

QCD from Kaon decays

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On behalf of NA48/2 collaboration

WIN09

LNGS

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Outline

- Introduction
- The decay $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ (Ke4)
 - Form factor extraction by NA48/II
 - Measurement of a_0 and a_2 scattering lengths
- The “cusp” effect on $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$
 - Description
 - Pionium
 - Measurement of a_0 and a_2 scattering lengths
 - Cusp on $K_L \rightarrow \pi^0 \pi^0 \pi^0$
- Combining Ke4 and cusp results
- Conclusions

Introduction

Kaon decays: what can be learned on QCD at Low Energy?

Semileptonic decay mode $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ (Ke4):

- Small BR (4.1×10^{-5})
- Only two charged pions, very clean environment
- 1.1 Million events analyzed from NA48/2
- Accessible $m_{\pi\pi}$ range from $\pi^+ \pi^-$ threshold to $(m_K - m_e) \cong m_K$

Preliminary

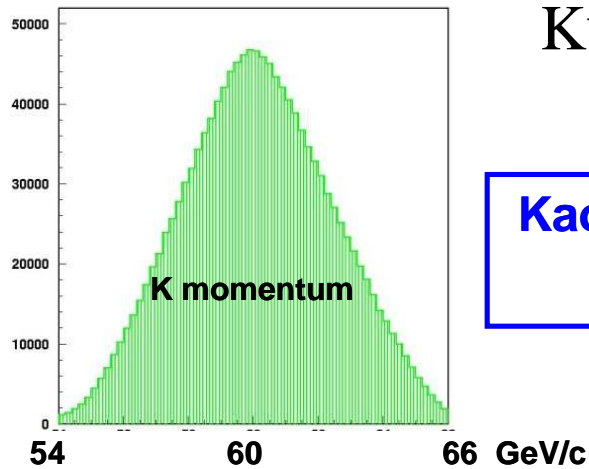
Hadronic decay mode into 3 pions $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$:

- Large BR: (1.7%)
- 60 Millions events now analyzed
- $\pi^0 \pi^0$ system + nearby hadron
- $m_{\pi\pi}$ range from $\pi^0 \pi^0$ threshold to $(m_K - m_\pi)$

In press

Two different but complementary approaches to $\pi\pi$ scattering near threshold to extract s-wave scattering lengths (a_0, a_2) for $I=0$ and $I=2$

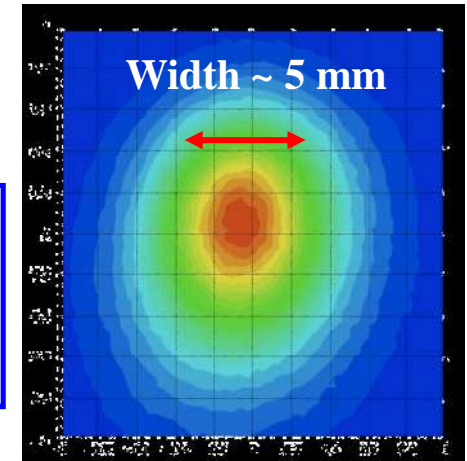
The simultaneous K^+/K^- beams



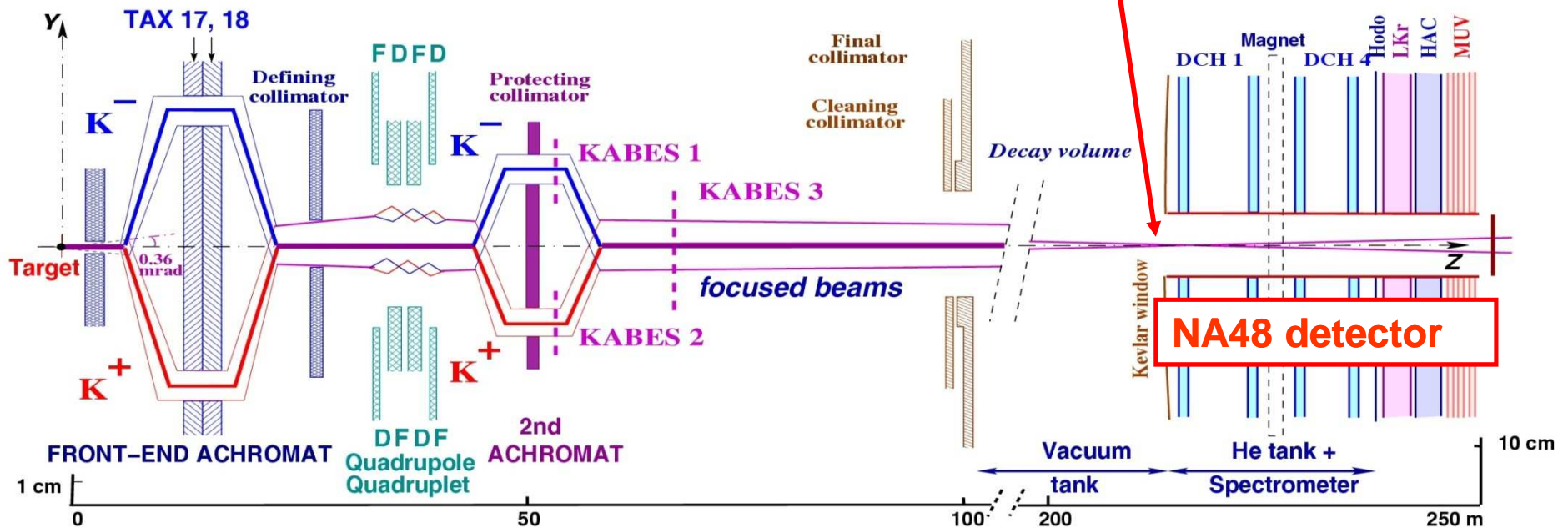
K^+ and K^- superimposed in space
 Flux ratio: $K^+/K^- \sim 1.8$

Kaon momentum:
 60 ± 3 GeV/c

Data taking:
 2003 – 50 days
 2004 – 60 days



Beams coincide within 1 mm



The NA48 detector

Magnetic spectrometer(4 DCHs):

4 views: redundancy \Rightarrow efficiency

$\sigma(p)/p = 1.0\% + 0.044\% p [\text{GeV}/c]$

Charged hodoscope (scintillators):

Fast trigger and precise time measurement (~ 250 ps on single track)

Liquid Krypton E.M. Calorimeter (LKr):

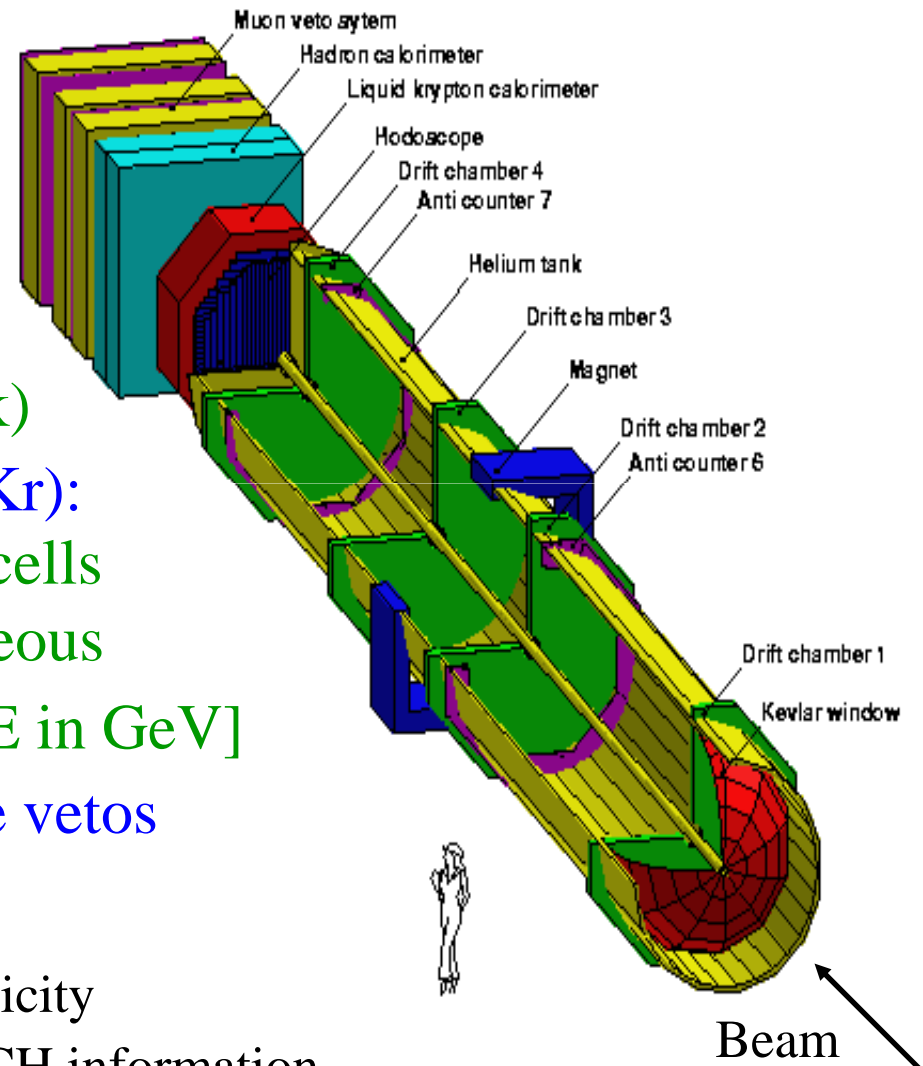
10 m^3 ($\sim 22 \text{ t}$), 1.25 m ($27 X_0$), 13212 cells

granularity: $2 \times 2 \text{ cm}^2$, quasi-homogeneous

$\sigma(E)/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\% [E \text{ in GeV}]$

Then hadronic calorimeter, large angle vetos and muon counter (scintillators)

Trigger: L1: Hodoscope, DCH multiplicity
L2: ON-line processing of DCH information



The Ke4 decay

- **Topology:**

- 3 tracks and 1 vertex
- 1 electron ($E_{LKr}/p_{DCH} \sim 1$)
- Two π of opposite charge
- P_K reconstruction \rightarrow pick the solution closer to 60 GeV/c
- (or alternatively the ν missing mass can be reconstructed under the $P_K=60$ GeV/c assumption)

- **Background sources:**

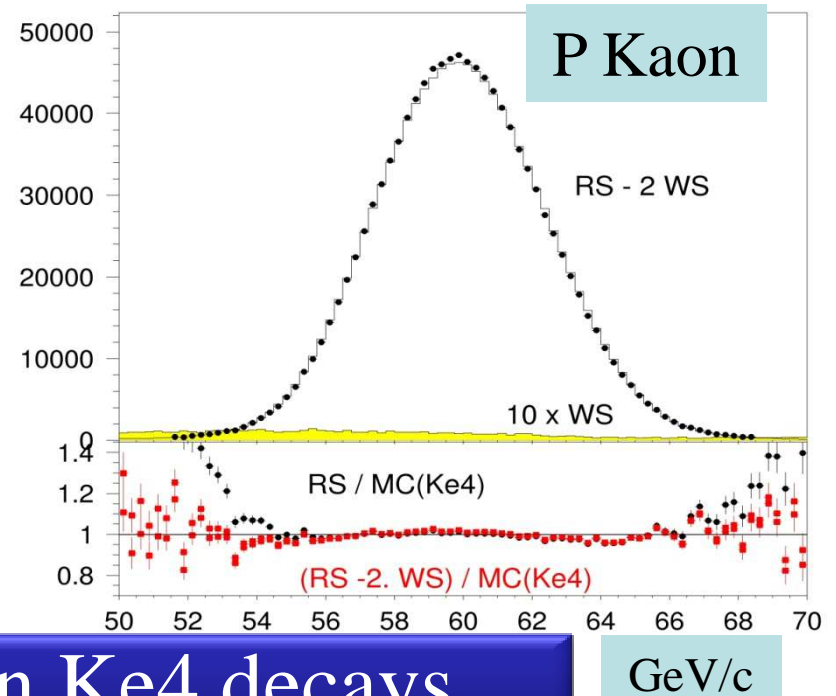
- $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ (dominant) with a $\pi \rightarrow e\nu$ decay or a misID π
- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ with $\pi^0 \rightarrow e^+ e^- \gamma$ decay and with electron misID as π

Total background is $\sim 0.5\%$ wrt signal, estimation from WS events confirmed by MC simulation



- **Background estimations :**

- “wrong sign” selection, with electron charge opposite to the charge of the vertex (kaon) (signal suppressed by a factor 10^{-10} by $\Delta S = \Delta Q$ rule)
- Need to be rescaled according to the relative misID probability:
 - RS/WS=2 if coming from $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$
 - RS/WS=1 if coming from $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$



Total statistics 1.13 Million Ke4 decays

Analysis procedure

5 kinematic variables [Phys.Rev. 137, B438 (1965)]:

$$S_\pi(M_{\pi\pi}^2) \quad S_e(M_{e\nu}^2) \quad \cos\theta_\pi \quad \cos\theta_e \quad \phi$$

3 form factors (F,G,H) – partial waves expansion [Pais-Treiman1968]:

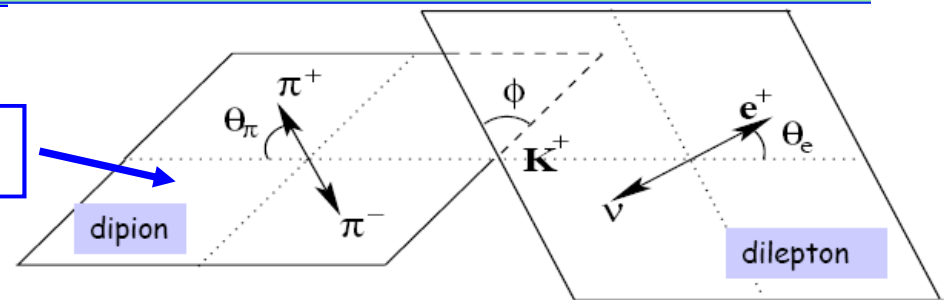
$$\begin{array}{l} \text{axial} \left\{ \begin{array}{l} F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi \\ G = G_p e^{i\delta_g} \end{array} \right. \\ \text{vector} \left\{ \begin{array}{l} H = H_p e^{i\delta_h} + d\text{-wave...} \end{array} \right. \end{array}$$

Expanding also in terms of $q^2 = (S_\pi/4m_\pi - 1)$ and S_e [Amoros-Bijnens1999]:

$$\begin{array}{l} F_s = f_s + f'_s q^2 + f''_s q^4 + f_e (S_e / 4m_\pi^2) + \dots \\ F_p = f_p + f'_p q^2 + \dots \\ G_p = g_p + g'_p q^2 + \dots \\ H_p = h_p + h'_p q^2 + \dots \end{array}$$

Find F_s, F_p, G_p, H_p and $\delta = \delta_s - \delta_p$ in q^2 bins
Extract the slopes from these q^2 dependences

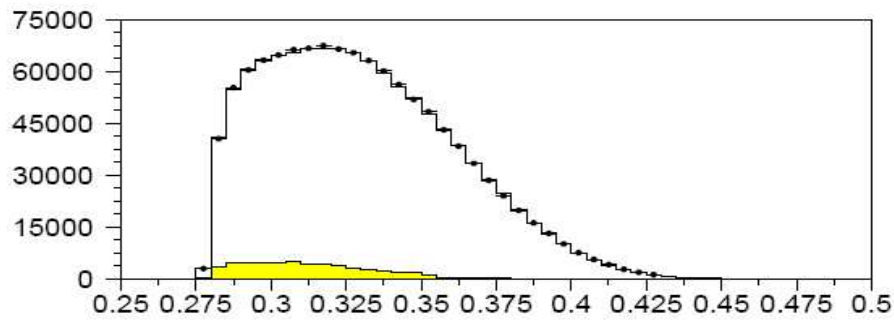
Extract a_0 from $\delta(q^2)$ dependence



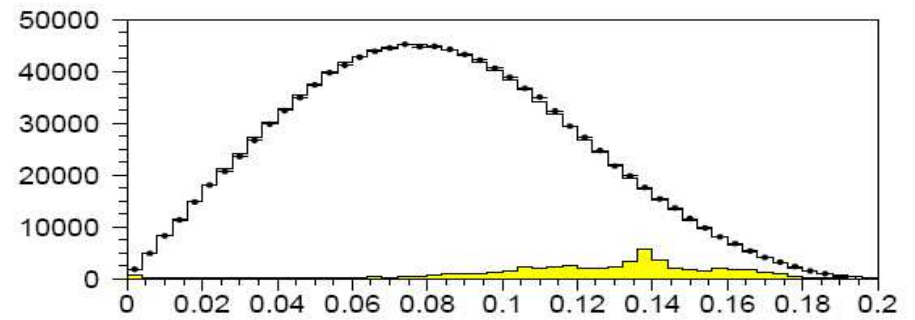
Procedure to fit the form factors:

- Define 10x5x5x5x12 iso populated bins in $(M_{\pi\pi}, M_{e\nu}, \cos\theta_\pi, \cos\theta_e, \phi)$
- 10 independent fits (one fit per $M_{\pi\pi}$ bin) for $F_s, F_p, G_p, H_p, \delta$ in 4D space
- The 4 sets of points are fitted with polynomials **in powers of q^2** (Taylor expansion valid in isospin symmetry limit)
- “Model independent” analysis** → no assumption is made on variation of δ and FF from one bin to the next
- Without the overall normalization from branching fraction, only **relative form factors** $(F_p, G_p, H_p)/F_s$ are measured

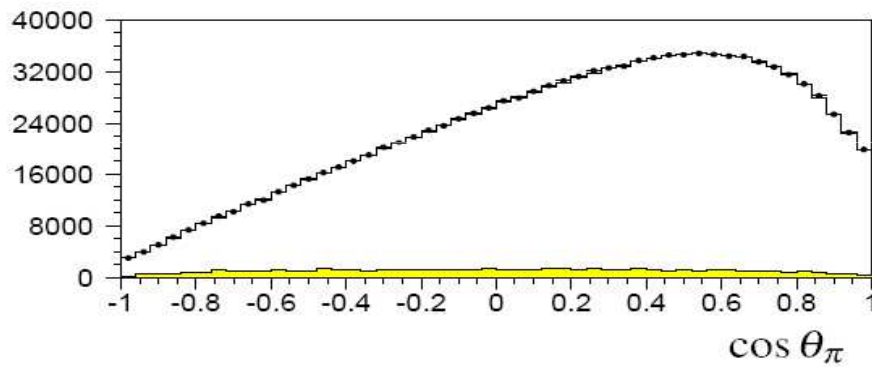
Ke4: kinematic variables



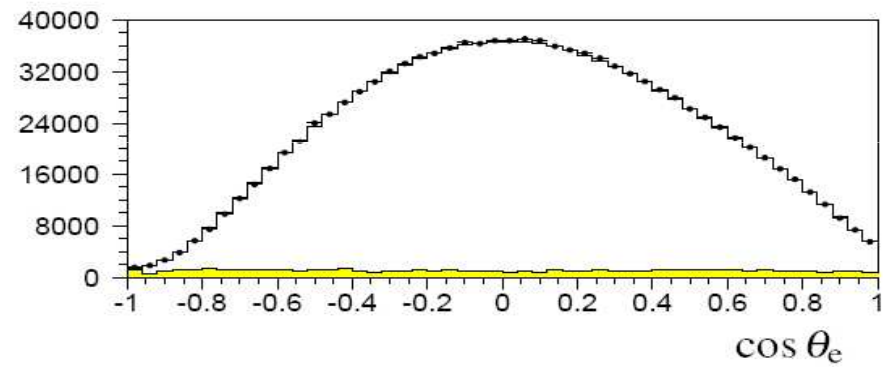
$M_{\pi\pi}$ (GeV/c^2)



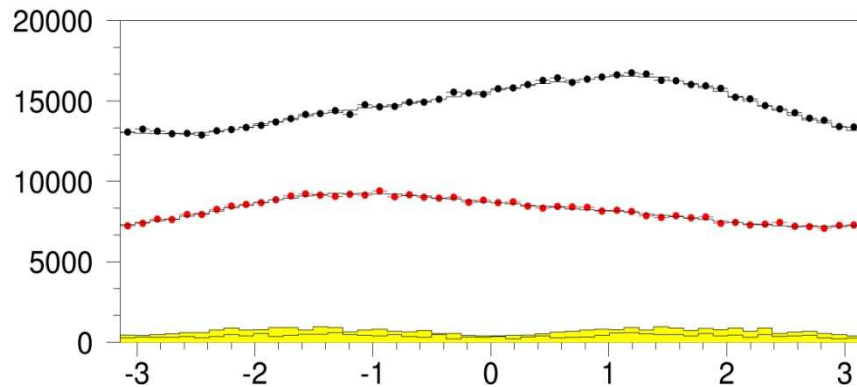
M_{ev} (GeV/c^2)



$\cos \theta_{\pi}$



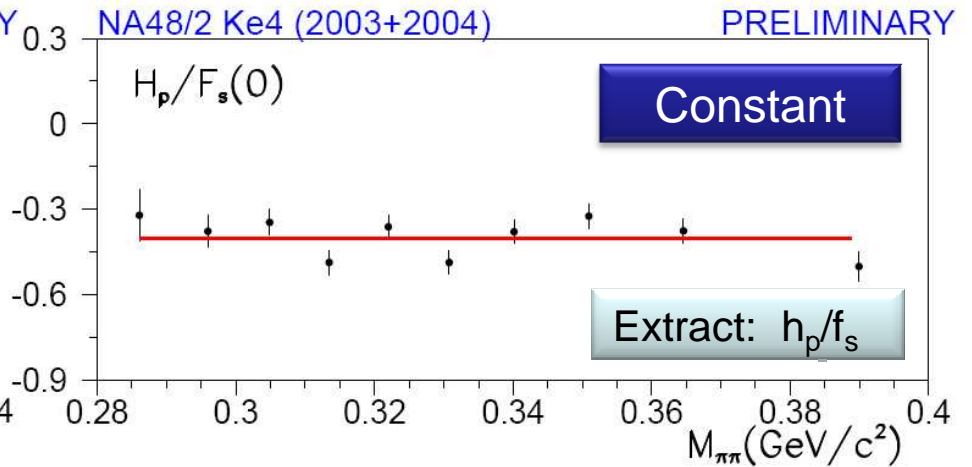
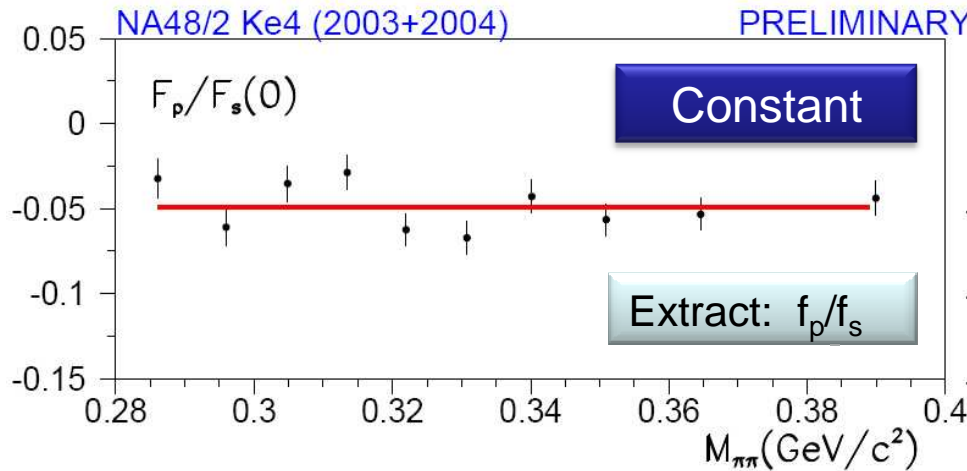
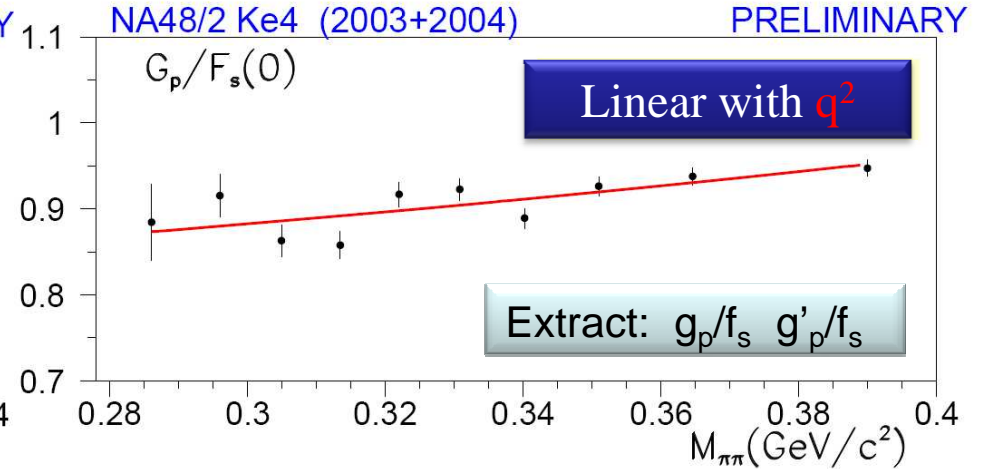
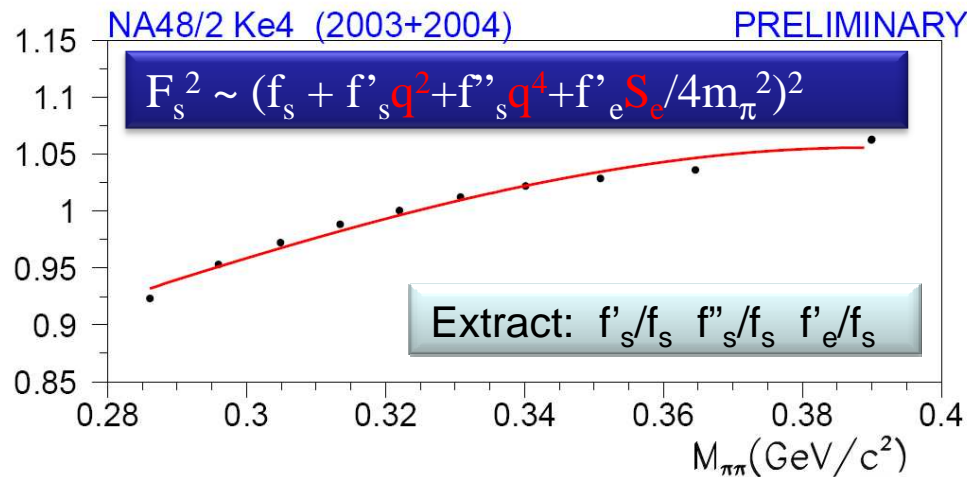
$\cos \theta_e$



	= Data
	= Simulation after fit
	= WS Background (x 10 to be visible)

CP symmetry :
 $(K^+) \phi$ distribution is opposite of $(K^-) \phi$ distribution

Form factors measurement



Results

Preliminary
2003+2004

f'_s/f_s	$0.152 \pm 0.007_{\text{stat}} \pm 0.005_{\text{syst}}$
f''_s/f_s	$-0.073 \pm 0.007_{\text{stat}} \pm 0.006_{\text{syst}}$
★ f'_e/f_s	$0.068 \pm 0.006_{\text{stat}} \pm 0.007_{\text{syst}}$
★ f_p/f_s	$-0.048 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}}$
g_p/f_s	$0.8698 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}}$
g'_p/f_s	$0.089 \pm 0.017_{\text{stat}} \pm 0.013_{\text{syst}}$
h_p/f_s	$-0.398 \pm 0.015_{\text{stat}} \pm 0.008_{\text{syst}}$

★ First evidence by NA48/2

- ◆ Results in agreement with the published 2003 data → **EPJ C54 (2008)**
- ◆ Systematics: mostly from background and from acceptance control, same size as statistical error or smaller, checked also particles ID, beam geometry and radiative corrections

From phase space to scattering lengths

$\pi\pi$ phases at threshold can be predicted from data above 0.8 GeV using Roy Equation (unitarity, analyticity, crossing symmetries).

The S-wave scattering lengths a_0 and a_2 can be extracted from the variation of $\delta = \delta_0^0 - \delta_1^1$

Numerical solutions have been developed (ACGL, DFGS) valid in the Isospin symmetry limit (Universal Band in the $[a_0, a_2]$ plane), but broken in the experimental world

Factorization of electromagnetic and mass effects:

Gamow factor x PHOTOS generator

x

Isospin corrections

Radiative effects: included in the simulation

Gamow factor: “classical” Coulomb attraction between the 2 charged pions

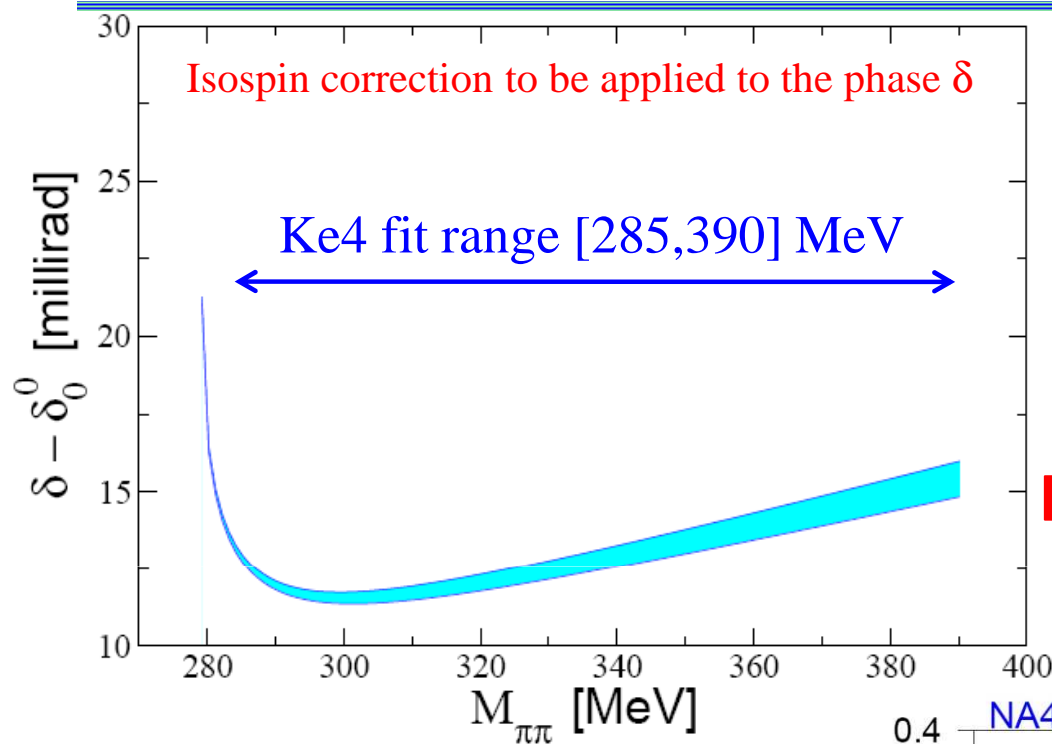
PHOTOS generator: real photon(s) are emitted and tracked in the simulation

→ Effect on event selection + possible bias on reconstructed quantities

Mass effects:

- Recently computed as a correction to the measurements
- Even larger than current experimental precision!
(CGR EPJ C59 (2009) 777, DK preliminary June 2008, in progress)

Effect of isospin correction



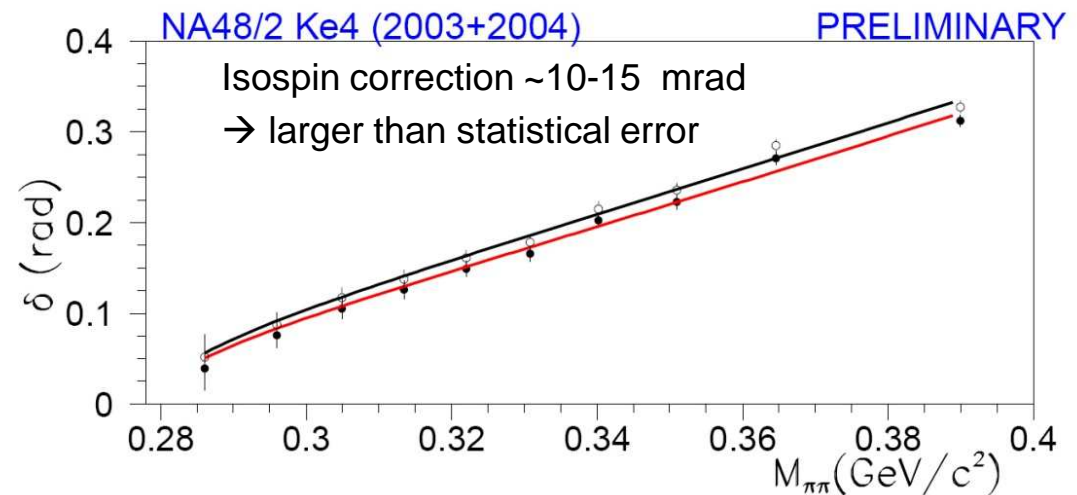
2p fit	Isospin corr OFF	Isospin corr ON
a_0	0.244 ± 0.013	0.218 ± 0.013
a_2	-0.0385 ± 0.0084	-0.0457 ± 0.0084



2 methods for fit:

2p fit → Extraction of a_0 and a_2 as independent parameters

1p fit → Extraction of a_0 assuming χ PT constraint



Results

Preliminary 2003+2004:

2p fit:

$$m_\pi a_0 = 0.2220 \pm 0.0128_{\text{stat}} \pm 0.0050_{\text{syst}} \pm 0.0037_{\text{theor}}$$

$$m_\pi a_2 = -0.0432 \pm 0.0086_{\text{stat}} \pm 0.0034_{\text{syst}} \pm 0.0028_{\text{theor}}$$

1p χ PT fit:

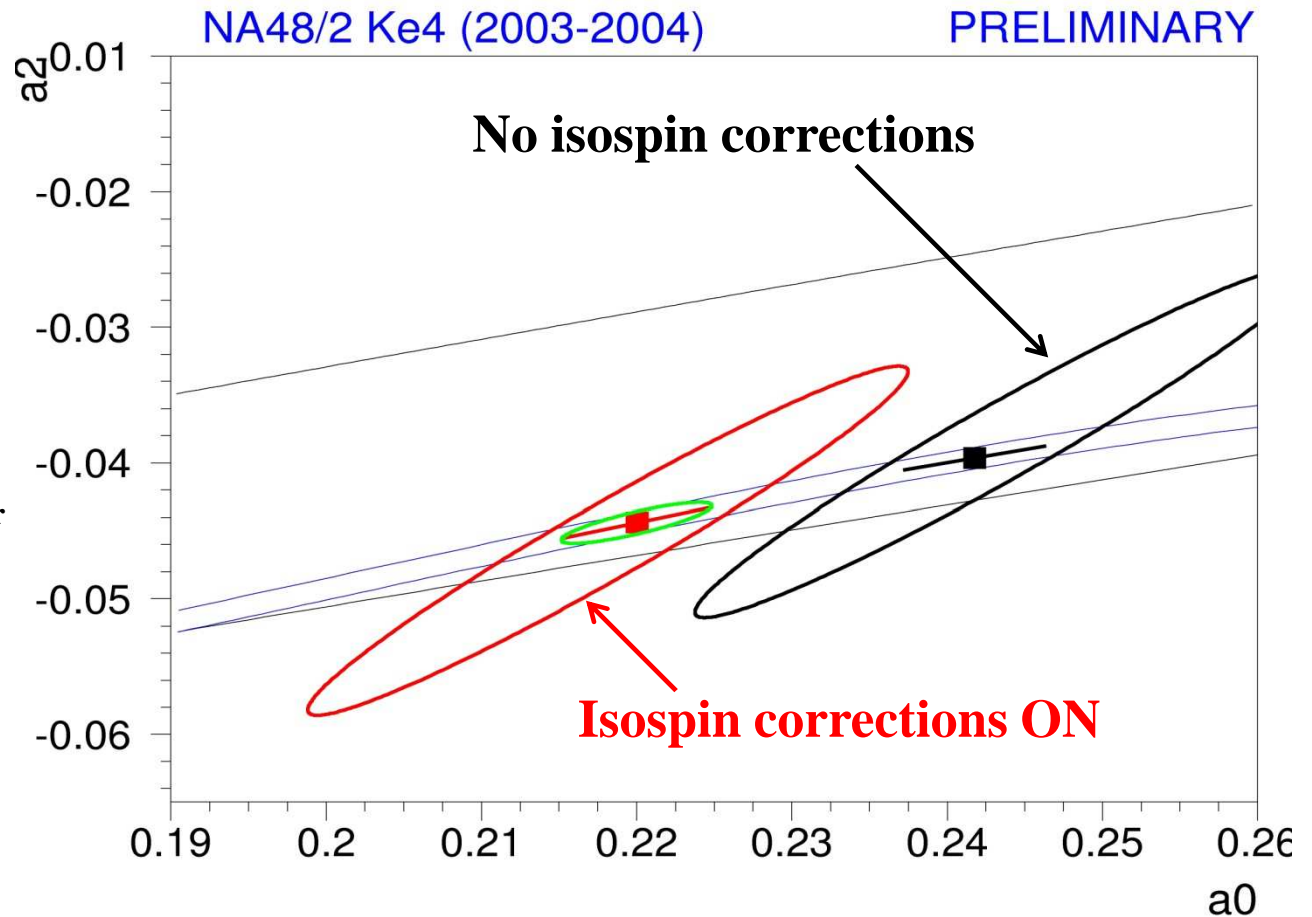
$$m_\pi a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{theor}}$$

χ PT prediction:

(CGL NPB 603(2001), PRL 86(2001))

$$m_\pi a_0 = 0.220 \pm 0.005$$

$$m_\pi a_2 = -0.0444 \pm 0.0008$$



Systematic error → Main contribution from acceptance

Theory error → Control of the isospin corrections & inputs to Roy equation numerical solutions

The $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decay

Assume $\pi^0 \rightarrow \gamma\gamma$ for each $\gamma\gamma$:

$$m_0^2 = 2E_i E_k (1 - \cos\alpha) \approx E_i E_k \alpha^2 = E_i E_k \frac{(D_{ik})^2}{(z_{ik})^2}$$

m_0^2 – mass of π^0

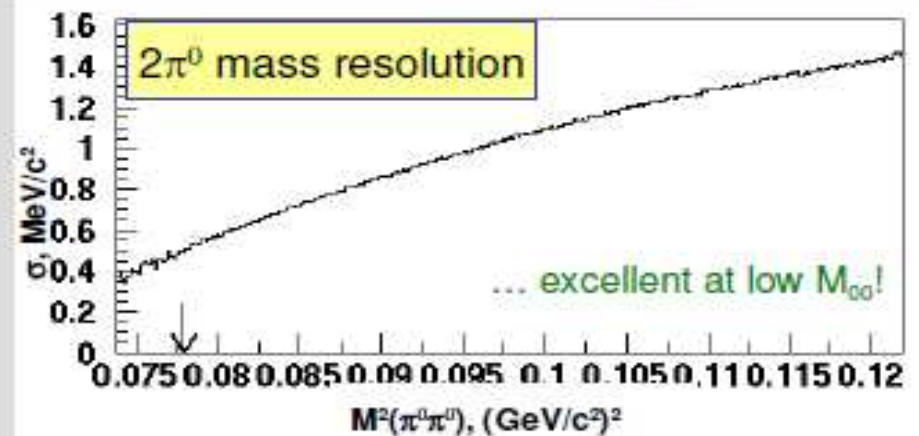
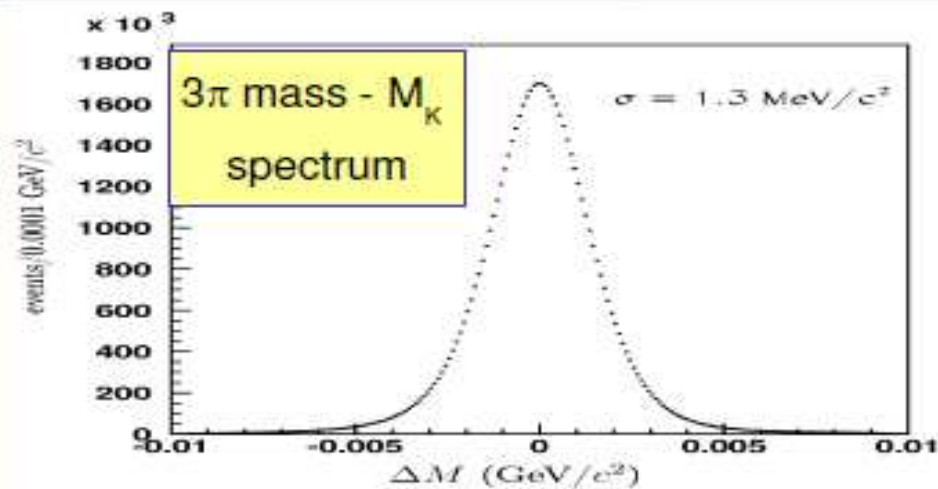
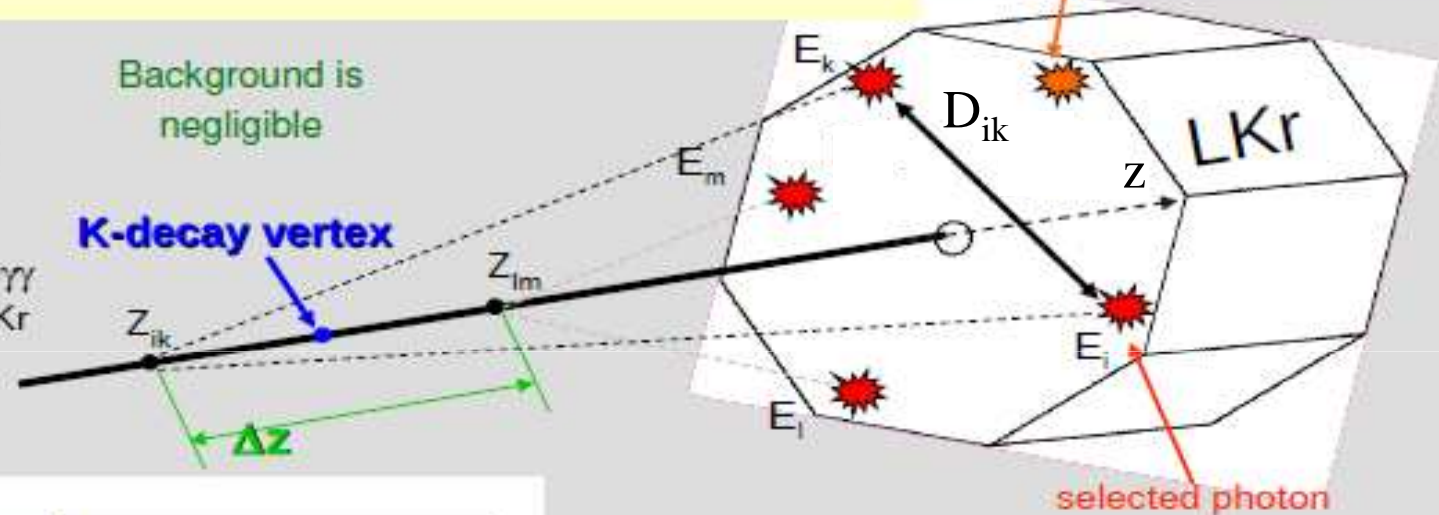
E_i, E_k – energy of γ_i, γ_k

D_{ik} – distance between γ_i and γ_k on LKr

z_{ik} – distance from $\pi^0 \rightarrow \gamma\gamma$ decay vertex to LKr

Background is negligible

K-decay vertex



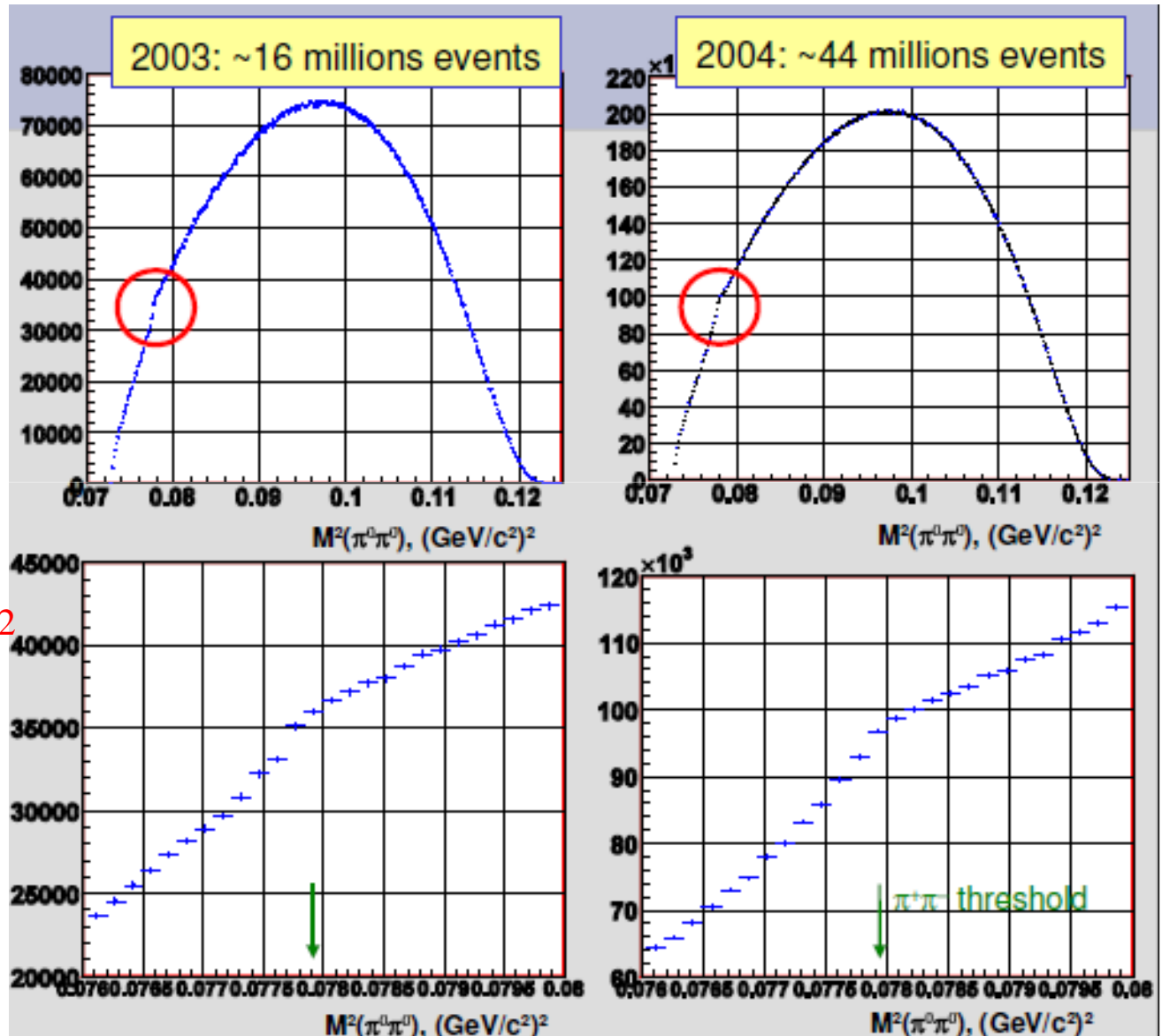
Cusp observation

First observation of the cusp was made on 2003 data

$M^2(\pi^0\pi^0)=M_{00}^2$
invariant mass between the two neutral pions

Cusp at $M^2(\pi^0\pi^0)=4m_\pi^2$

Confirmed by 2004 data (statistics increased by a factor 3.7)



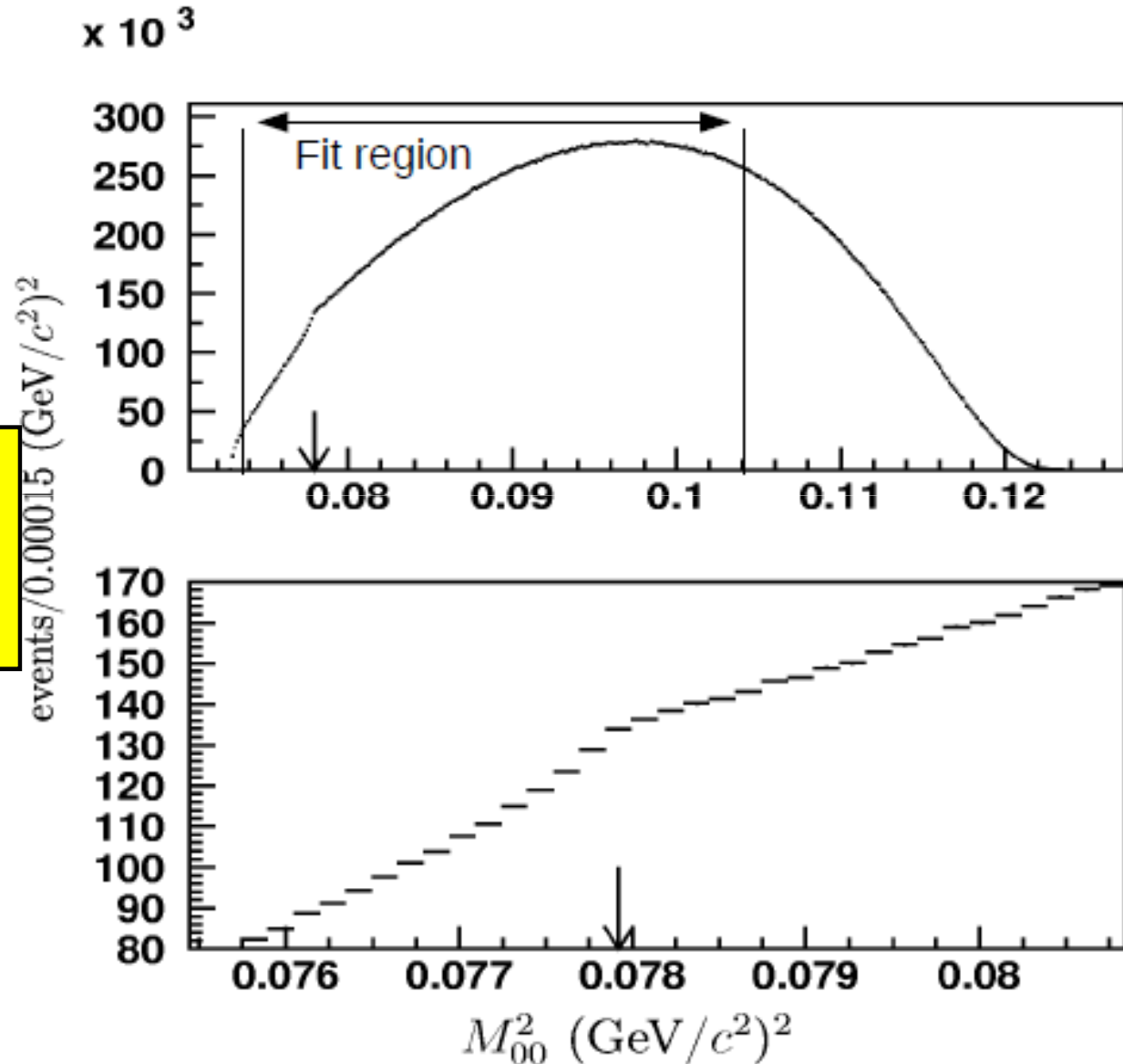
Full statistics

Uniform conditions
for selection and
Data/MC statistics, so
the 2003 and 2004
data were merged for
a joint analysis

2003+2004

60.31 millions events

Fit region is chosen to
reach a minimum total
error (systematical
contribution grows
with upper M_{00})



Theory: final state rescattering

First interpretation by N. Cabibbo, PRL 93 (2004) 121801

$$M(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$$

Direct emission ($k_0, h_0 \ll g$):

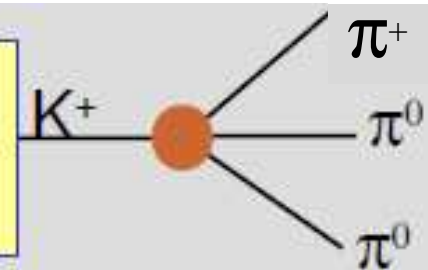
$$M_0 = A_0(1 + g_0 u/2 + h_0 u^2/2 + k_0 v^2/2)$$

$$M_+ = A_+(1 + g_+ u/2 + h_+ u^2/2 + k_+ v^2/2)$$

Rescattering amplitude:

$$M_1 = -2/3(a_0 - a_2)m_+ M_+ \sqrt{1 - \left(\frac{M_{00}}{2m_+}\right)^2}$$

Kaon rest frame:
 $u = 2m_K \cdot (m_K/3 - E_{\text{odd}})/m_\pi^2$
 $v = 2m_K \cdot (E_1 - E_2)/m_\pi^2$

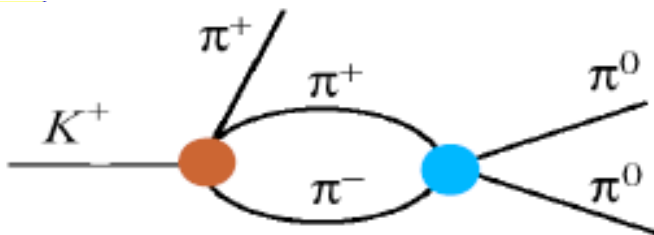


Neg. interference under threshold

Combination of S wave scattering lengths

Amplitude for K^\pm into 3 charged pions

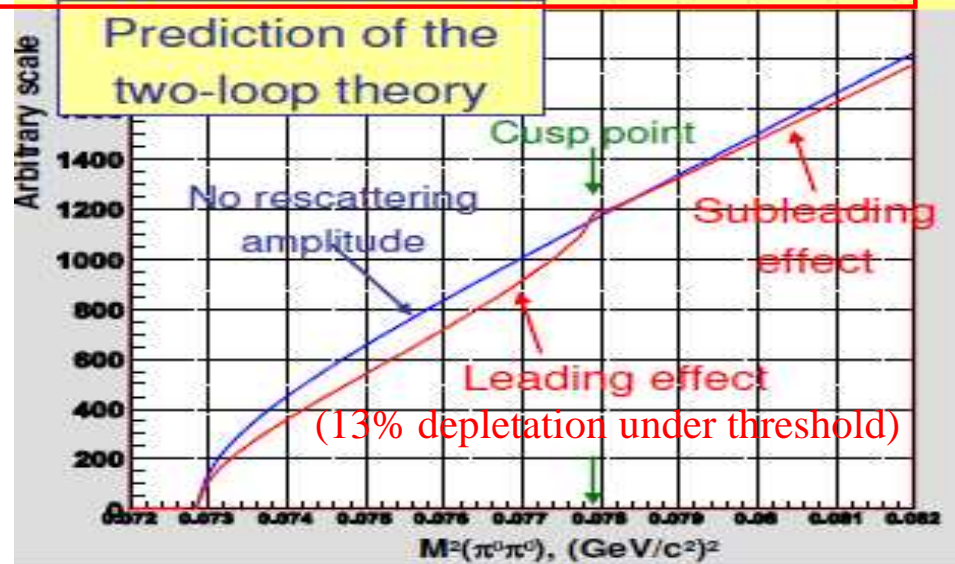
One loop diagram



Improvement: two loop diagram

N. Cabibbo, G. Isidori (CI) JHEP 503 (2005) 21

- 2π scattering
- Irreducible 3π scattering
- Reducible 3π scattering



An alternative view: effective fields theory

G. Colangelo, J. Gasser, B Kubis, A Rusetsky (Bern-Born group, **BB**)
Phys. Lett. B 638 (2007) 187

Non-relativistic Lagrangian for effective fields:

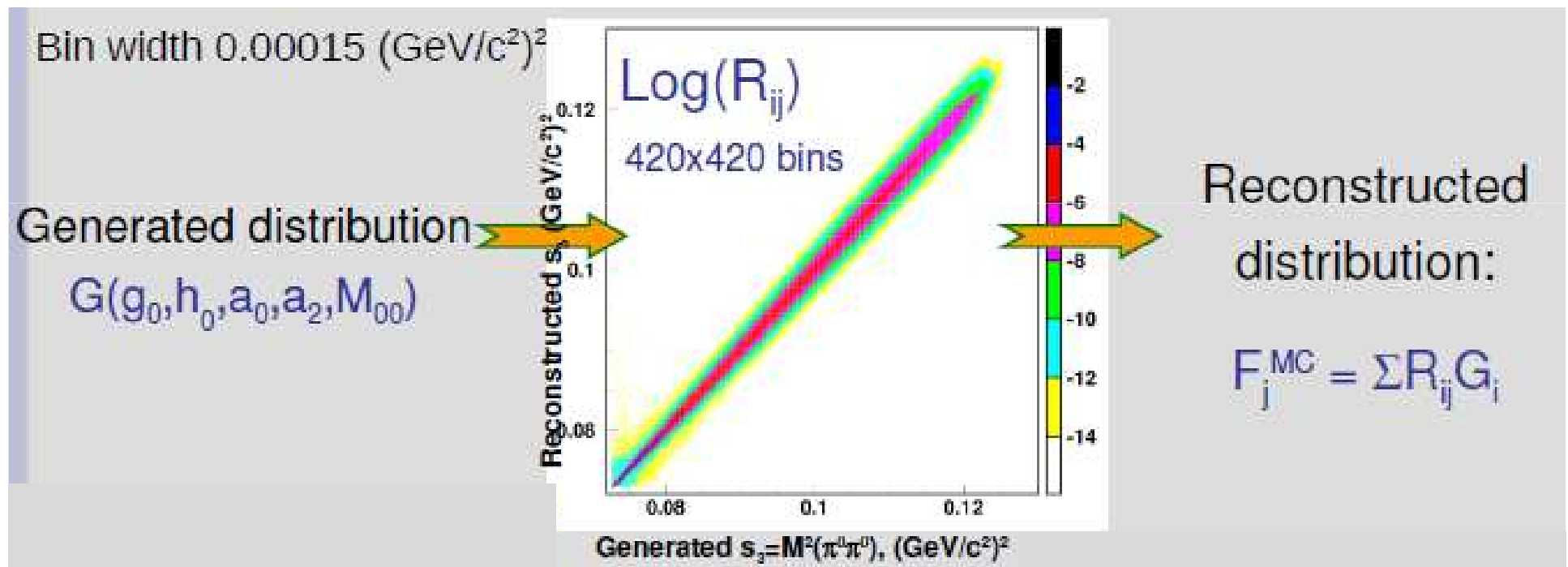
- Valid in the whole decay region
- Another (in comparison with CI) part of the amplitude is absorbed in the polynomial terms
- At two loop, algebraically different formulae for amplitudes

Fortran code for numerical calculation provided by authors to NA48/II collaboration

Fitting procedure

1-dimensional fit of the M_{00}^2 projection

Detector response matrix R_{ij} obtained with a GEANT3-based MonteCarlo simulation



Minuit minimization of χ^2 with data/MC shapes

fixing a_2 or linking it to a_0 by some relation, gives a more precise result for $(a_0 - a_2)$

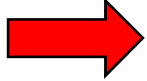
Amplitude parameters for $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

To proceed with the fit amplitude parameters for the decay $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ must be used.

For the **CI** fit we take the parameters measured by NA48/2:

$$M_+ = A_+(1 + g_+ u/2 + h_+ u^2/2 + k_+ v^2/2):$$
$$g_+ = -0.21117(15); \quad h_+ = 0.00671(26); \quad k_+ = -0.00477(8)$$

For the **BB** case the parameters are similar but not the same, due to presence of additional rescattering terms on M_+

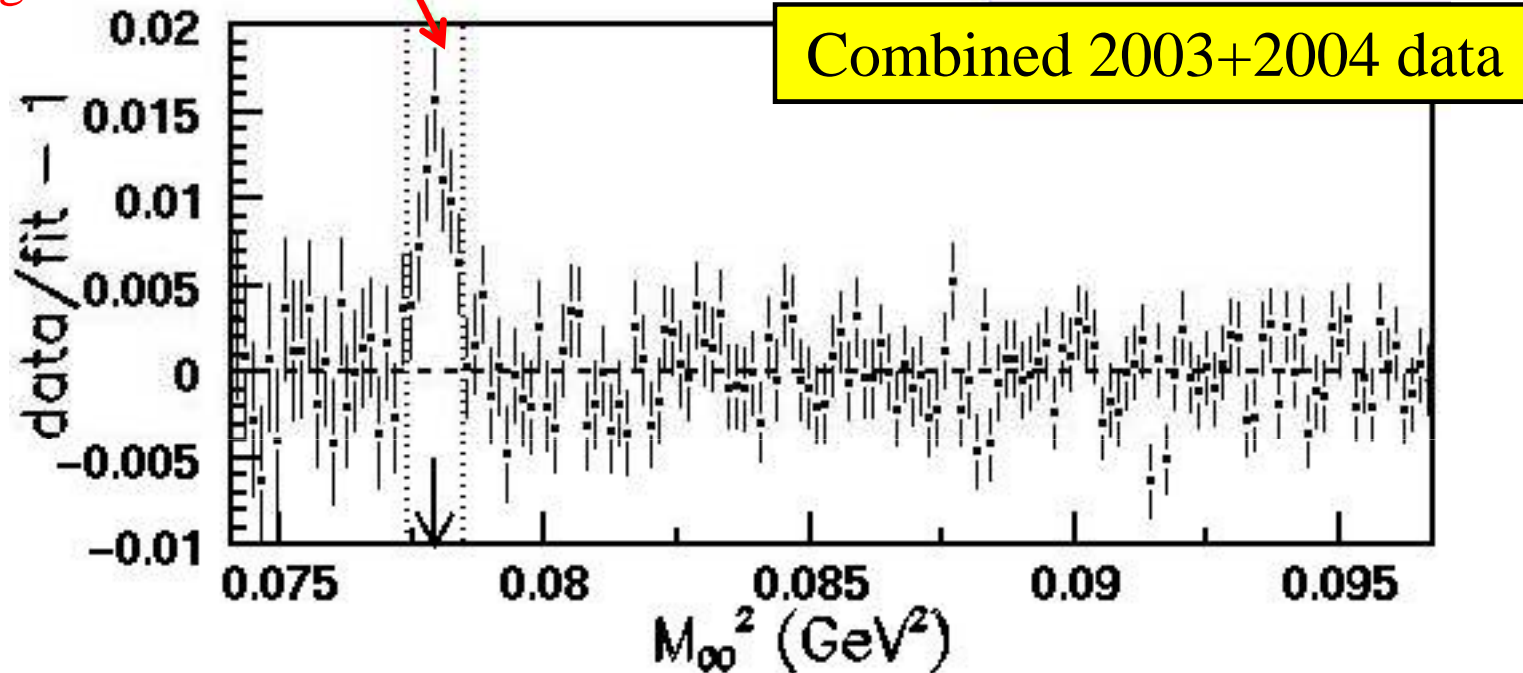
To redefine and measure them  simultaneous fit of the NA48/2 $K^\pm \rightarrow 3\pi$ and $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ Dalitz plots implemented with the BB code:

$$g_+ = -0.1894(4); \quad h_+ = 0.00003(20); \quad k_+ = -0.0077(19)$$

BB radiative corrections to pion rescattering amplitude for 3π decay also included
(M. Bisseger, A. Fuhrer, J. Gasser, B. Kubis, A. Rusetsky Nucl. Phys. B 806, 2009, 178.)

Pionium signature

7 data points excluded from the fit around the threshold due to absence of electromagnetic corrections in the model



Excess of events in the excluded interval (**CI** fit), interpreted as due to pionium decaying as $A_{2\pi} \rightarrow \pi^0 \pi^0$ and giving:

$$R = \Gamma(K^\pm \rightarrow \pi^+ A_{2\pi}) / \Gamma(K^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = (1.8 \pm 0.3) \times 10^{-5}$$

Prediction [Z.K. Silagadze, JETP Lett. 60 (1994) 689] $R = 0.8 \times 10^{-5}$

Discrepancy explained by addition $\pi^+ \pi^-$ unbound states with resonance structure Phys. Lett. B649 (2007)

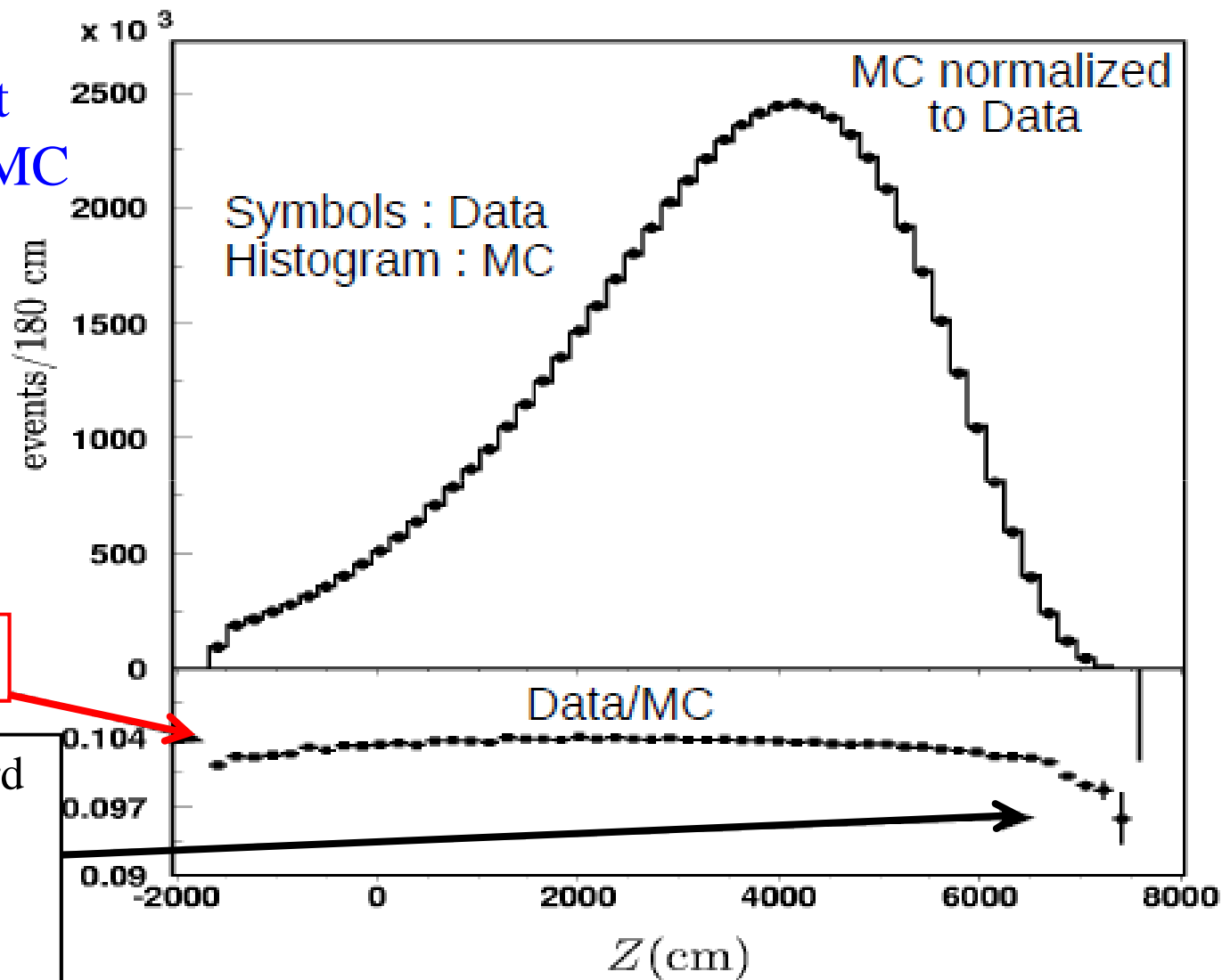
Systematics for independent a_0 and a_2

Source	g_0	h_0	a_0	a_2	$a_0 - a_2$	f_{atom}
Acceptance(Z)	31	21	16	20	4	0
Acceptance(V)	6	1	7	8	1	4
Trigger efficiency	26	22	29	39	10	13
LKr resolution	10	9	21	29	9	60
LKr nonlinearity	34	36	56	67	12	1
P_K spectrum	12	11	18	32	13	10
MC(T)	2	1	4	1	5	25
k_0 error	5	5	4	6	2	1
Hadronic showers	2	4	8	18	10	20
Total systematic	56	50	72	94	25	70
Statistical	47	46	92	129	48	97

In units of 10^{-4}

Systematics from reconstructed vertex

Good agreement
between data and MC



Trigger effect

Difference in forward
decays acceptance
(cut on radius track
and beam shape)

NA48/2 results for independent a_0 and a_2

$$m_\pi(a_0 - a_2) = 0.2571 \pm 0.0048_{\text{stat.}} \pm 0.0025_{\text{syst.}} \pm 0.0014_{\text{ext.}}$$

$$m_\pi a_2 = -0.024 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.002_{\text{ext.}}$$

External uncertainty:

- From the knowledge of the ratio of the decay rates of $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ and $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$
- Additional theoretical uncertainty is estimated from the difference between the **CI** fit and the **BB** one, assuming different next-order contributions:

$$\delta(a_0 - a_2) = 0.088 \quad \delta(a_2) = 0.015$$

The relative errors for a_2 are essentially larger than $(a_0 - a_2)$ ones, as a_2 corresponds to second order contributions to cusp shape

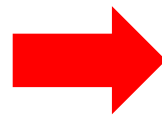
Systematics with χ PT constraint

Source	g_0	h_0	a_0	a_2	$a_0 - a_2$	f_{atom}
Acceptance(Z)	24	14	4	1	3	9
Acceptance(V)	8	4	2	1	2	0
Trigger efficiency	14	16	9	2	7	8
LKr resolution	0	1	2	1	2	46
LKr nonlinearity	12	13	13	3	10	31
P_K spectrum	0	0	2	1	2	5
MC(T)	2	2	6	1	4	24
k_0 error	7	7	0	0	0	2
Hadronic showers	5	3	4	1	3	17
Total systematic	33	26	18	4	14	64
Statistical	9	9	32	8	24	77

In units of 10^{-4}

Results with χ PT constraint

χ PT constraint
(Colangelo et al., PRL 86 (2001) 5008)



$$a_0^2 = -.0444 \pm .0008 + .236 (a_0^0 - .22) - 0.61 (a_0^0 - .22)^2 - 9.9 (a_0^0 - .22)^3$$

$$M_\pi(a_0 - a_2) = 0.2633 \pm 0.0024_{\text{stat.}} \pm 0.0014_{\text{syst.}} \pm 0.0019_{\text{ext.}}$$

External uncertainty from theory is estimated to be:

$$\delta(a_0 - a_2)m_+ = 0.0053 (2\%) \quad (\text{S.Gallorini 2008})$$

$$\chi\text{PT prediction: } a_0 - a_2 = 0.265 \pm 0.004$$

Experimental precision is comparable with the theoretical one!

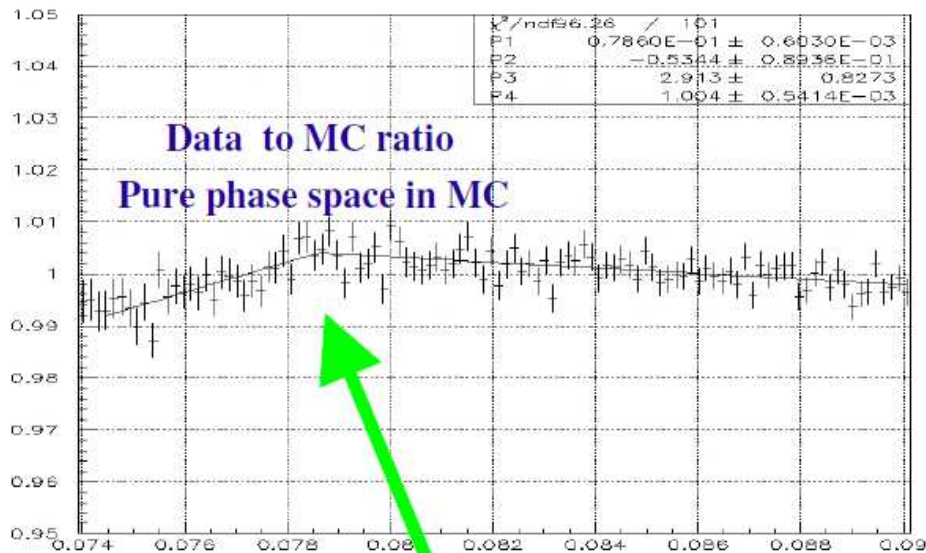
Cusp on $K_L \rightarrow \pi^0 \pi^0 \pi^0$

Cusp also visible on $K_L \rightarrow \pi^0 \pi^0 \pi^0$
(suppressed by a factor 3)

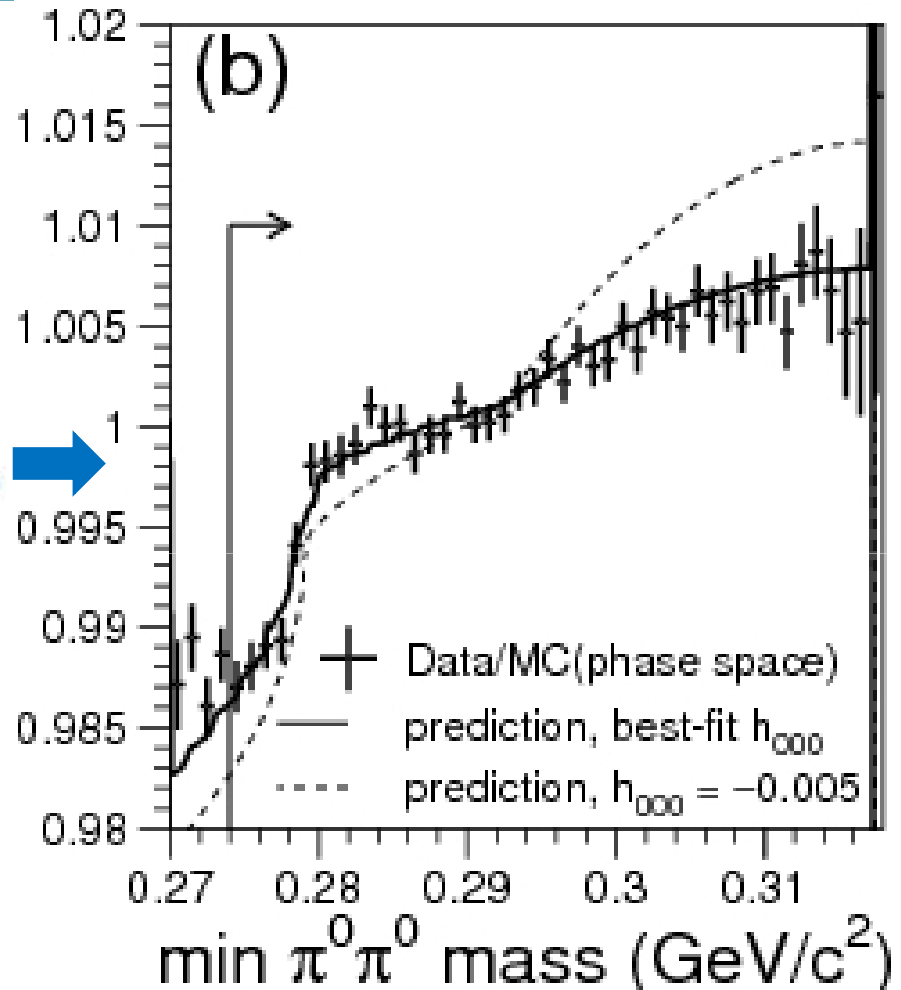
KTeV has collected 68.3 M decays
between 1996 and 1999

Phys. Rev. D 78, 032009 (2008)

$$m_{\pi^+}(a_0 - a_2) = 0.215 \pm 0.014_{stat} \pm 0.025_{syst} \pm 0.006_{ext}$$

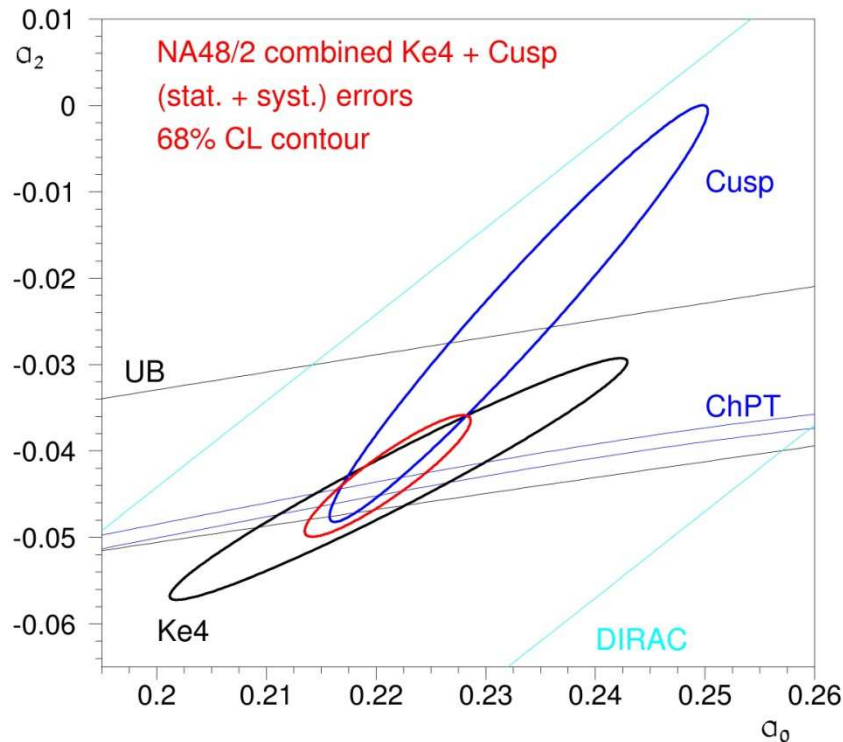


Fitted CUSP position at 0.0786 GeV²

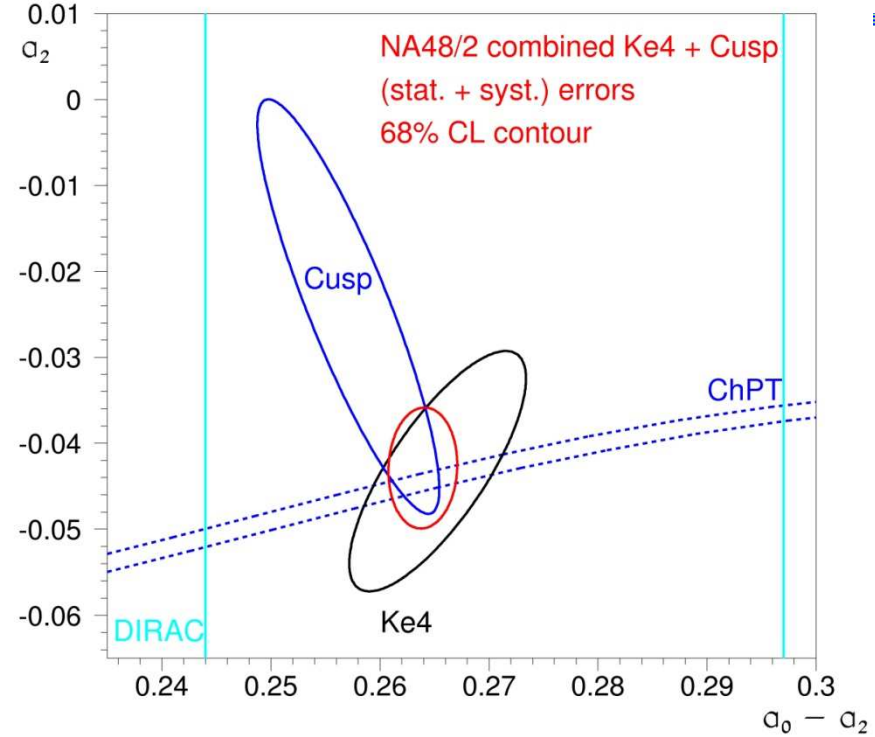


From NA48 100 M events collected
on 2000, work in progress

Combined results from ke4 and cusp



$a_0 = 0.2210 \pm 0.0047 \pm 0.0015 \pm 0.0049$
 $a_2 = -0.0429 \pm 0.0044 \pm 0.0016 \pm 0.0030$
Correlation 0.912
 Total errors $\Delta a_0: \pm 0.0070$ (3% rel. precision)
 $\Delta a_2: \pm 0.0055$ (13% rel. precision)



$a_0 - a_2 = 0.2639 \pm 0.0020 \pm 0.0004 \pm 0.0021$
 $a_2 = -0.0429 \pm 0.0044 \pm 0.0016 \pm 0.0030$
Correlation 0.277
 Total errors $\Delta (a_0 - a_2): \pm 0.0030$ (1% rel. precision)
 $\Delta a_2: \pm 0.0055$ (13% rel. precision)

Including the ChPT constraint:

	stat	syst	theo
$a_2 = -0.0444 \pm 0.007 \pm 0.005 \pm 0.0012$			
$a_0 = 0.2196 \pm 0.0027 \pm 0.0021 \pm 0.0048$	or	$a_0 - a_2 = 0.2640 \pm 0.0020 \pm 0.0017 \pm 0.0035$	
Total error $\Delta a_0: \pm 0.0059$	$\Delta a_2: \pm 0.0015$	$\Delta (a_0 - a_2): \pm 0.0044$	

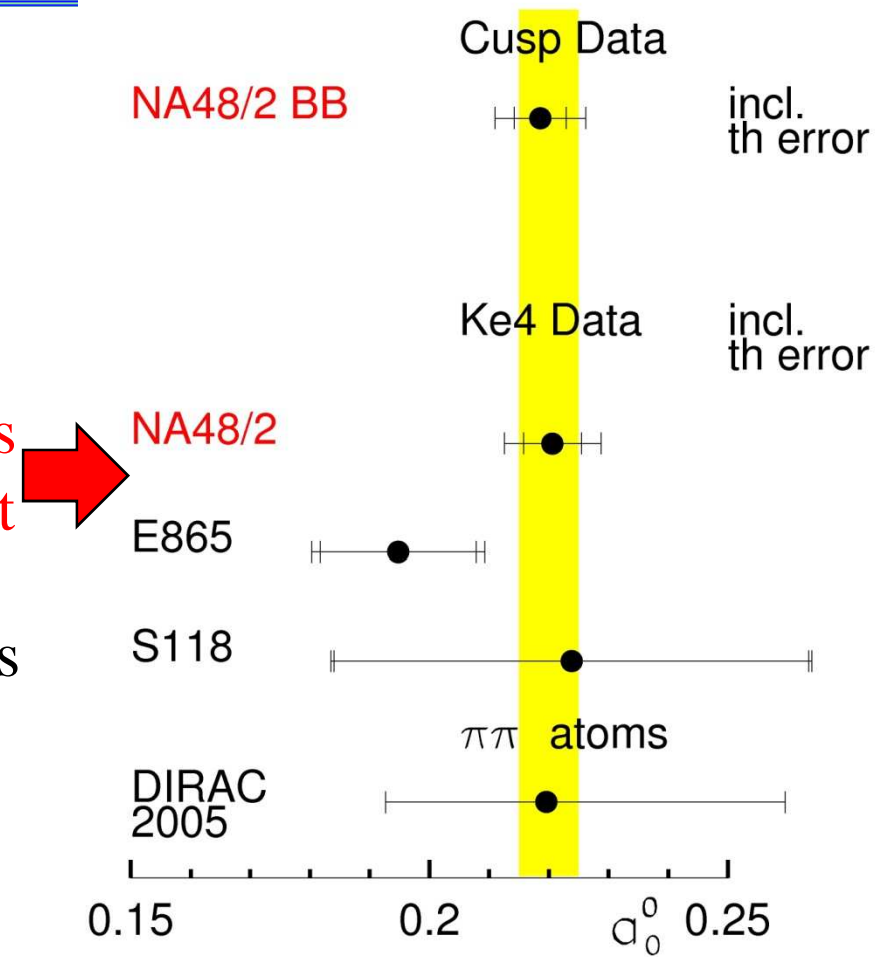
Conclusions

Measurements from the data collected in 2003 and 2004 by the NA48/2 experiment have been shown:

- Ke4 form factors
- $\pi\pi$ scattering lengths from ke4 phases
- $\pi\pi$ scattering lengths from cusp effect

Good agreement of the measured values with χ PT predictions (yellow band):

$$m_\pi a_0 = 0.220 \pm 0.005$$



The experimental error on the values of the scattering amplitudes is now at the same level of the theoretical one