New physics limits from kaon decays

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Abstract

Searches for lepton flavour violation and lepton number violation in kaon decays by the NA62 and NA48/2 experiments at CERN are presented. A new measurement of the ratio of charged kaon leptonic decay rates $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$ to sub-percent relative precision is discussed. An improved upper limit on the lepton number violating $K^{\pm} \to \pi^{\mp} \mu^{\pm} \mu^{\pm}$ decay rate is also reported. The future 10% precision measurement of the branching ratio of the ultra-rare kaon decay $K^+ \to \pi^+ \nu \overline{\nu}$ with the NA62 experiment is finally reviewed.

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1 Introduction

In the Standard Model (SM) the decays of charged pseudo scalar mesons (P) into lepton neutrino are helicity suppressed.

Supersymmetric new physics models, like certain 2-Higgs doublet models (e.g. 2HDM type II) [1], predict sizable deviations from the SM via new physics contributions already at tree level. In these frameworks the supersymmetric parameters $\tan \beta$ (the ratio of the two Higgs vacuum expectation values) and M_{H^+} (the mass of the charged Higgs) usually describe the new physics contributions. The dependence of these decay rates on $\tan^2 \beta$ and $(M_P/M_{H^+})^2$ naturally enhances the sensitivity of the B mesons to new physics, like the $B^+ \to \tau^+ \nu_\tau$ decay. Although similar new physics contributions for $K^+ \to l^+ \nu_l$ could result in 100 times smaller effects, leptonic kaon decays still offer the opportunity to search for new physics thanks to a very high experimental precision [2]. However, precision measurements in this sectors clash with the poor knowledge of the hadronic matrix elements which severely limits the theoretical prediction of $\Gamma(P^+ \to l^+ \nu_l)$. This uncertainty largely cancels in the ratio of the rates of these decays into different lepton families (e.g. with $l = e, \mu$), like the parameter $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$. Processes with $e\nu$ or $\mu\nu$ in the final state, on the other side, are experimentally accessible only in the π and K sector.

The SM prediction for R_K inclusive of internal bremsstrahlung (IB) radiation is [3]:

$$R_K^{SM} = \left(\frac{M_e}{M_\mu}\right)^2 \left(\frac{M_K^2 - M_e^2}{M_K^2 - M_\mu^2}\right)^2 (1 + \delta R_{QED}) = (2.477 \pm 0.001) \times 10^{-2}, \quad (1)$$

where δR_{QED} is an electromagnetic correction due to the IB and structure-dependent effects. Deviations of R_K from the SM require new physics models with sources of lepton flavour violation (LFV) [4, 5]. Within the MSSM, for example, the LFV sources appear at the one loop level via the exchange of the charged Higgs boson coupled with a right-handed slepton loop. The dominant contribution is

$$R_K^{LFV} \simeq R_K^{SM} \left[1 + \left(\frac{M_K}{M_H} \right)^4 \left(\frac{M_\tau}{M_e} \right)^2 |\Delta_R^{31}|^2 \tan^6 \beta \right], \tag{2}$$

where $|\Delta_R^{31}|$ is the mixing parameter between the superpartners of the right-handed leptons, which can reach values up to 10^{-3} . After an appropriate tuning of the new physics parameters, the effect on R_K could be up to % level without contradicting any experimental constraints. Already in the 1970s several experiments measured R_K [6, 7, 8], while the present PDG value [9], $R_K = (2.493 \pm 0.031) \times 10^{-5}$, is largely dominated by a recent result from KLOE [10]. A new measurement of R_K based on a part of a data sample collected by the NA62 experiment (phase I) at CERN in 2007 [11] is reported here (section 3).

Kaon decays may also contribute to the search for lepton number violation via decays like $K^{\pm} \to \pi^{\mp} \mu^{\pm} \mu^{\pm}$. They violate the lepton number by 2 units and can proceed only if the ν is a Majorana particle: consequently the limit on their branching ratio provide constraints on the effective Majorana neutrino mass [12]. Such processes were already studied experimentally by the BNL E865 experiment in 1997 [13]. The NA48/2 experiment at CERN collected a $\pi\mu\mu$ sample about 8 times larger than the one from E865. It allows improving the limits on the $K^{\pm} \to \pi^{\mp} \mu^{\pm} \mu^{\pm}$ process significantly [14] (section 4).

Among the many rare flavour changing neutral current K and B decays, the ultra rare decays $K \to \pi \nu \overline{\nu}$ play a key role in search for new physics through underlying mechanisms of flavour mixing. The SM branching ratio can be computed to an exceptionally high degree of precision and the prediction for the $K^+ \to \pi^+ \nu \overline{\nu}$ channel is $(7.81 \pm 0.75 \pm 0.29) \times 10^{-11}$ [15]. The first error comes from the uncertainty on the CKM matrix elements, the second one is the pure theoretical uncertainty. The extreme theoretical cleanness of these decays remain also in new physics scenarios like Minimal Flavour Violation (MFV) [16] or non-MFV models [17] and even not large deviations from the SM value (for example around 20%) can be considered signals of new physics. The decay $K^+ \to \pi^+ \nu \overline{\nu}$ has been observed by the experiments E787 and E949 at the Brookhaven National Laboratory and the measured branching ratio is $1.73^{+1.15}_{-1.05} \times 10^{-10}$ [18]. However only a measurement of the branching ratio with at least 10% accuracy can be a significant test of new physics. This is the main goal of the NA62 experiment at CERN-SPS [19] (section 5).

2 The NA48/2 and NA62 (phase I) Experiments

The NA48/2 and NA62 (phase I) experiments at CERN collected data in 2003-04 and 2007-08 using the same beam line and experimental set-up, respectively. NA48/2 aimed to the study of the CP violation in the decay of the charged kaons into three pions [20], NA62 to the measurement of the above defined R_K ratio. They were fixed target experiments which used a 400 GeV/c primary proton beam, extracted from the SPS accelerator at CERN, which produced a secondary charged kaon beam after impinging on a beryllium target. A 100 m long beam line selected the momentum of the secondary beam to (60 ± 3) GeV/c in 2003-04 and (75 ± 2) GeV/c in 2007-08. Finally the beams entered a decay volume, housed in a 100 m long vacuum tank. With a primary beam intensity of about 7×10^{11} protons per SPS spill of 4.8 s duration, the positive (negative) beam flux at the entrance of the decay volume was 3.8×10^7 (2.6×10^7) particles per pulse. The fraction of kaons decaying in the decay volume was about 20%, depending on the beam energy.

The detector was designed to see the charged and neutral products of the kaons decaying in the vacuum region. A magnetic spectrometer tracked the charged particles.

It was housed in a tank containing He and separated from the vacuum region by a Kevlar window. An aluminum beam pipe of 16 cm diameter with vacuum inside, traversed the spectrometer and allowed the not decayed beam particles passing through without touching the sensitive detector volume. The spectrometer consisted of four drift chambers (DCH) separated by a dipole magnet, which gave to the charged particles an horizontal transverse momentum kick of 120 MeV/c (265 MeV/c) 2003-04 (2007-08). The momentum resolution was $\sigma_p/p = (1.02 \oplus 0.044 \times p)\%$ in 2003-04 and $\sigma_p/p = (0.48 \oplus 0.009 \times p)\%$ (p in GeV/c) in 2007-08.

An hodoscope (HOD), made of two orthogonal planes of 64 plastic scintillator slabs each, followed the magnetic spectrometer. It provided the time reference for the other detectors and the main trigger for the events with charged particles.

An electromagnetic calorimeter (LKr), placed after the hodoscope, was used for photon detection and particle identification. It was a quasi-homogeneous calorimeter with liquid kripton as active material. A system of Cu-Be ribbons electrodes allowed the collection of the ionization signal. In total 13248 projective cells segmented the active volume transversely to the beam axis. The total length of the detector corresponded to about 27 X_0 . The measured energy resolution was $\sigma(E)/E = 0.032/\sqrt(E) \oplus 0.09/E \oplus 0.0042$ (E in GeV).

An hadronic calorimeter (HAC) and a muon detector (MUV) followed the electromagnetic calorimeter.

A detailed description of the NA48/2 layout can be found elsewhere [21].

3 Measurement of R_K with NA48/2

The measurement of R_K has been performed using 40% of the data collected in 2007 by NA62. The analyzed data contained only positive kaons.

The measurement relied on counting the numbers of reconstructed K_{e2} and $K_{\mu 2}$ candidates collected simultaneously. Consequently R_K did not depend on the absolute kaon flux and the ratio allowed for a first order cancellation of several systematic effects, like reconstruction and trigger efficiencies and time dependent biases. The basic formula is:

$$R_K = \frac{1}{D} \cdot \frac{N(K_{e2}) - N_B(K_{e2})}{N(K_{\mu 2}) - N_B(K_{\mu 2})} \cdot \frac{A(K_{\mu 2}) f_{\mu} \epsilon(K_{\mu 2})}{A(K_{e2}) f_{e} \epsilon(K_{e2})} \cdot \frac{1}{f_{LKr}}.$$
 (3)

Here $N(K_{l2})$ and $N_B(K_{l2})$ are the number of the selected K_{l2} events and the expected number of background events, respectively; D is the downscaling factor applied to the $K_{\mu 2}$ trigger; $A(K_{l2})$ the geometrical acceptance of the selected K_{l2} ; f_e and f_{μ} the identification efficiencies of electrons and muons, respectively; $\epsilon(K_{l2})$ the trigger efficiencies for the selected K_{l2} ; f_{LKr} the global LKr efficiency.

A detailed Monte Carlo simulation (MC) was developed, including beam line optics and time-dependent detector inefficiencies. The computation of the acceptance correction $A(K_{\mu 2})/A(K_{e2})$ and the geometrical part of the acceptance entering in the background computation relied on MC. The particle identification efficiencies, the readout and the trigger efficiencies, instead, were measured directly on data. Because both the signal acceptance and the background depended strongly on the lepton momentum, the measurement was performed in bins of this observable by dividing the range between 13 and 65 GeV/c in 10 intervals.

A large part of the selection was in common between K_{e2} and $K_{\mu 2}$, because of the similar single-track topology. Exactly one positive track in the final state, reconstructed in the spectrometer and whose extrapolation passed through the downstream detector acceptance, was required; it had to have a momentum within 13 and 65 GeV and the reconstructed longitudinal position of the kaon decay vertex had to be located within the fiducial decay region. Events with deposits of energy greater than 2 GeV not associated with the charged tracks in the LKr calorimeter were rejected in order to further suppress backgrounds with photons in the final state.

The event kinematics and the lepton identification were effective to separate K_{e2} and $K_{\mu 2}$. The $M_{miss}^2 = (P_K - P_l)^2$ characterized completely the kinematics of the single track decays. Here P_K and P_l are the kaon and lepton 4-momenta respectively; the average P_K was measured on spill basis using the $K^+ \to \pi^+ \pi^+ \pi^-$ decays; P_l was computed in the electron or muon mass hypothesis. A cut around the M_{miss}^2 peak, according to the M_{miss}^2 resolution and dependent on the lepton momentum, selected the K_{l2} candidates. The ratio E/p of the track energy deposit in the LKr calorimeter to its momentum measured by the spectrometer, identified positrons (0.95 < E/p < 1.1) and muons (E/p < 0.85).

The numbers of selected K_{e2} and $K_{\mu 2}$ candidates were 59813 and 1.803×10^7 , respectively. The total background was $8.71 \pm 0.24\%$. The $M_{miss}^2(K_{e2})$ and the background contamination as a function of lepton momentum are shown in figure 1.

The background was strongly momentum dependent. In particular for momenta higher than 35-40 GeV/c, the K_{e2} kinematics resembled more and more the $K_{\mu 2}$ one and $K_{\mu 2}$ with a muon mis-identified as a positron became the largest background source. The accuracy of its evaluation was critical to keep the total systematic uncertainty smaller than the statistical one. The 'catastrophic' bremsstrahlung in or in front of the LKr was the dominant source of the muon-positron mis-identification. The corresponding probability $P_{\mu e}$ was measured on data as a function of lepton momentum. During a first period of the 2007 run, data were taken with a 9.2 X_0 lead wall in front of the LKr, covering about 20% of the total geometrical acceptance. This set-up allowed the collection of a muon sample free from the about 10^{-4} contamination due to $\mu \to e$ decays. The sample used for the measurement of R_K , however, was taken without the lead wall. Let $P_{\mu e}^{Pb}$ be the probability of muon mis-identification in presence of the lead wall and $P_{\mu e}$ the one without: because of ionization energy

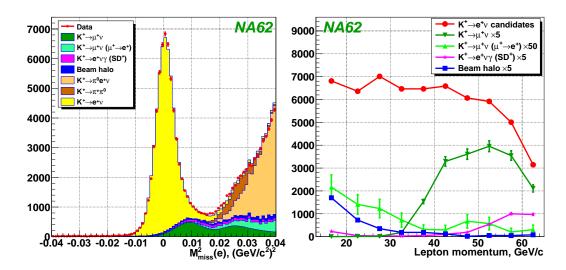


Figure 1: Left: reconstructed $M_{miss}^2(K_{e2})$ for K_{e2} and backgrounds. Right: lepton momentum distributions of the K_{e2} candidates and the dominant backgrounds.

loss and bremsstrahlung in lead $P_{\mu e}$ and $P_{\mu e}^{Pb}$ differed significantly. Consequently, the value measured with the wall was corrected for using a dedicated simulation based on Geant4 [22]. The measured $P_{\mu e}^{Pb}$ varied in the range of $(3-5)\times 10^{-6}$ according to the muon momentum and was in agreement with the simulation within the uncertainties (about 10% from simulation). The correction f_{Pb} varied from +10% to -20% of $P_{\mu e}^{Pb}$ depending on lepton momentum. Its uncertainty was around 2%. The $K_{\mu 2}$ background in the K_{e2} sample integrated over lepton momentum was $(6.11\pm 0.22)\%$. It was computed using the $P_{\mu e}^{Pb}$ measured on data and f_{Pb} from simulation. This combination allowed the minimization of the total uncertainty.

The other background components coming from kaon decays were evaluated using the MC simulation. The beam halo background induced by halo muons undergoing decay in flight or mis-identified, was measured by reconstructing positive K_{e2} among data collected from K^- beam with the K^+ beam blocked while its halo was not. The size of the control sample limited the evaluation of the background uncertainty.

The acceptance correction was evaluated using MC. The contribution to the K_{e2} acceptance due to the radiative $K^+ \to e^+ \nu \gamma$ inner bremsstrahlung process was taken into account following [23, 24, 25]. The bremsstrahlung suffered by the positrons in the material upstream of the spectrometer magnet, induced about 6% loss of K_{e2} acceptance, as a consequence of the M_{miss}^2 cut. The effect was computed by studying spectra and rates of bremsstrahlung photons produced by 25 GeV/c (40 GeV/c) electron (positron) beam steered into the DCH acceptance, collected by NA48/2 in 2006 (2004). The knowledge of the helium purity in the spectrometer tank was the second largest source of systematic uncertainty.

The R_K value was extracted from a χ^2 fit to the measurements in the lepton

momentum bins, taking into account the bin-to-bin correlations between the systematic uncertainties. Table 1 summarizes the uncertainties. All the assigned systematic errors were checked a posteriori by varying the selection criteria and the analysis procedure.

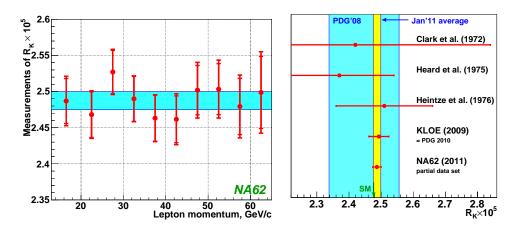


Figure 2: Left: measurements of R_K in lepton momentum bins. The band indicates the average R_K and its total uncertainty. Right: the new world average including the present result.

Source	$\delta R_K \times 10^5$
Statistical	0.011
K_{μ} 2 background	0.005
Other background from K decays	0.001
Beam halo background	0.001
Helium purity	0.003
Acceptance correction	0.002
Spectrometer alignment	0.001
Positron identification efficiency	0.001
1-track trigger efficiency	0.002
LKr readout inefficiency	0.001
Total systematic	0.007
Total	0.013

Table 1: Summary of the uncertainties on R_K .

The result is $(\chi^2/\text{ndf} = 3.6/9)$:

$$R_K = (2.487 \pm 0.011_{stat} \pm 0.007_{syst}) \times 10^{-5} = (2.487 \pm 0.013) \times 10^{-5}.$$
 (4)

The individual results in lepton momentum bins and the new world average are presented in figure 2.

4 Search for lepton number violation with NA48/2

The search for $K^{\pm} \to \pi^{\mp} \mu^{\pm} \mu^{\pm}$ decay was performed on the NA48/2 2003-04 data, using the $K^{\pm} \to \pi^{\pm} \pi^{+} \pi^{-}$ ($K_{3\pi}$) decay as a normalization channel. A three-track event topology was required, with tracks compatible with pion (E/p < 0.95) or muon hypothesis (E/p < 0.2). Hits in the muon veto in time with the tracks, identified the presence of muons in the final state. The muon identification inefficiency was measured to be below 2% for momentum greater than 10 GeV/c.

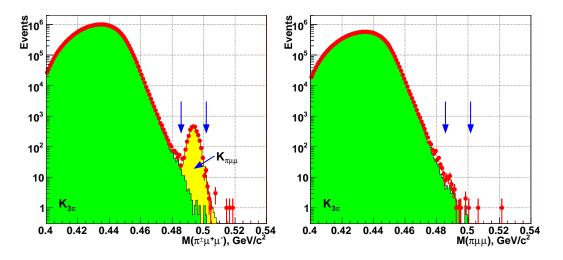


Figure 3: Reconstructed $M_{\pi\mu\mu}$ spectra for candidates with different (left) and same sign (right) muons. Dots are the data.

Figure 3 shows the invariant mass spectra of the reconstructed $\pi^{\pm}\mu^{\pm}\mu^{\mp}$ and $\pi^{\mp}\mu^{\pm}\mu^{\pm}$ candidates. The $K^{\pm}\to\pi^{\pm}\mu^{\pm}\mu^{\mp}$ decay was studied separately [14]. Simulations showed that 52.6 ± 19.8 events with same sign muons were expected in the signal region of the invariant mass spectrum ($|M_{\pi\mu\mu}-M_K|<8$ MeV/c²), mainly due to $K_{3\pi}$ decays: 52 were found in data. The Feldman-Cousins method [26] was applied for the evaluation of the confidence interval, also taking into account the systematic uncertainty of the background evaluation. This lead to an upper limit of $BR(K^{\pm}\to\pi^{\mp}\mu^{\pm}\mu^{\pm})<1.1\times10^{-9}$ at 90% CL, which improves the best previous limit by almost of a factor 3.

5 The ultra-rare kaon decays: future prospects with the NA62 experiment

The goal for the future of the NA62 experiment is the measurement of the branching ratio of the $K^+ \to \pi^+ \nu \overline{\nu}$ decay with 10% precision. Therefore NA62 aims to collect of the order of 100 $K^+ \to \pi^+ \nu \overline{\nu}$ events in about two years of data taking and to keep the total systematic uncertainty small. To this purpose, at least $10^{13}~K^+$ decays are required, assuming a 10% signal acceptance and a $K^+ \to \pi^+ \nu \overline{\nu}$ branching ratio of 10^{-10} . To keep the systematic uncertainty small requires a rejection factor for generic kaon decays of the order of 10^{12} , and the possibility to measure efficiencies and background suppression factors directly from data. In order to match the above required kaon intensity, signal acceptance and background suppression, new detectors must replace the existing NA62 apparatus.

The CERN-SPS extraction line, already used for NA48, can deliver the required intensity, asking for 30% more SPS protons on target only. Consequently the NA62 experiment will be housed in the CERN North Area High Intensity Facility (NAHIF) where NA48 was located. Considerations about signal acceptance drive the choice of a 75 GeV/c charged kaon beam with 1% momentum bite. The use of a decay-in-flight technique to identify K^+ decay products is the experimental principle of NA62.

The experimental set-up is close to the one used for NA48: a 100 m beam line to select the appropriate beam, a 80 m evacuated decay volume and detectors downstream which measure the secondary particles from the kaon decays occurring in the decay volume.

The signature of the signal is one track in the final state matched to one K^+ track in the beam. The integrated rate upstream is about 800 MHz (only 6% of the beam particles are kaons, the other are π^+ and protons). The rate seen by the detector downstream is about 10 MHz, mainly due to K^+ decays. Timing and spatial information are needed to match the upstream and downstream track.

Backgrounds come from all the kaon decays with one track left in the final state and from accidental tracks reconstructed downstream matched by chance to a track upstream. The background suppression profits from the high momentum of the kaon beam. Different techniques have to be employed in combination in order to reach the required level of rejection. Schematically they can be divided into: kinematic rejection, precise timing, high efficient photon and muon vetoes, precise particle identification systems to distinguish π^+ , μ^+ and positrons.

The above requirements drove the design and the construction of the subdetectors systems. The main subdetectors forming the NA62 layout are: a differential Cerenkov counter on the beam line to identify the K^+ in the beam; a Si-pixel beam tracker; a guard-ring counters surrounding the beam tracker to veto catastrophic interactions of particles; a downstream spectrometer made by straw chambers in vacuum; a RICH to

distinguish pions and muons; a charged hodoscope; a system of photons veto including a series of annular lead glass calorimeters surrounding the decay and detector volume, the NA48 LKr calorimeter and a small angle calorimeter to keep the hermetic coverage for photons emitted at zero angle; a muon veto detector.

The design of the experimental apparatus and the R&D of the new subdetectors was completed in 2010. The experiment is under construction and the first technical run is foreseen at the end of 2012.

6 Conclusion

Kaon decays exhibit good sensitivity to new physics thanks to the high experimental precision achieved. In most of the cases the sensitivity is complementary to the one obtained measuring B decays.

The NA62 experiment at CERN provided in 2007 the most precise measurement of the lepton flavour parameter R_K : $R_K = (2.487 \pm 0.013) \times 10^{-5}$. It is consistent with the SM value and can be used to constrain multi-Higgs new physics scenario. NA48/2 improved the upper limit on the branching ratio of the lepton number violating decay $K^{\pm} \to \pi^{\mp} \mu^{\pm} \mu^{\pm}$, which is now 1.1×10^{-9} .

The ultra-rare $K \to \pi \nu \overline{\nu}$ decay is a unique environment where to search for new physics. The NA62 experiment at CERN-SPS proposes to follow this road by collecting O(100) events of the $K^+ \to \pi^+ \nu \overline{\nu}$ decay. The experiment has been approved and funded. After three years of successful R&D program, the NA62 experiment is now under construction.

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