# Results from NA48/2
# Ke4 decays: Form factors and $\pi\pi$ scattering lengths

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On behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

# Outline

- The NA48/2 experiment : for an introduction to beams, detector and performances, refer to previous NA48 talks (R.Wanke, D.Madigozhin..)
- Ke4 results : Form Factors and phase shift : Results from 2003 Data analysis published as EPJC 54 3 (2008) 411
- Ke4 decays ($K^{\pm} \to e^{\pm} \nu \pi^{+}\pi^{-}$) : short reminder about kinematic variables and Form Factors
- Event selection, reconstruction and form factor extraction
- Interpretation in terms of $\pi\pi$ scattering lengths
- Summary

# The NA48/2 experiment: an introduction

The primary goal :

Search for CP-violating charge asymmetries ( $K^+ - K^-$) in $K^\pm \rightarrow 3\pi$ decays

Two measurements : "charged" $\pi^\pm \pi^+\pi^-$ and "neutral" $\pi^\pm \pi^0\pi^0$ asymmetries

both modes with large BR's of $(2\text{-}5)\ 10^{-2}$

but also

Study of many rare and/or radiative decays which properties may be compared to theoretical predictions from ChPT

The "charged" Ke4 ($\pi^+\pi^- e^\pm \nu$ final state,) with small BR's of ~$4\ 10^{-5}$ is a unique laboratory to study $\pi\pi$ scattering at threshold in a clean environment

In the $\pi\pi$ scattering process, it is possible to relate amplitudes with different Isospin using dispersion relations (Roy equations) which depend essentially on two parameters, the scattering lengths $a_0^0$ and $a_0^2$ .

Chiral PT predictions for low energy $\pi\pi$ interaction introduce further constrains between $a_0^0$ and $a_0^2$ which are related to the size of the quark condensate.

# NA48/2 Ke4 published results (EPJC 54-3 (2008) 411

Based on 2003 Data (~50 days)

| | | stat | syst |
|---|---|---|---|
| f's/fs(0) | = | 0.172 | ± 0.009 ± 0.006 |
| f''s/fs(0) | = - | 0.090 | ± 0.009 ± 0.007 |
| f'e/fs(0) | = | 0.081 | ± 0.008 ± 0.008 |
| fp/fs(0) | = - | 0.048 | ± 0.004 ± 0.004 |
| gp/fs(0) | = | 0.873 | ± 0.013 ± 0.012 |
| g'p/fs(0) | = | 0.081 | ± 0.022 ± 0.015 |
| hp/fs(0) | = | -0.411 | ± 0.019 ± 0.008 |

Fit with 2 free parameters :

a0 = 0.233 ± 0.016 (stat) ± 0.007 (syst)

a2 = -0.0471 ± 0.011 (stat) ± 0.004 (syst)

The preliminary result triggered many discussions with theorists

2004 Run ~60 days

Potentially more statistics available, but trigger was modified after first month to allow more $\pi\,\pi 0\pi 0$ triggers at the price of a downscaling of Ke4 triggers …

Additional statistics only 70% more but valuable to improve further the statistical precision on the scattering lengths

More work going on in several groups, some already well advanced on how to include Isospin symetry breaking effects

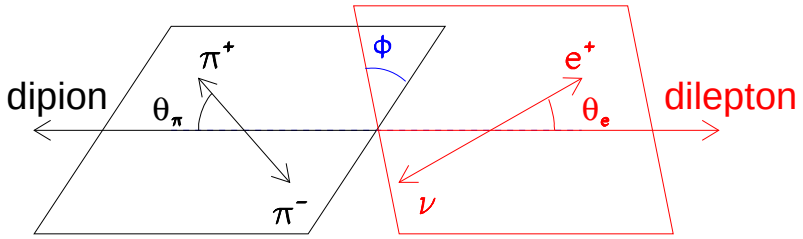
# Ke4 charged decays : 4-body decay formalism

Five kinematic variables

(Cabibbo-Maksymowicz):

2 masses: $S_\pi$ ($M^2_{\pi\pi}$), $S_e$ ($M^2_{e\nu}$),

3 angles : $\cos\theta_\pi$, $\cos\theta_e$ and $\phi$.




partial wave expansion of the amplitude:

F, G = Axial Form Factors

$F = F_s\, e^{i\delta s} + F_p\, e^{i\delta p} \cos\theta_\pi$ + d-wave term...

$G = G_p\, e^{i\delta g}$ + d-wave term...

H = Vector Form Factor

$H = H_p\, e^{i\delta h}$ + d-wave term...

expansion in powers of $q^2$ , $Se/4m\pi^2$

($q^2 = (S_\pi/4m_{\pi^2} - 1)$)

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e\left(S_e/4m_{\pi^2}\right) + ..$$

$$F_p = f_p + f_p' q^2 + ..$$

$$G_p = g_p + g_p' q^2 + ..$$

$$H_p = h_p + h_p' q^2 + ..$$

The fit parameters are : $F_s$ $F_p$ $G_p$ $H_p$ and $\delta = \delta_s - \delta_p$

# Ke4 decays: event selection and background rejection

Signal ( $\pi^+\pi^-e^\pm\nu$) Topology : 3 charged tracks , two opposite sign pions, 1 electron (LKr info E/p ~ 1), some missing energy and $p_T$ (neutrino)

Background : main sources

$\pi^\pm\ \pi^+\ \pi^-$ decay followed by $\pi \rightarrow e\ \nu$ (dominant) or $\pi$ misidentified as e

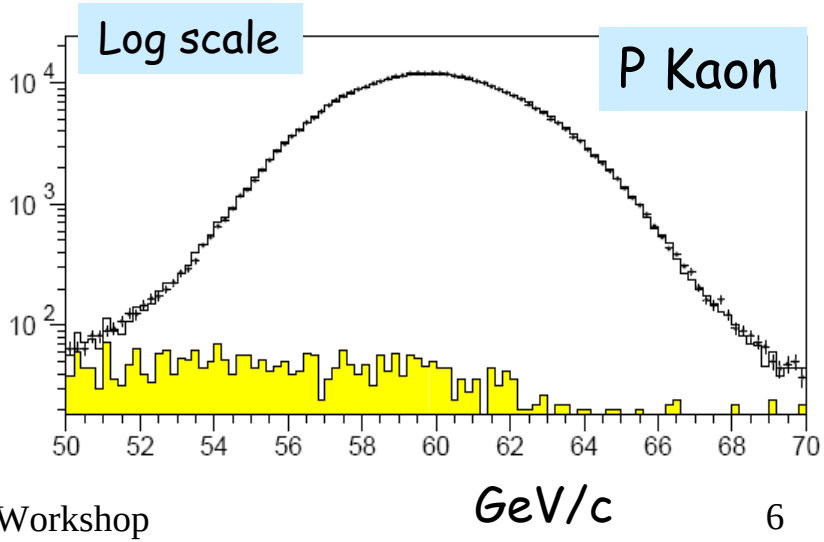
$\pi^\pm\ \pi^0(\pi^0)$ decay + $\pi^0$ Dalitz decay $(e^+e^-\gamma)$ + e misidentified as $\pi$ and $\gamma$ (s) undetected

Total background level can be kept at ~ 0.5 % relative level

Control from data sample : Wrong Sign

-total charge (±1) as Right Sign events
-electron charge opposite to total charge

In RS events:
twice the rate if coming from K3$\pi$
same rate if coming from K2$\pi$($\pi$0)



Ke4 charged decays : Total Data sample (2003 + 2004) = 1.15 M events !

Using iso-populated bins in the 5-dimension space of the C.M. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and $\phi$) one defines a grid of

10x5x5x5x12=15000 boxes.

The set of Form Factor values is used to minimize a log-likelihood estimator taking into account the statistics of the data and simulation in each box.

| | |
|---|---|
| $K^+$ sample (739 500 events) | 49 events/box |
| $K^-$ sample (411 600 events) | 27 events/box |
| MC K+ sample (17.7 Millions events) | ~1180 events/box |
| MC K- sample (9.8 Millions events) | ~653 events/box |

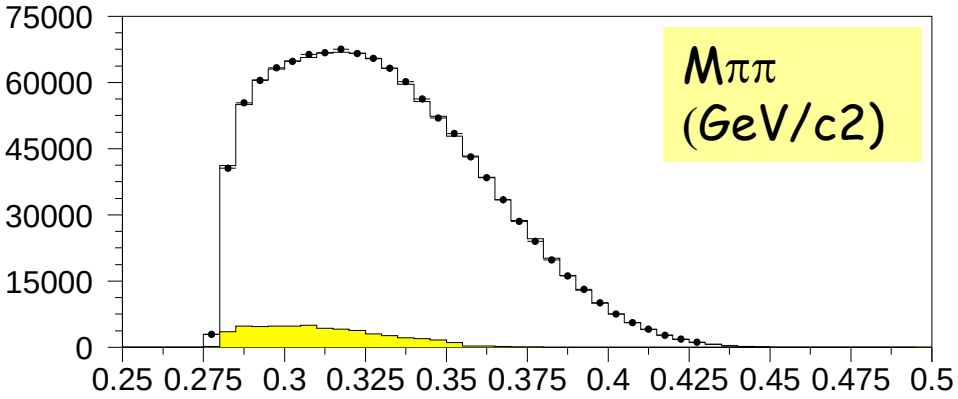
Ratio K+/K- ~ 1.8 both in Data and MC (run by run basis)
Ratio MC/Data ~ 24. both for K+ and K- (run by run basis)

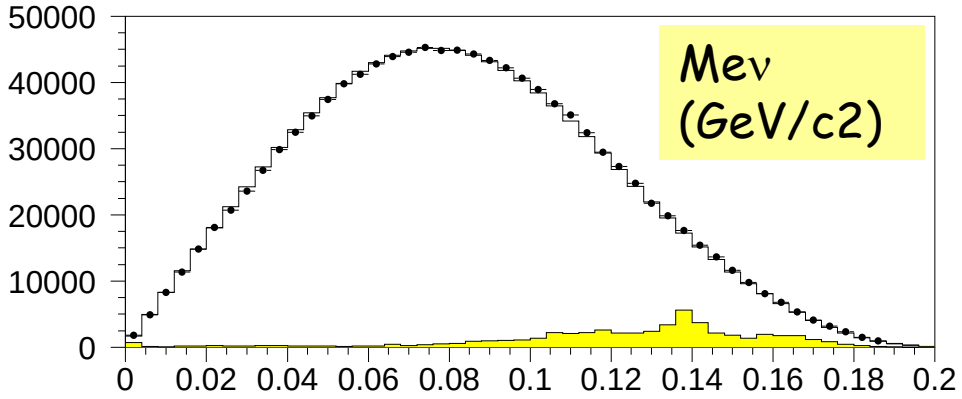
# Ke4 charged decays : the mass and cosθ distributions

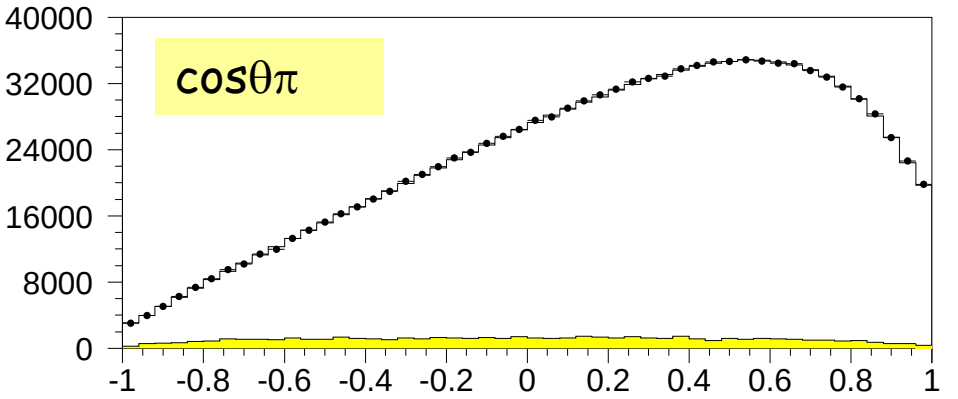
$K^+$ and $K^-$ samples fitted separately, results combined

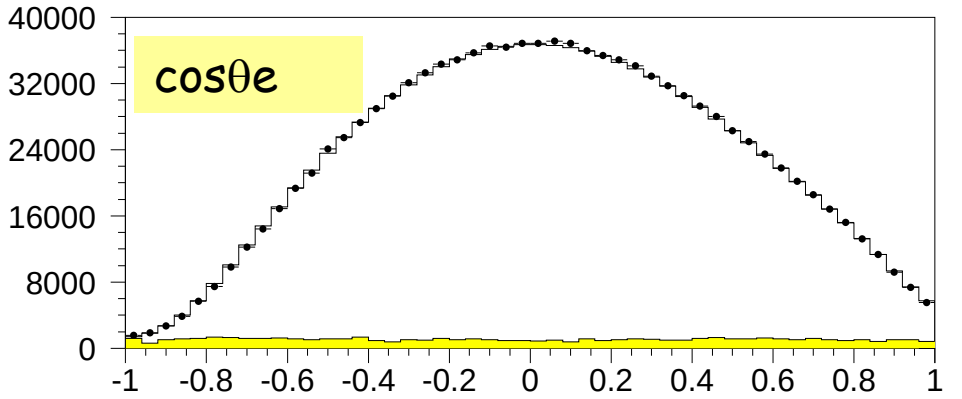
data (symbols), simulation after fit (hist) and background ( x 10 to be visible)

Background flat in cosθπ, cosθe, ϕ, concentrated at low Mππ and Meν= mπ



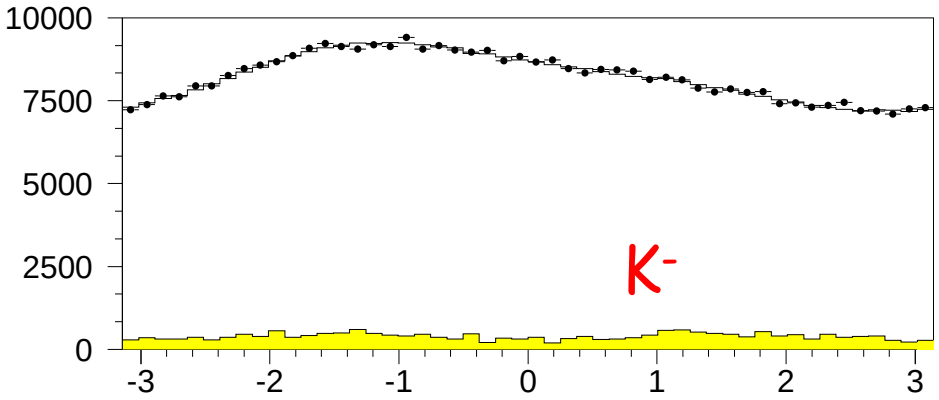



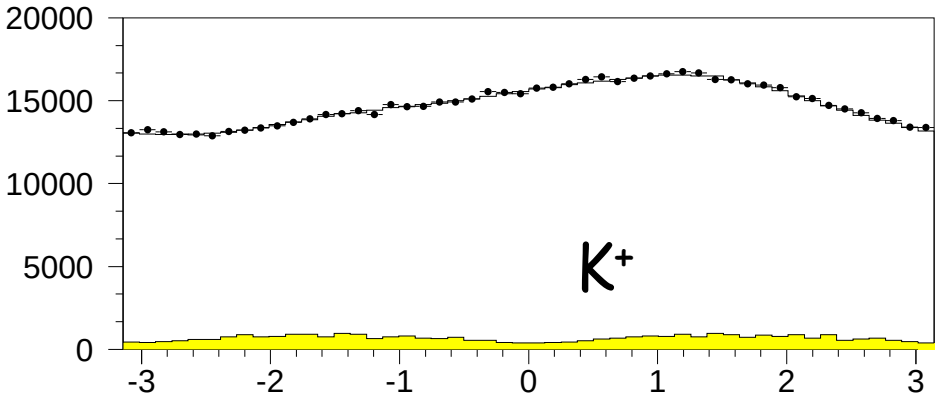



# Ke4 charged decays : the $\phi$ distributions

CP symmetry : (K+) $\phi$ distribution is opposite of (K-) $\phi$ distribution




$\phi$ (rad)

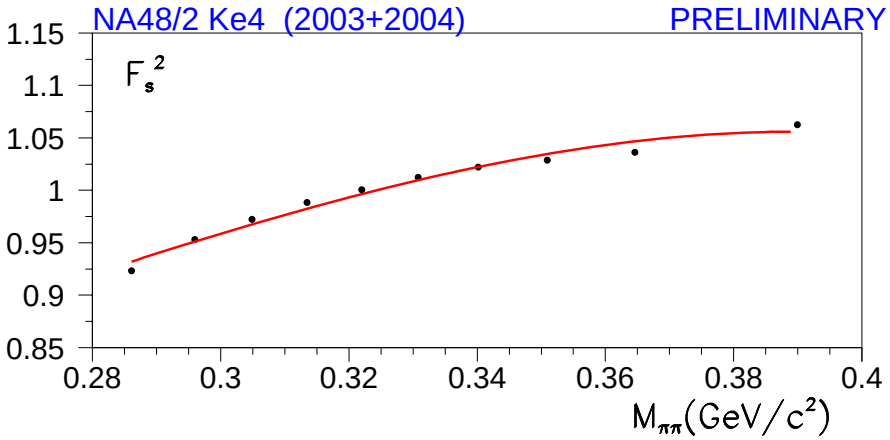
Ratio K+/K- = 1.8 as expected from beam composition

# Ke4 charged decays : getting F,G,H form factors and phase shift

Ten independent fits, one in each $M_{\pi\pi}$ bin, assuming ~constant form factors over each box. This allows a model independent analysis.

Without the overall normalization (Branching fraction), one can quote relative form factors and their variations with $q^2$,$q^4$ ($q^2 = (S_\pi/4m_\pi^2 - 1)$ and $Se/4m_\pi^2$

- $Fs^2$ is obtained from the relative bin to bin normalization Data/MC after fit
- if projected along Meν, a residual variation is observed.
- a 2-dimension fit of the normalization is performed to get the 3 slopes



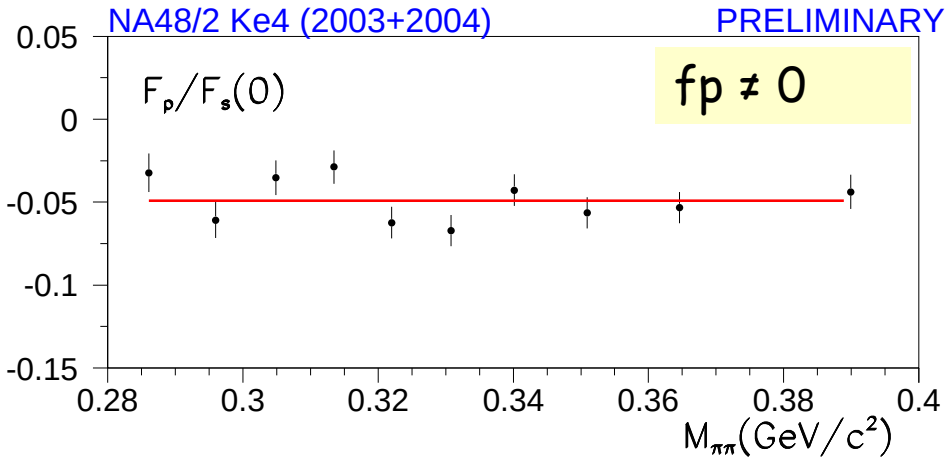

The 3 slopes are correlated

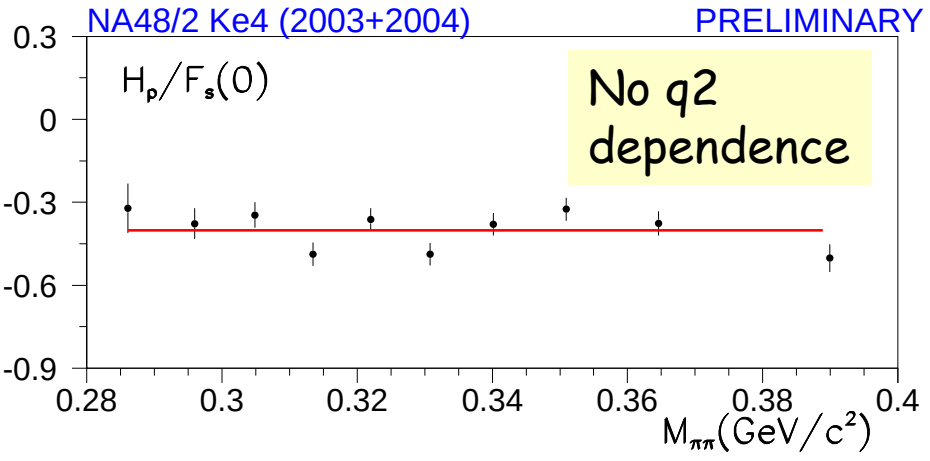
$Fs^2 \propto (1 + f's\, q^2 + f''s\, q^4 + f'e\, Se/4m\pi^2)^2$

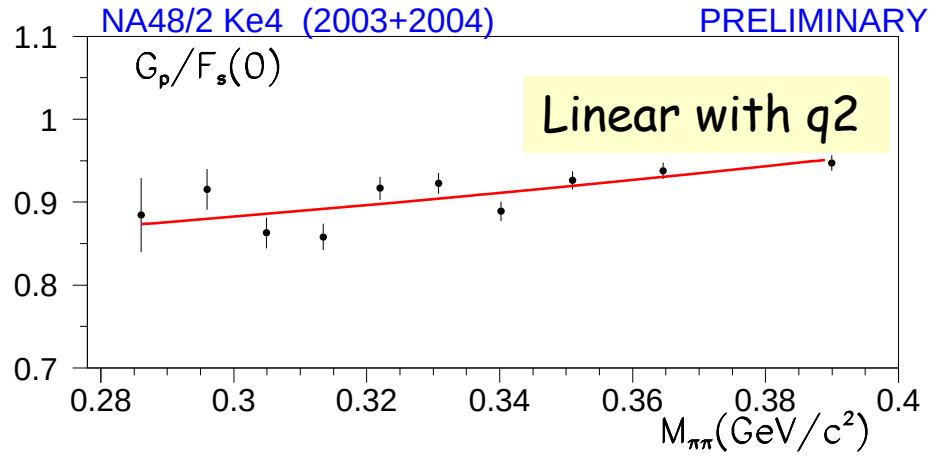
| | f″s | f′e |
|---|---|---|
| f′s | -0.95 | 0.08 |
| f″s | | 0.02 |

Other parametrizations could be easily tried

# Getting Fp, Gp, Hp




Correlation

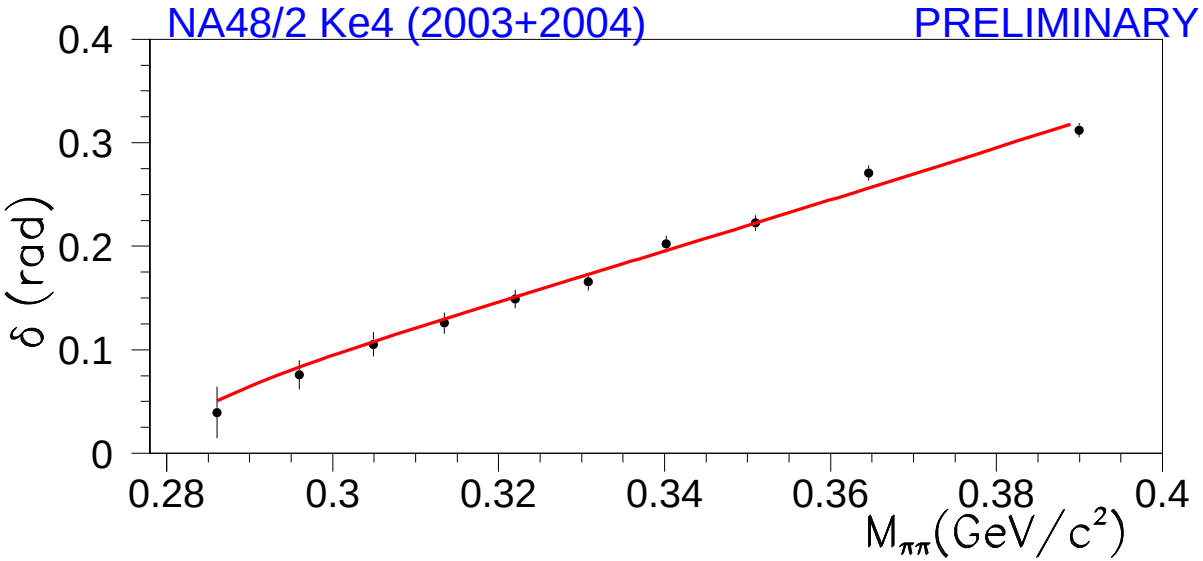
$g'_p$

$g_p(0)$ -0.914

# Ke4 charged decays : $\delta$ form factor and ($a_0^0$ ,$a_0^2$ )

To extract information from the $\delta = (\delta_0^0 - \delta_1^1)$ variation, some external data (I=2 $\pi\pi$ data @Higher energy) and theoretical work are needed :

- numerical solutions of Roy equations (ACGL Phys. Rep.353 (2001), DFGS EPJ C24 (2002) ) relate $\delta$ and (a0, a2 )
- the Universal Band centre line parameterization corresponds to a 1-p fit with a fixed relation a2 = f(a0) and a large uncertainty ( ± 0.088)
- the ChPT constrain band (CGL NPB603(2001)): reduced uncertainty(± 0.008)
- a 2-parameter fit can also be performed with 2 free parameters
- Isospin symetry breaking effects can be considered ( "Bern" arxiv:hep-ph/0710.3048 by JG)




2 free parameters
isospin corrections ON

$\Delta a0 = \pm 0.013$ (stat)

$\Delta a2 = \pm 0.0084$ (stat)

# Ke4 charged decays : isospin corrections to $\delta$

Using "Bern" corrections: 11 to 15 mrad over the fitted $M\pi\pi$ range

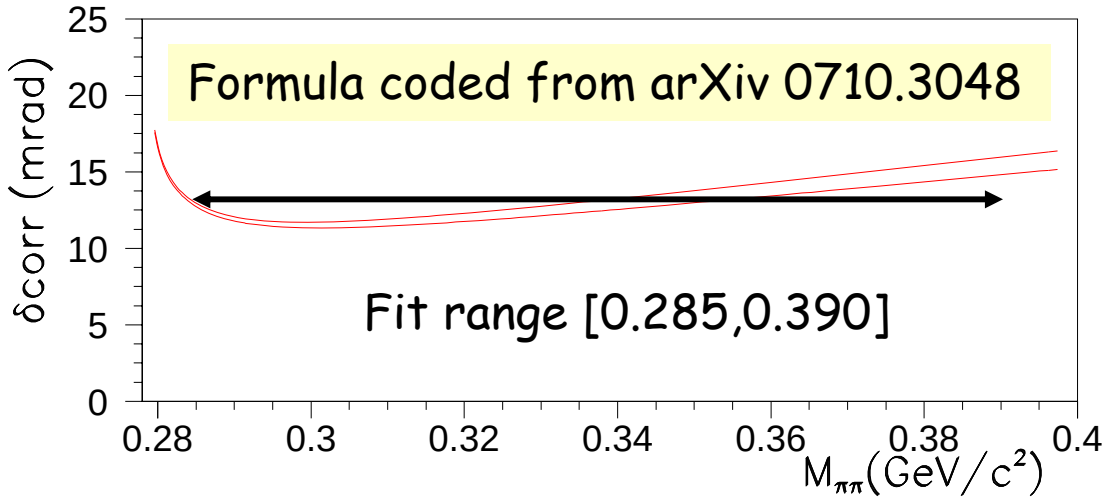
Uncertainty $R \pm \Delta R = 37 \pm 4$ ( R=ms-mud/(md-mu))

$\Delta R$ translates into $\Delta\delta corr$ = at most $\pm$ 0.5 mrad @ 0.390 GeV/c2

Could be quoted later as a 'theoretical precision', but marginal effect

Well adapted to the analysis which applies radiative and Isospin symetry breaking effects in a multiplicative way :

Gamow factor x PHOTOS generator x Isospin corrections



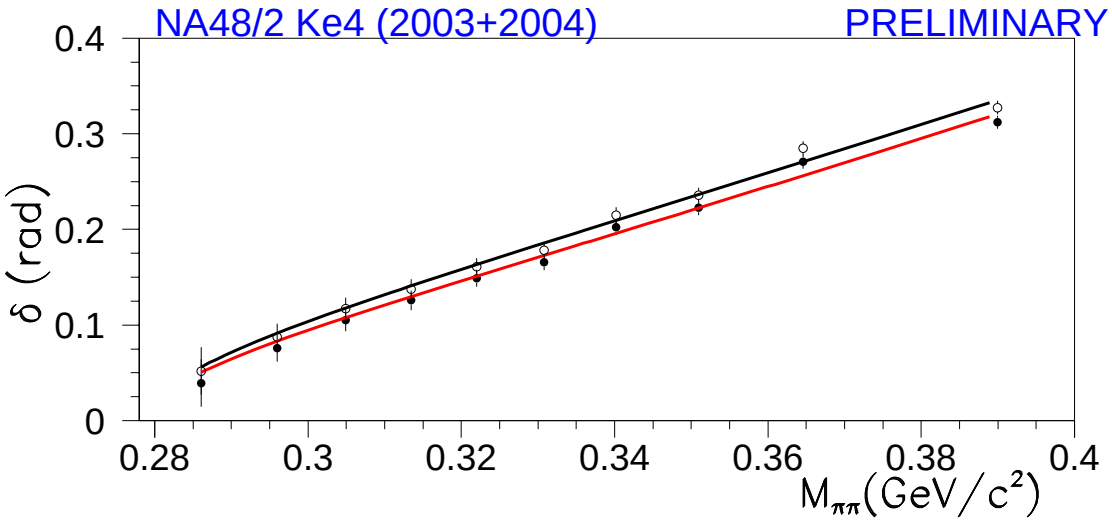
# Ke4 charged decays : isospin corrections to δ

All Data K+ and K- combined

Open = Isospin corr OFF

Closed = Isospin corr ON

Corection (~10mrad) is larger than the statistical error on each point above 0.3 GeV/c2 (7-8 mrad)




| 2p fit | Isospin corr OFF | Isospin corr ON |
|---|---|---|
| a00 | 0.244 ± 0.013 | 0.218 ± 0.013 |
| a02 | -0.0385 ± 0.0084 | -0.0457 ± 0.0084 |

## Ke4 charged decays : Form Factors results (1 151 100 decays)

relative Form Factors = FF/Fs(0)

- measured separately for $K^+$ and $K^-$,
- combined according to stat. errors,
- Fs obtained from bin to bin norm.
- Fp, Gp, Hp de-convoluted from observed Fs(q2,Se) variation .

Scattering length a0 now measured with ~2% relative precision (ChPT 1p fit),

Improved precision for a0 (~6%) and a2 (~18%) in a 2-free parameter fit

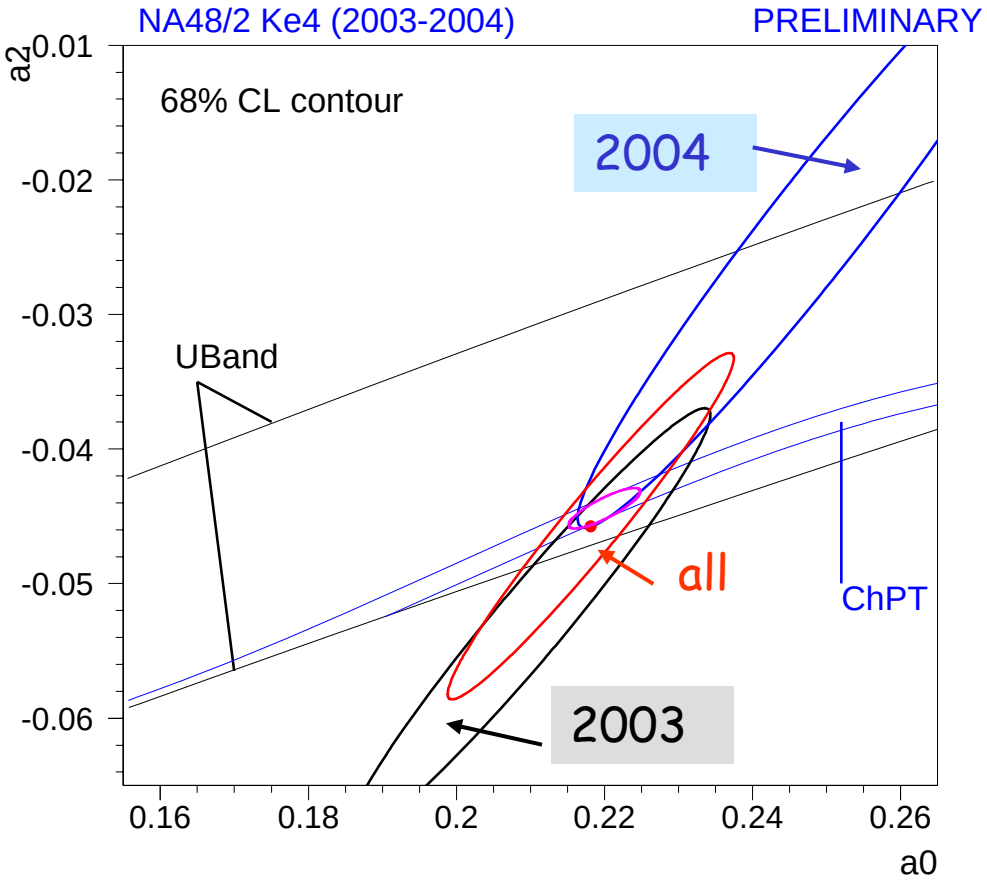
Systematic uncertainty as for 2003 (conservative, will be revisited) but~0.5 x (stat) for a0,a2

value ± stat.

| | 2003 EPJC 54(2008) | 2003 + 2004 preliminary |
|---|---|---|
| f's/fs | 0.172 ± 0.009 | 0.158 ± 0.007 |
| f"s/fs | -0.090 ± 0.009 | -0.078 ± 0.007 |
| f'e/fs | 0.081 ± 0.008 | 0.067 ± 0.006 |
| fp/fs | -0.048 ±0.004 | -0.049 ± 0.003 |
| gp/fs | 0.873 ± 0.013 | 0.869 ± 0.010 |
| g'p/fs | 0.081 ± 0.022 | 0.087 ± 0.017 |
| hp/fs | -0.411± 0.019 | -0.402 ±0.014 |
| a0 ChPT 1p fit a2=f(a0) | 0.223 ±0.006 (-0.0437 ±0.0015) | **0.220 ± 0.005** (-0.0444 ±0.0011) |
| a0 free a2 free | 0.209 ± 0.016 -0.0529±0.0105 | **0.218 ± 0.013** **-0.0457±0.0084** |

# Ke4 charged decays : towards $\pi\pi$ scattering lengths

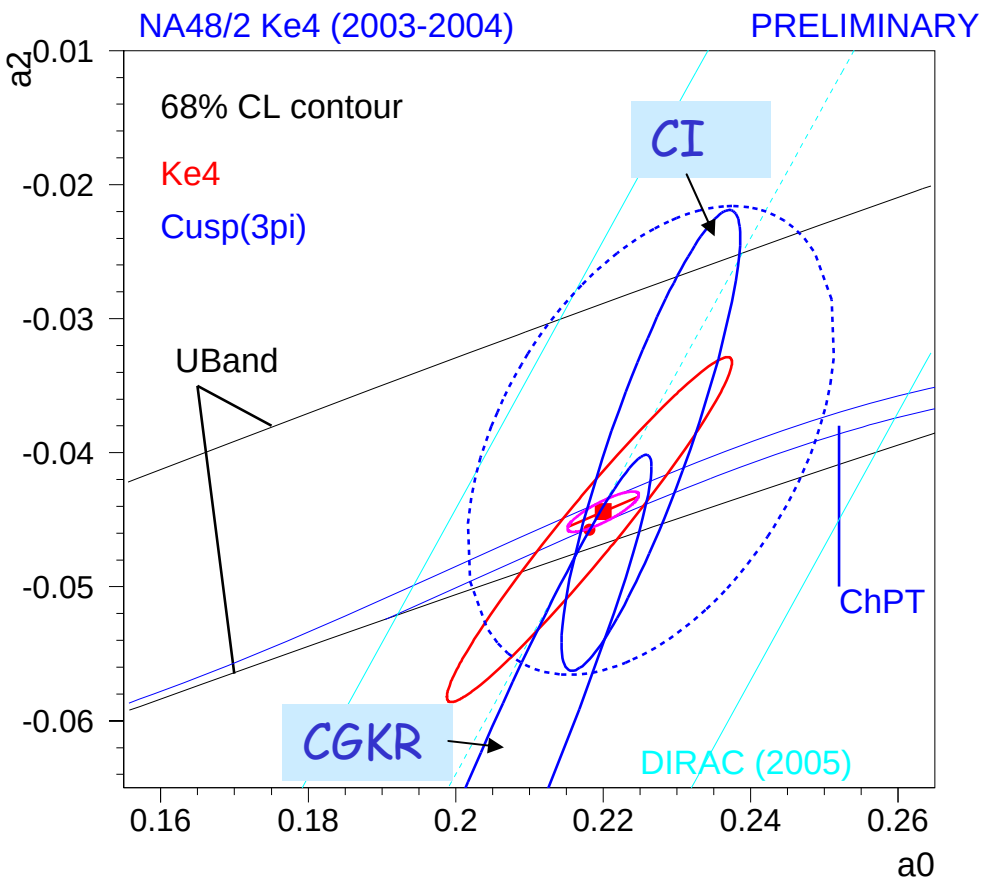
Isospin corrections ON (Bern prescription)




| 2p fit | 2003 EPJC 54(2008) | **2003 + 2004 preliminary** |
|---|---|---|
| a0 free | 0.209 ± 0.016 stat ± 0.007 syst | **0.218 ± 0.013stat ± 0.007 syst** |
| a2 free | -0.0529 ± 0.0105 stat ± 0.0041 syst | **-0.0457 ± 0.0084 stat ± 0.0041 syst** |
| a0 ChPT 1p fit | 0.223 ± 0.006 stat ± 0.002 syst | **0.220 ± 0.005 stat ± 0.002 syst** |

Precise ChPT predictions : a2 = -0.0444 ± 0.0008 and a0 = 0.220 ± 0.005

# scattering lengths measurements by NA48




Two statistically independent measurements by NA48/2:

Cusp in K ($\pi\,\pi 0\,\pi 0$) : 1p fit within 2 models (CI, CGKR), as presented by D. Madigozhin

Ke4 : "Bern" Isospin corrections ON

- 2p fit using Roy equations
- 1p fit with ChPT constraint

Impressive agreement with ChPT predictions

DIRAC $\pi\pi$ atom life time (PLB 619 (2005) value and errors)

# Comparing with other measurements

Ke4 : apply isospin corrections to all 3 experiments (from published phase points) and perform a 1p ChPT fit

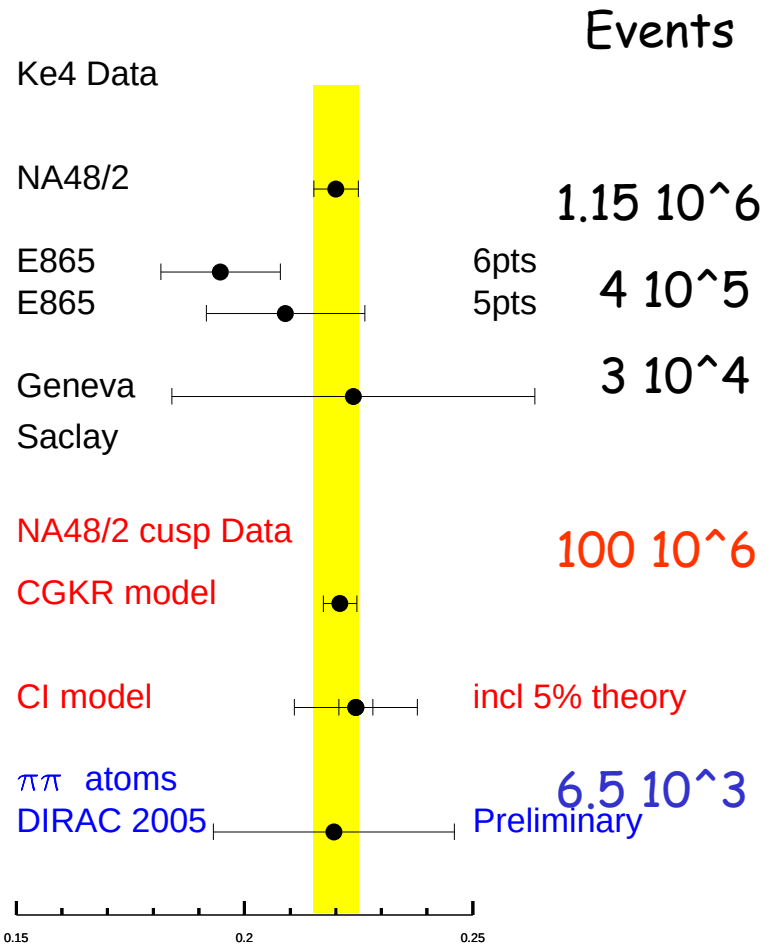
Note : E865 number dominated by highest energy data point, otherwise compatible

Cusp : use 1p ChPT fit and 2 models

DIRAC : |a0-a2| non-symetric errors from PLB619 (2005), use ChPT constraint, still being revisited

Yellow band is ChPT prediction

NA48/2 experimental precision now at the same level !




a0 from 1p ChPT fit, isospin corr ON

# Ke4 analysis in NA48/2 : Summary and prospects

NA48/2 has been analyzing ~1.15 M Ke4 events (2003+2004):

- Axial and vector Form Factors have been measured with an improved precision, including their variation with the dipion and dilepton masses
- using a more elaborated theory input, the scattering lengths can be extracted, giving a consistent picture with other measurements and an impressive agreement with predictions from ChiralPT.
- an improved precision on a0 ($\pm$ 0.005 stat $\pm$ 0.002 syst) can be reached
- systematic uncertainty will be revisited for the full data sample

The very positive collaboration with several theory groups (Bern, Dubna, Orsay, Madrid, Marseille ..) was/still is invaluable in understanding how to extract scattering lengths from Ke4 phase shift data.

I hope the discussions will continue to make the best use of the available Kaon data .

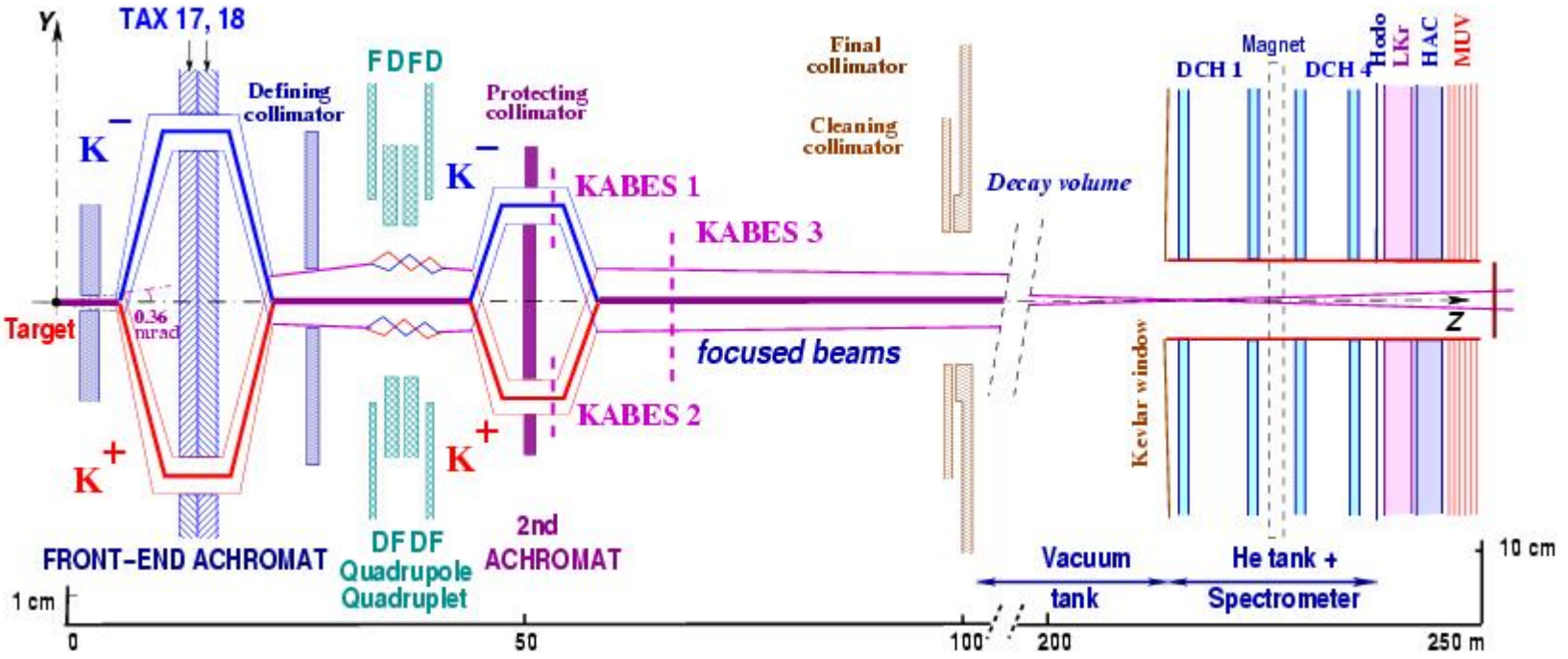
The Capri workshop is a perfect opportunity,

thanks to the FlaviANet Network and our Italian hosts

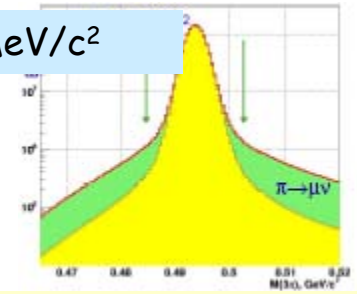
# SPARES

# The NA48/2 experiment: beams and detector

Simultaneous K+/K- beams : (60 ± 3 ) GeV/c



# The NA48/2 experiment: detector performances

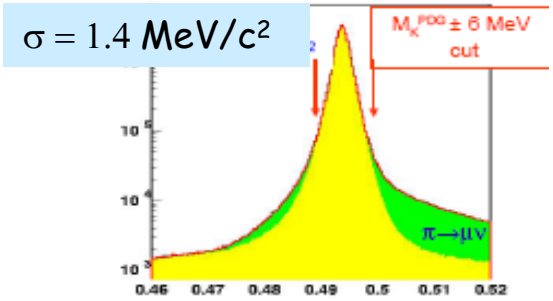



$(\pi^{\pm}\,\pi^{+}\pi^{-})$ mass GeV/$c^2$

- Most important components for Ke4 analysis :

- Magnetic spectrometer : 4 high-resolution DCH's
- $\Delta p/p = (1.02 \oplus 0.044\ p)\%$ ( p in GeV/c)
- Very good resolution for charged invariant masses (Kaon)

- LKr electromagnetic calorimeter : quasi-homogenous and high granularity
- $\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ ( E in GeV)
- $\sigma x = \sigma y \sim 1.5$ mm for E=10 GeV
- Very good resolution for neutrals (p0)
- → E/p ratio for e/**p** discrimination




$(\pi^{\pm}\,\pi^{0}\pi^{0})$ mass GeV/$c^2$

# Ke4 charged decays : systematic uncertainties (2003 data)

- Two independent analyses with slightly different approaches ( binning, trigger efficiency, fit method..) for part of the statistics,

- more cross-checks on 2004 data selection
- Possible bin to bin correlations investigated and taken into account in the overall fit procedure ( non diagonal covariance matrix)

Table 1: Contribution of the systematic errors (in $10^{-3}$ units) to each of the form factors. The background level and $S_e$ dependence contributions are 100% bin-to-bin correlated.

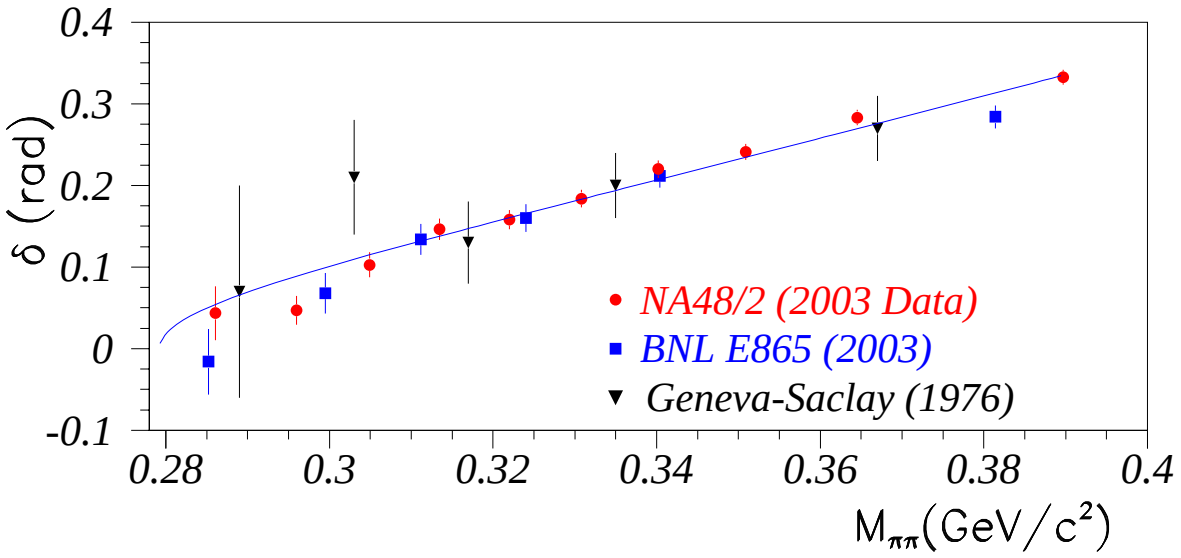
| $\times 10^3$ | fit method | trigger | accep-tance | back. shape | e-ident. | rad. corr. | back. level | $S_e$ dep. | total syst. |
|---|---|---|---|---|---|---|---|---|---|
| $f_s'/f_s$ | 0.4 | 2.5 | 1.2 | 2.5 | 1.8 | 1.1 | 3.1 | 2.6 | $\pm 5.9$ |
| $f_s''/f_s$ | 0.2 | 2.3 | 1.2 | 2.2 | 1.5 | 1.4 | 2.4 | 5.2 | $\pm 7.0$ |
| $f_e'/f_s$ | 1.6 | 0.1 | 1.0 | 2.7 | 0.2 | 2.7 | 6.8 | 4.1 | $\pm 9.0$ |
| $f_p/f_s$ | 0.4 | 0.4 | 0.6 | 1.5 | 0.8 | 0.9 | 3.0 | 2.4 | $\pm 4.3$ |
| $g_p/f_s$ | 1.3 | 0.9 | 2.0 | 5.5 | 2.8 | 3.5 | 8.5 | 4.4 | $\pm 12.2$ |
| $g_p'/f_s$ | 1.7 | 1.3 | 2.6 | 6.8 | 3.4 | 6.9 | 9.3 | 4.5 | $\pm 14.9$ |
| $h_p/f_s$ | 1.0 | 1.0 | 3.8 | 2.7 | 5.1 | 2.1 | 1.6 | 2.6 | $\pm 8.0$ |
| $a_0^0$ (1p fit) | 0.4 | 0.2 | 1.3 | 0.9 | 0.9 | 0.5 | 0.9 | 0.5 | $\pm 2.2$ |
| $a_0^0$ (2p fit) | 1.3 | 0.6 | 3.9 | 3.3 | 2.8 | 1.6 | 2.5 | 1.3 | $\pm 6.9$ |
| $a_0^2$ (2p fit) | 0.7 | 0.4 | 2.6 | 1.8 | 1.7 | 1.0 | 1.2 | 0.7 | $\pm 4.1$ |

# Ke4 charged decays : towards $\pi\,\pi$ scattering lengths

Comparison with previous published Ke4 results :

- CERN/PS Geneva-Saclay ~30000 decays ($K^+$) (Phys. Rev. D15 (1977))
- BNL E865 ~390 000 decays ($K^+$) (PRL 87 (2001), Phys. Rev. D67 (2003))
- CERN/SPS NA48/2 : ~677 500 decays ($K^+$+$K^-$) (EPJC 54 3 (2008))
  significant acceptance at larger $m_{\pi\pi}$ values, high resolution and low background level.

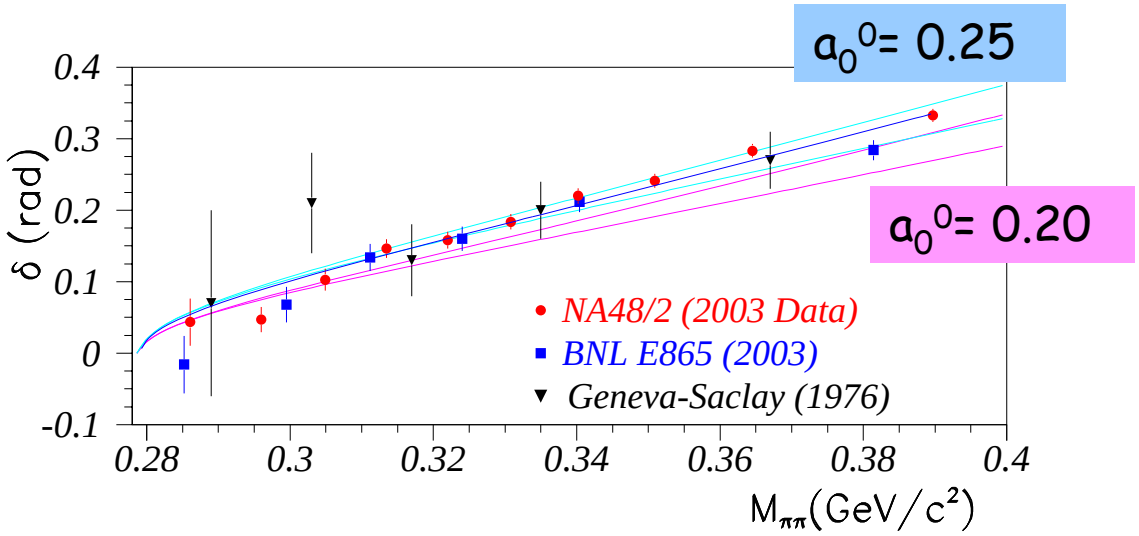
Universal Band centre line shown (stat. + experimental syst. errors added)



# Ke4 phase shift measurements : are they compatible ?

Thanks to the "independent bin" analysis, one can replay the scattering length extraction with more elaborated models and combine results from the various experiments even after completion of collaborations !

E865 quotes various values extracted from their Ke4 phase measurements, ranging from $a_0^0 = 0.203$ to $a_0^0 = 0.237$
NA48/2 seems to prefer slightly higher values




Last point of E865 seems somewhat inconsistent with other points (Could that be a problem with the mass value quoted for the last bin …?)