [Colangelo, Gasser, Leutwyler (2000)]

$$a_0 m_{\pi^+} = 0.220 \pm 0.005 ,$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

# Br ( $K^\pm \rightarrow \pi^\pm \gamma\gamma$ )

- **A model independent** measurement of **Br** in preparation:

Br measured in bins of  $m_{\gamma\gamma}$  (or  $z = m_{\gamma\gamma}^2 / m_K^2$ ) (including acceptances, efficiencies and background) and integrated over whole  $d\Gamma_{\pi\gamma\gamma}/dz$  range.

⇒ This allows to extract  $\hat{\mathbf{c}}$  using measured  $\text{Br}(m_{\gamma\gamma})$  and fitting to absolute O(6) ChPT prediction [G. Ecker, A. Pich and E. de Rafael, Nucl.Phys. B386 (1996), 403]

Other approach (step by step):

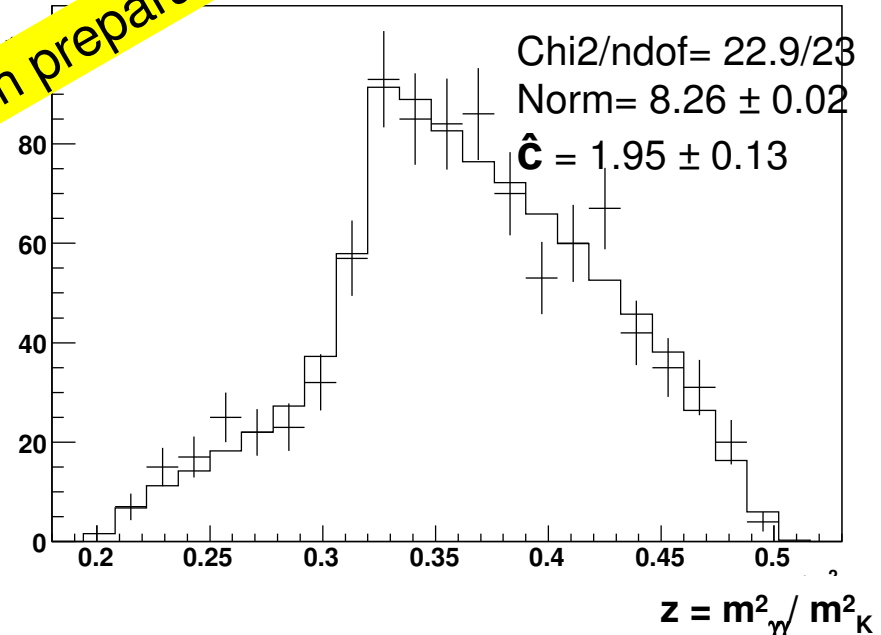
- 1.- Assume no vector mesons and discard poles

⇒ only  $\hat{\mathbf{c}}$  free parameter [G. D'Ambrosio, J. Portolés. Nucl.Phys. B386 (1996), 403].

- 2.- Test for **poles**: include their contribution and fit simultaneously  $\hat{\mathbf{c}}$  and **G8s** ⇒ exclude some ranges for G8s [M. Gerard, C. Smith, S. Trine, hep-ph/0508189].

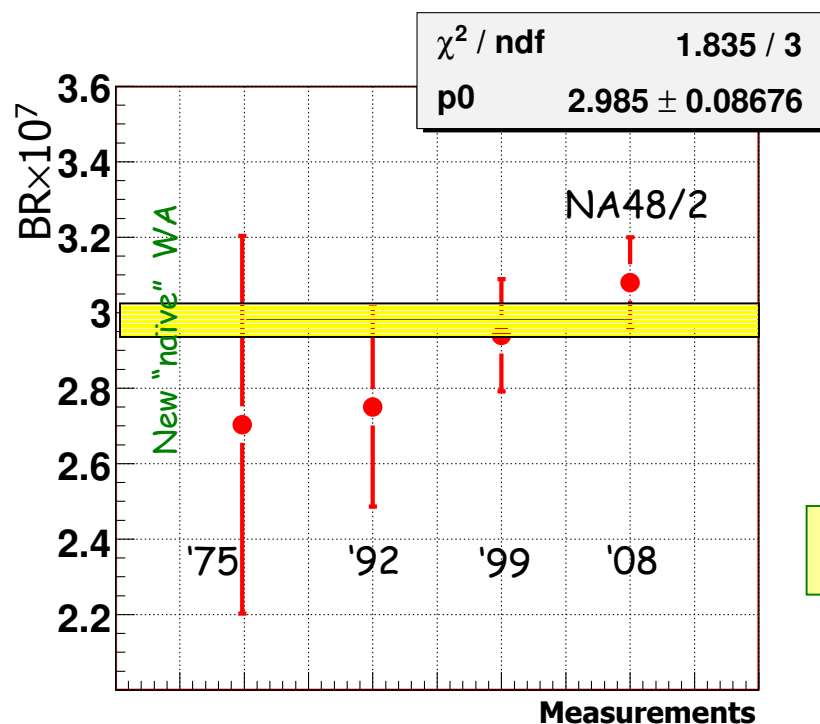
- 3.- To test for **vector mesons**, include the  $\eta_i$ , see if they come out large in fit, and if they approximately follow the VMD ansatz.

In preparation



# BR(K<sup>±</sup>→π<sup>±</sup>e<sup>+</sup>e<sup>-</sup>)

Measurement	Sample	BR×10 <sup>7</sup>
Bloch et al., PL 56 (1975) B201	41 (K <sup>+</sup> )	2.70±0.50
Alliegro et al. [E777], PRL 68 (1992) 278	500 (K <sup>+</sup> )	2.75±0.26
Appel et al. [E865], PRL 83 (1999) 4482	10,300 (K <sup>+</sup> )	2.94±0.15
NA48/2 preliminary (2008)	7,100 (K <sup>±</sup> )	3.08±0.12



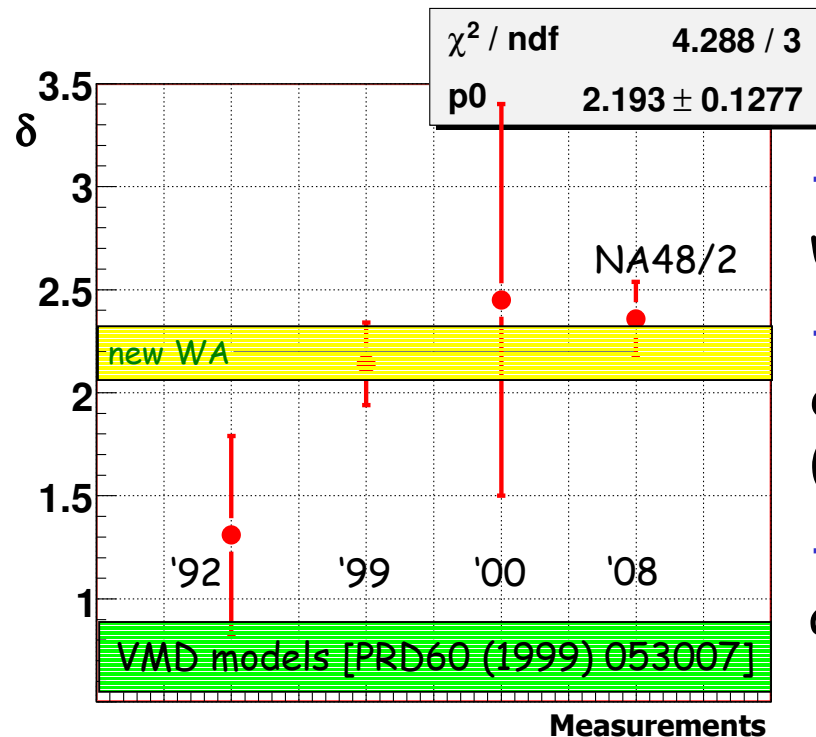
Comparison of E865 vs NA48/2 results:  
 dismissing correlated errors  
 (normalization and model dependence)  
 and using the same PDG BR(K→2π)

BR [E865, linear FF] = 2.88±0.11;  
 BR [NA48, linear FF] = 3.056±0.056.

δ(BR) = (0.176±0.123) → 1.4σ difference  
 [15% probability]

# $\delta$ -Form Factor of $K^\pm \rightarrow \pi^\pm e^+ e^-$

Measurement	Process	Result
Alliegro et al. [E777], PRL 68 (1992) 278	$K^+ \rightarrow \pi^+ e^+ e^-$	$1.31 \pm 0.48$
Appel et al. [E865], PRL 83 (1999) 4482	$K^+ \rightarrow \pi^+ e^+ e^-$	$2.14 \pm 0.20$
Ma et al. [E865], PRL 84 (2000) 2580	$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	$2.45^{+1.30}_{-0.95}$
NA48/2 preliminary (2008)	$K^\pm \rightarrow \pi^\pm e^+ e^-$	$2.35 \pm 0.18$



- NA48/2 measurement of  $\delta$  is compatible with the earlier results, has good precision;
- A contradiction of the data to the meson dominance models observed earlier ( $\delta = (m_K/m_\rho)^2$ ) is further confirmed;
- NA48/2 values of  $(f_0, a_+, b_+)$  are in agreement with BNL E865 ones.

**Leptonic and semileptonic modes**

$\Gamma_1$	$e^+ \nu_e$	$(1.55 \pm 0.07) \times 10^{-5}$	
$\Gamma_2$	$\mu^+ \nu_\mu$	$(63.44 \pm 0.14) \%$	S=1.2
$\Gamma_3$	$\pi^0 e^+ \nu_e$ Called $K_{e3}^+$ .	$(4.98 \pm 0.07) \%$	S=1.3
$\Gamma_4$	$\pi^0 \mu^+ \nu_\mu$ Called $K_{\mu 3}^+$ .	$(3.32 \pm 0.06) \%$	S=1.2
$\Gamma_5$	$\pi^0 \pi^0 e^+ \nu_e$	$(2.2 \pm 0.4) \times 10^{-5}$	
$\Gamma_6$	$\pi^+ \pi^- e^+ \nu_e$	$(4.09 \pm 0.09) \times 10^{-5}$	
$\Gamma_7$	$\pi^+ \pi^- \mu^+ \nu_\mu$	$(1.4 \pm 0.9) \times 10^{-5}$	
$\Gamma_8$	$\pi^0 \pi^0 \pi^0 e^+ \nu_e$	$< 3.5 \times 10^{-6}$	CL=90%

**Hadronic modes**

$\Gamma_9$	$\pi^+ \pi^0$	$(20.92 \pm 0.12) \%$	S=1.1
$\Gamma_{10}$	$\pi^+ \pi^0 \pi^0$	$(1.757 \pm 0.024) \%$	S=1.1
$\Gamma_{11}$	$\pi^+ \pi^+ \pi^-$	$(5.590 \pm 0.031) \%$	S=1.1

**Leptonic and semileptonic modes with photons**

$\Gamma_{12}$	$\mu^+ \nu_\mu \gamma$	[a,b] $(6.2 \pm 0.8) \times 10^{-3}$	
$\Gamma_{13}$	$\mu^+ \nu_\mu \gamma(\text{SD}^+)$	[c] $< 3.0 \times 10^{-5}$	CL=90%
$\Gamma_{14}$	$\mu^+ \nu_\mu \gamma(\text{SD}^+\text{INT})$	[c] $< 2.7 \times 10^{-5}$	CL=90%
$\Gamma_{15}$	$\mu^+ \nu_\mu \gamma(\text{SD}^- + \text{SD}^-\text{INT})$	[c] $< 2.6 \times 10^{-4}$	CL=90%
$\Gamma_{16}$	$e^+ \nu_e \gamma(\text{SD}^+)$	[c] $(1.52 \pm 0.23) \times 10^{-5}$	
$\Gamma_{17}$	$e^+ \nu_e \gamma(\text{SD}^-)$	[c] $< 1.6 \times 10^{-4}$	CL=90%
$\Gamma_{18}$	$\pi^0 e^+ \nu_e \gamma$	[a,b] $(2.69 \pm 0.20) \times 10^{-4}$	
$\Gamma_{19}$	$\pi^0 e^+ \nu_e \gamma(\text{SD})$	[c] $< 5.3 \times 10^{-5}$	CL=90%
$\Gamma_{20}$	$\pi^0 \mu^+ \nu_\mu \gamma$	[a,b] $(2.4 \pm 0.8) \times 10^{-5}$	
$\Gamma_{21}$	$\pi^0 \pi^0 e^+ \nu_e \gamma$	$< 5 \times 10^{-6}$	CL=90%

**Hadronic modes with photons**

$\Gamma_{22}$	$\pi^+ \pi^0 \gamma$	[a,b] $(2.75 \pm 0.15) \times 10^{-4}$	
$\Gamma_{23}$	$\pi^+ \pi^0 \gamma(\text{DE})$	[b,d] $(4.4 \pm 0.7) \times 10^{-6}$	
$\Gamma_{24}$	$\pi^+ \pi^0 \pi^0 \gamma$	[a,b] $(7.6 \pm 5.6 \pm 3.0) \times 10^{-6}$	
$\Gamma_{25}$	$\pi^+ \pi^+ \pi^- \gamma$	[a,b] $(1.04 \pm 0.31) \times 10^{-4}$	
$\Gamma_{26}$	$\pi^+ \gamma \gamma$	[b] $(1.10 \pm 0.32) \times 10^{-6}$	
$\Gamma_{27}$	$\pi^+ 3\gamma$	[b] $< 1.0 \times 10^{-4}$	CL=90%

**Leptonic modes with  $\ell\bar{\ell}$  pairs**

$\Gamma_{28}$	$e^+ \nu_e \nu \bar{\nu}$	$< 6 \times 10^{-5}$	CL=90%
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