FLAVIAnet Kaon Workshop 2008

Unitarity and analyticity in K_{e4}

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G. Colangelo, J.G., A.Rusetsky, work in progress

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INTRODUCTION

Let us consider $K^+ \to \pi^+\pi^- e^+\nu_e$ decays

The decay amplitude contains the axial current matrix element

$$\langle \pi^+\pi^- | A_\mu | K^+ \rangle \quad = \quad \frac{-i}{M_K}[P_\mu F + Q_\mu G + L_\mu R]$$

In the isospin symmetry limit, the form form factors F, G, R contain the $\pi\pi$ phases $\to$ can measure these and compare with the predictions:

ChPT + Roy equations + data above 800 MeV $\Rightarrow$

$$a_0 = 0.220 \pm 0.005, \;\; a_0 - a_2 = 0.265 \pm 0.004$$

$a_{0 \leftarrow}$isospin

Colangelo, J.G., Leutwyler 2001

By the way...

The phases of F, G concern $I = 0, 1$ channels – why can NA48/2 determine $a_{I=2}$?

Solution of the puzzle:

Roy equations contain 2 subtraction constants $a_{0,2}$

$$a_{0,2} \Leftrightarrow \delta_0^0 - \delta_1^1$$

However

- Predictions of $\pi\pi$ scattering lengths concern paradise world, where $\alpha_{\rm QED} = 0, m_u = m_d, M_{\pi^+} = M_{\pi^0} = 139.6$ MeV
- Measurements in K_{e4} decays are performed in the real world, where $\alpha_{\rm QED} \neq 0, m_u \neq m_d, M_{\pi^+} = 139.6$ MeV, $M_{\pi^0} = 135$ MeV
- $\Rightarrow$ How can one translate measured phase shifts to paradise world, such that a comparison with the predictions becomes meaningful?

In the following, I discuss general aspects of this problem, not relying on a specific Lagrangian

The framework

Assumption:

- Isospin breaking=PHOTOS $\times$ Coulomb factor$\times$ mass effects

In the following, I discuss mass effects. Let us imagine a world described e.g. by

$$
\begin{aligned}
\mathcal{L} &= \frac{F_0^2}{4}\langle D_\mu U D^\mu U^\dagger + B_0 \mathcal{M}(U+U^\dagger)\rangle + Z\langle QUQU^\dagger\rangle + \cdots \\
Q &= \text{diag}(2,-1,-1)/3
\end{aligned}
$$

Specific form of Lagrangian does not matter

Alternative

- Do full radiative corrections of decay amplitude
- Set up new event generator
- Redo data analysis, from scratch [how?]

Is long term project

ANALYTICITY, UNITARITY

Analyticity, Unitarity

To simplify the discussion: consider scalar form factor of the pion

J.G., KAON07; G. Colangelo, J.G.,A. Rusetsky, work in progress; S.Descotes-Genon and M. Knecht, work in progress

$$
\begin{aligned}
-F_c(s) &= \langle 0|j(0)|\pi^+(p_1)\pi^-(p_2)\rangle \\
F_0(s) &= \langle 0|j(0)|\pi^0(p_1)\pi^0(p_2)\rangle\,, \qquad s=(p_1+p_2)^2
\end{aligned}
$$

j: Lorentz scalar; isoscalar in isospin limit

Unitarity:

$$
\begin{aligned}
\mathrm{Im}\, F(s) &= T(s)\rho(s)F^*(s) \\
\mathrm{Im}\, T(s) &= T(s)\rho(s)T^*(s)
\end{aligned}
$$

$$
F=\begin{pmatrix} F_c \\ F_0 \end{pmatrix}, \; T=\begin{pmatrix} t_{cc} & -t_{c0} \\ -t_{c0} & t_{00} \end{pmatrix}, \; \rho=\mathrm{diag}[2\sigma\theta(s-4M_\pi^2), \sigma_0\theta(s-4M_{\pi^0}^2)]
$$

$$
\sigma=\sqrt{1-4M_\pi^2/s}\,,\ \sigma_0=\sqrt{1-4M_{\pi^0}^2/s}\,, \qquad t_{ik}\leftrightarrow \pi\pi\to\pi\pi
$$

Isospin symmetry limit

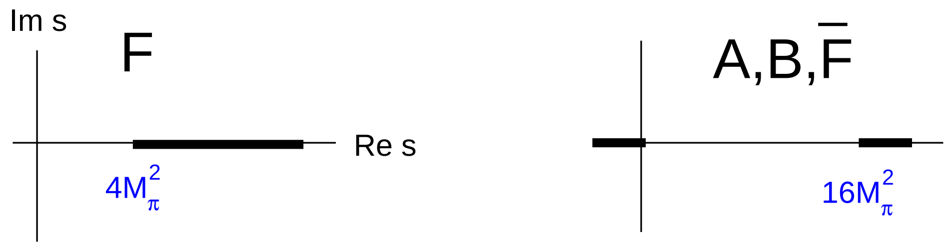
$F_c = F_0 \doteq F\,;$ unitarity relations decouple:

$$\text{Im}\, F = t_0^0\,\sigma\,F^*\,; \qquad \text{Im}\, t_0^I = \sigma |t_0^I|^2\,;\; I = 0, 2$$

Consequences

i) Threshold behaviour : $F(s) = A(s) + i\sigma B(s)$

ii) Remove the phase: $\tan\delta \doteq \sigma B/A\,,\; \bar{F}(s) \doteq e^{-i\delta} F(s)$



W. Zimmermann 1961

iii) δ is $I = 0$ S-wave phase shift in $\pi\pi \to \pi\pi$ [Fermi-Watson]

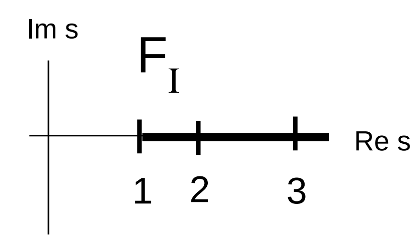
iv) $\bar{F}(s) = c_0 + c_1 q^2 + c_2 q^4 + \ldots$; $q^2 = s - 4M_\pi^2$; c_i real

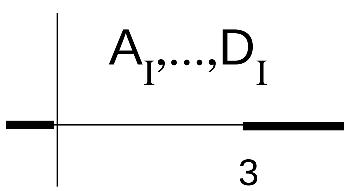
i)-iv) are modified in case of isospin breaking

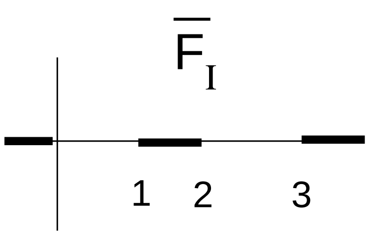
Isospin broken

i') $F_I = A_I + i\sigma B_I + i\sigma_0 C_I + \sigma\sigma_0 D_I \; ; \; I = c, 0$

ii') $\bar{F}_I \doteq e^{-i\delta_I} F_I$







1: $4M^2_{\pi^0}$

2: $4M^2_{\pi}$

3: $16M^2_{\pi^0}$

$$\sigma = \sqrt{1 - 4M^2_\pi/s}\,,\; \sigma_0 = \sqrt{1 - 4M^2_{\pi^0}/s}$$

iii') • δ_I not fixed by $\pi\pi$ scattering amplitude alone

• $\delta_I \not\to 0\,, \quad s \to 4M_\pi^2$

iv') $\bar{F}_I = c_0 + c_1\, q + c_2\, q^2 + \mathcal{O}(q^3)\,,\ s \to 4M_\pi^2$

$\Rightarrow$ $\bar{F}_I$ is Taylor series in q, not in q^2

$\Rightarrow$ c_1 is two-loop effect

$\Rightarrow$ analogous statements hold for form factors in K_{e4}

$\Rightarrow$ size of c_1 in K_{e4} remains to be worked out

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CONSEQUENCES FOR K_{e4}

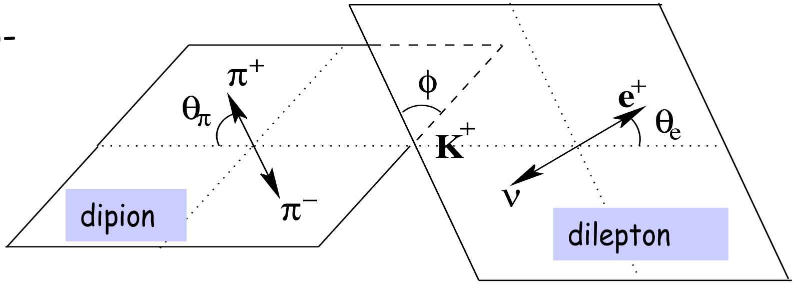
Consequences for K_{e4} analysis I

Ke4 charged decays : formalism

Five kinematic variables (Cabibbo-Maksymowicz):

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$),

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



partial wave expansion of the amplitude:

F, G = Axial Form Factors

$F = F_s\, e^{i\delta s} + F_p\, e^{i\delta p} \cos\theta_\pi$ + d-wave term…

$G = G_p\, e^{i\delta g}$ + d-wave term…

H = Vector Form Factor

$H = H_p\, e^{i\delta h}$ + d-wave term…

expansion in powers of q^2 , $Se/4m\pi^2$

($q^2 = (S_\pi / 4m_\pi^2 - 1)$)

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e (S_e/4m_\pi^2) + ..$$

$$F_p = f_p + f_p' q^2 + ..$$

$$G_p = g_p + g_p' q^2 + ..$$

$$H_p = h_p + h_p' q^2 + ..$$

The fit parameters are : F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

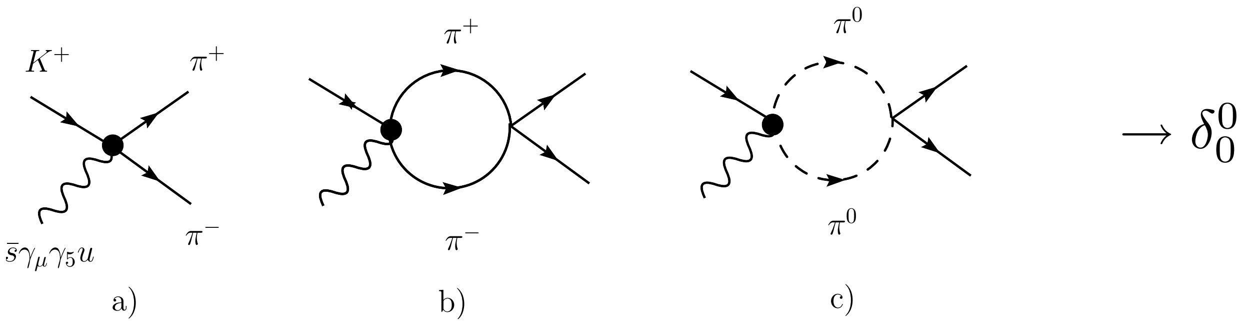
Brigitte Bloch-Devaux KAON ‘ 07 6

Taken from Brigitte Bloch-Devaux, KAON07

Consequences for K_{e4} analysis II. Need Lagrangian

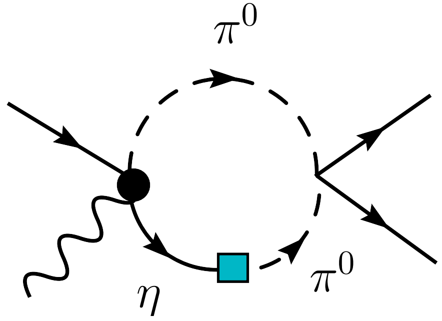
$m_u = m_d; e = 0$

Lowest order:



$m_u \neq m_d; e \neq 0$

Analytic structure of diagrams change. Additional diagram:



Phase is changed: $\delta_0^0 \rightarrow \delta$. Experiment measures δ. Effect is large.

Working out these diagrams, one finds that the phase becomes in the elastic region

$$\delta_0^0 \to \delta = \frac{1}{32\pi F_0^2}\left\{(4\Delta_\pi + s_\pi)\sigma + (s_\pi - M_{\pi^0}^2)\left(1 + \frac{3}{2R}\right)\sigma_0\right\} ,$$

with

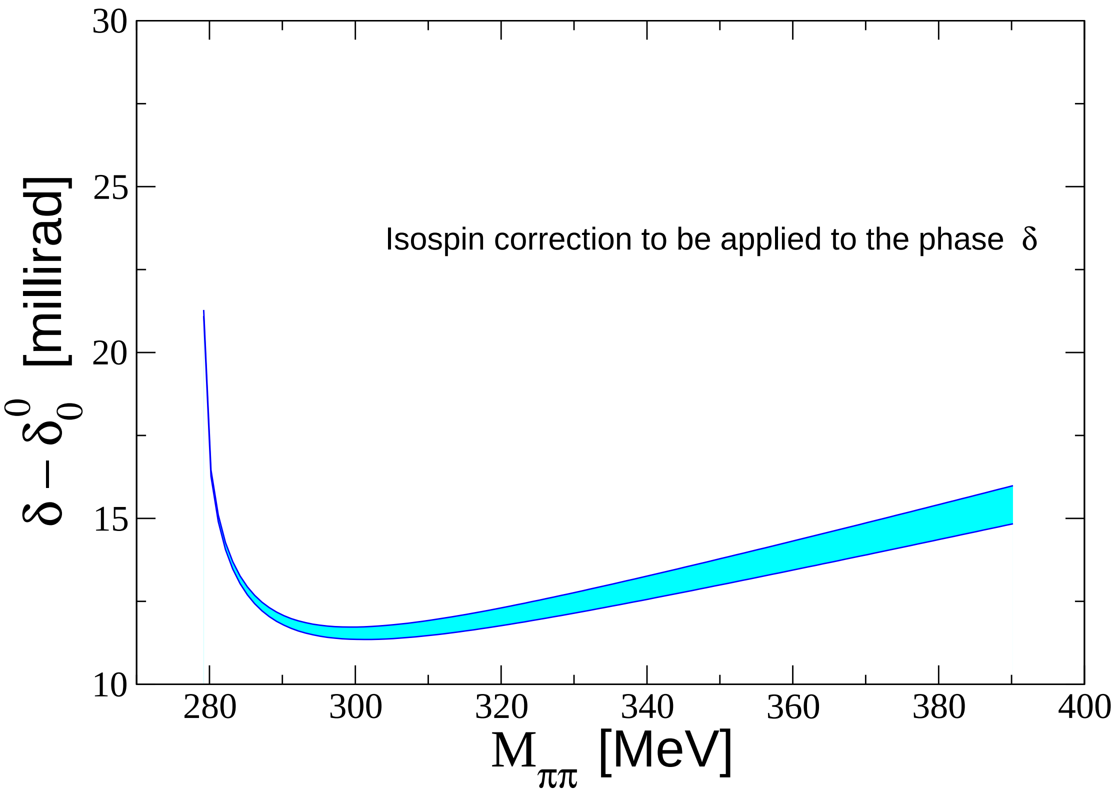
$$\Delta_\pi = M_\pi^2 - M_{\pi^0}^2 , \quad \sigma_0 = \sqrt{1 - 4M_{\pi^0}^2/s_\pi} , \quad R = \frac{m_s - \hat{m}}{m_d - m_u} .$$

See also Cuplov, Nehme 2003,2004,2005; Gevorkyan et al., 2007

For the analysis:

$$\delta_0^0 = \delta - (\delta - \delta_0^0) \Rightarrow \text{need to know} \qquad \delta - \delta_0^0$$

The correction for the phase



This part must be subtracted from the measured phase before a comparison with the prediction can be made.

Note: the correction displayed stems from a one–loop calculation.

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Cuplov, Nehme 2003-2005

SUMMARY

SUMMARY

Experiments are performed in the real world, described by the Standard Model, where

$$\alpha \neq 0, \; m_u \neq m_d$$

Note that
$$K^+ \not\to \pi^+\pi^0\pi^0$$
$$K^+ \not\to \pi^+\pi^-e^+\nu_e$$

zero probability that these processes occur in the laboratory

Bloch, Nordsieck 1937

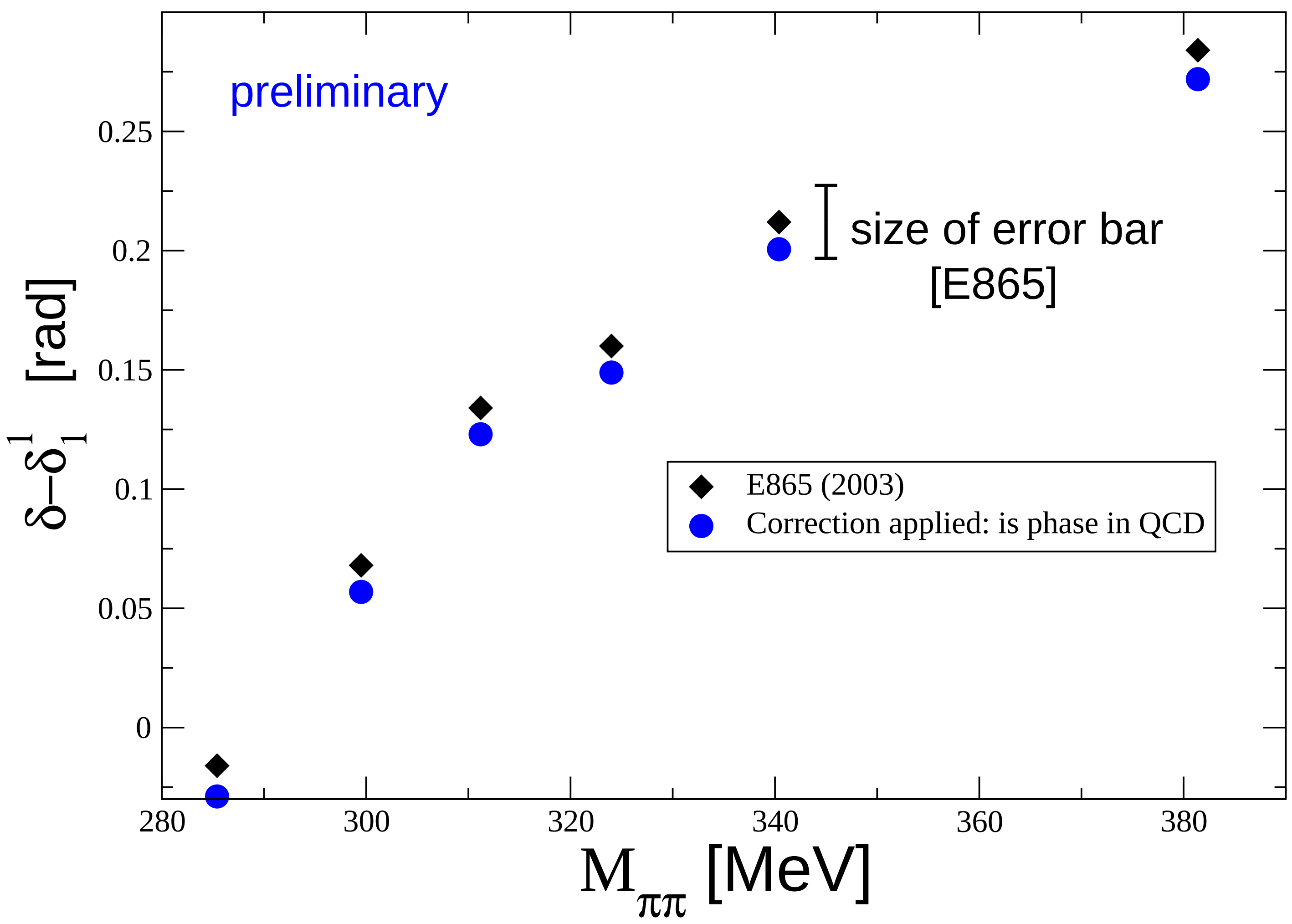
$\Rightarrow$ Need a careful analysis of the situation in e.g. K_{e4}:

- δ not fixed by $\pi\pi \to \pi\pi$ amplitude alone
- correction $\delta - \delta_0^0$ is substantial
- phase removed form factor is not holomorphic at threshold

Because NA48/2 data are so precise, these corrections matter!

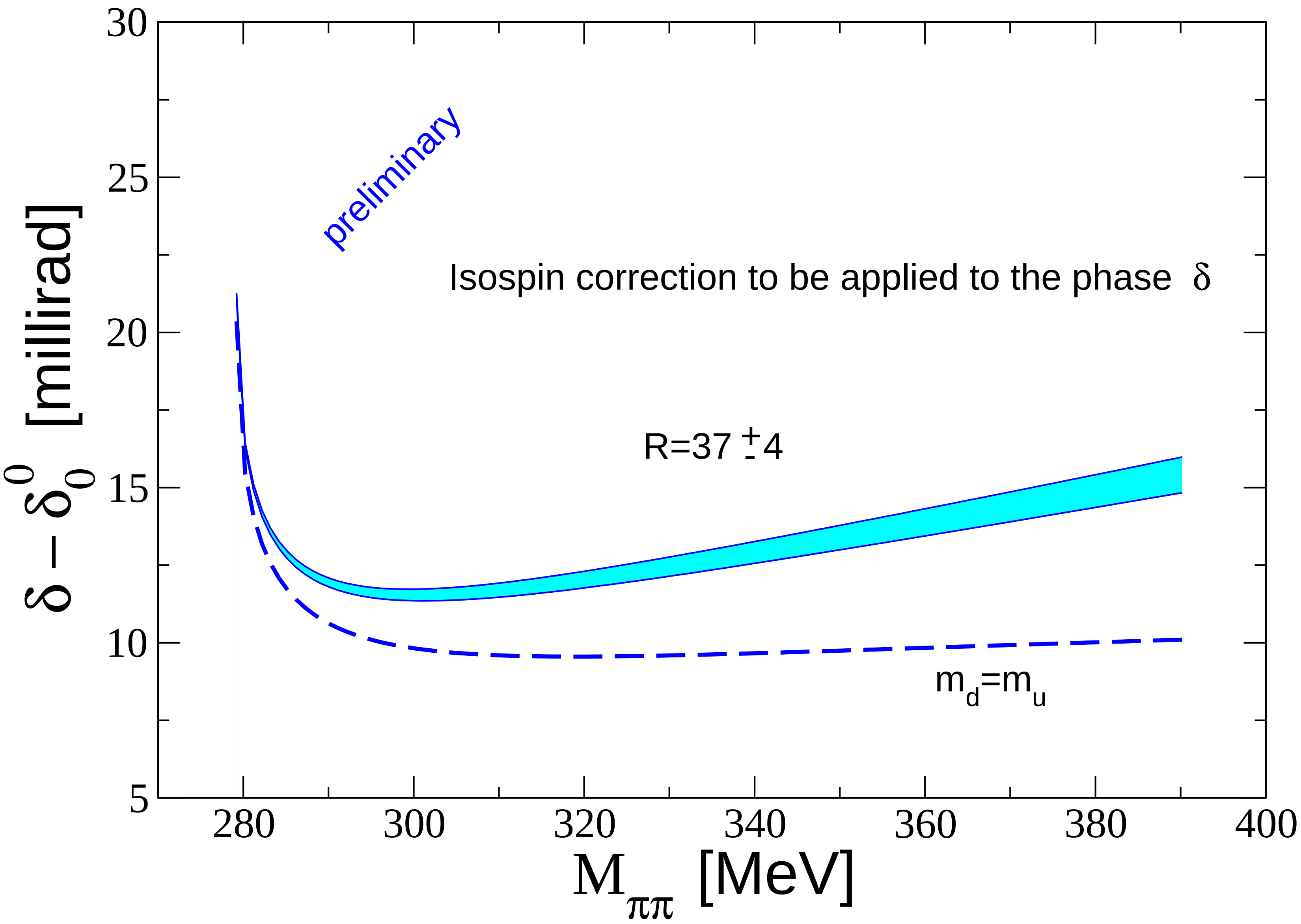
SPARES

Occam's razor at work [from my KAON07 talk]



Two-loop calculation: G. Colangelo, J.G., A. Rusetsky

$m_u = m_d$



30
25
20
15
10
5
280
300
320
340
360
380
400
δ − δ₀⁰ [millirad]
M_ππ [MeV]
preliminary
Isospin correction to be applied to the phase δ
R=37 ± 4
m_d=m_u