# Review of $\pi\pi$ scattering measurements in K decay

## 1. $\pi\pi$ scattering in $K \to 3\pi$ decay

- $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$
  - Summary of results on $\pi\pi$ scattering lengths
  - Comparison with the measurement of pionium lifetime
- $K_L \to \pi^{\circ}\pi^{\circ}\pi^{\circ}$
  - Evidence for $\pi^+\pi^- \to \pi^{\circ}\pi^{\circ}$ in $K_L \to \pi^+\pi^-\pi^{\circ}$ decay
- $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$
  - Is $\pi\pi$ scattering observable by looking only at $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$ decay ?

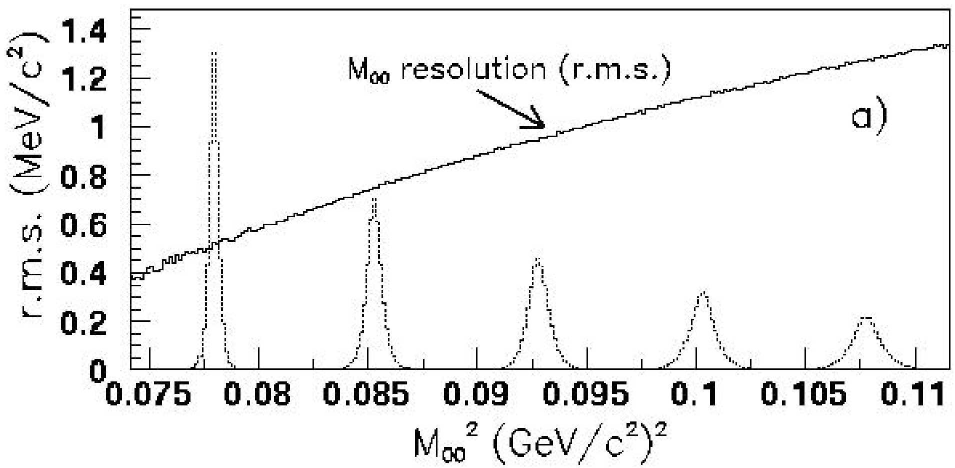
## 2. $\pi\pi$ scattering in $K^{\pm} \to \pi^+\pi^- e^{\pm} \overset{(-)}{\nu}$ decays

## 3. Conclusions

*L. DiLella, KAON 2007, May 23rd*

# $\pi\pi$ scattering in $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decay

**Observation of a cusp structure in the $\pi^{\circ}\pi^{\circ}$ invariant mass distribution at $M_{\pi^{\circ}\pi^{\circ}} = 2m_{\pi+}$ : an unexpected discovery from the NA48/2 high statistics study of $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decay with good $M_{\pi^{\circ}\pi^{\circ}}$ resolution**




$M_{\pi^{\circ}\pi^{\circ}}$ ( ≡ $M_{00}$) resolution optimized for low $M_{00}$ values

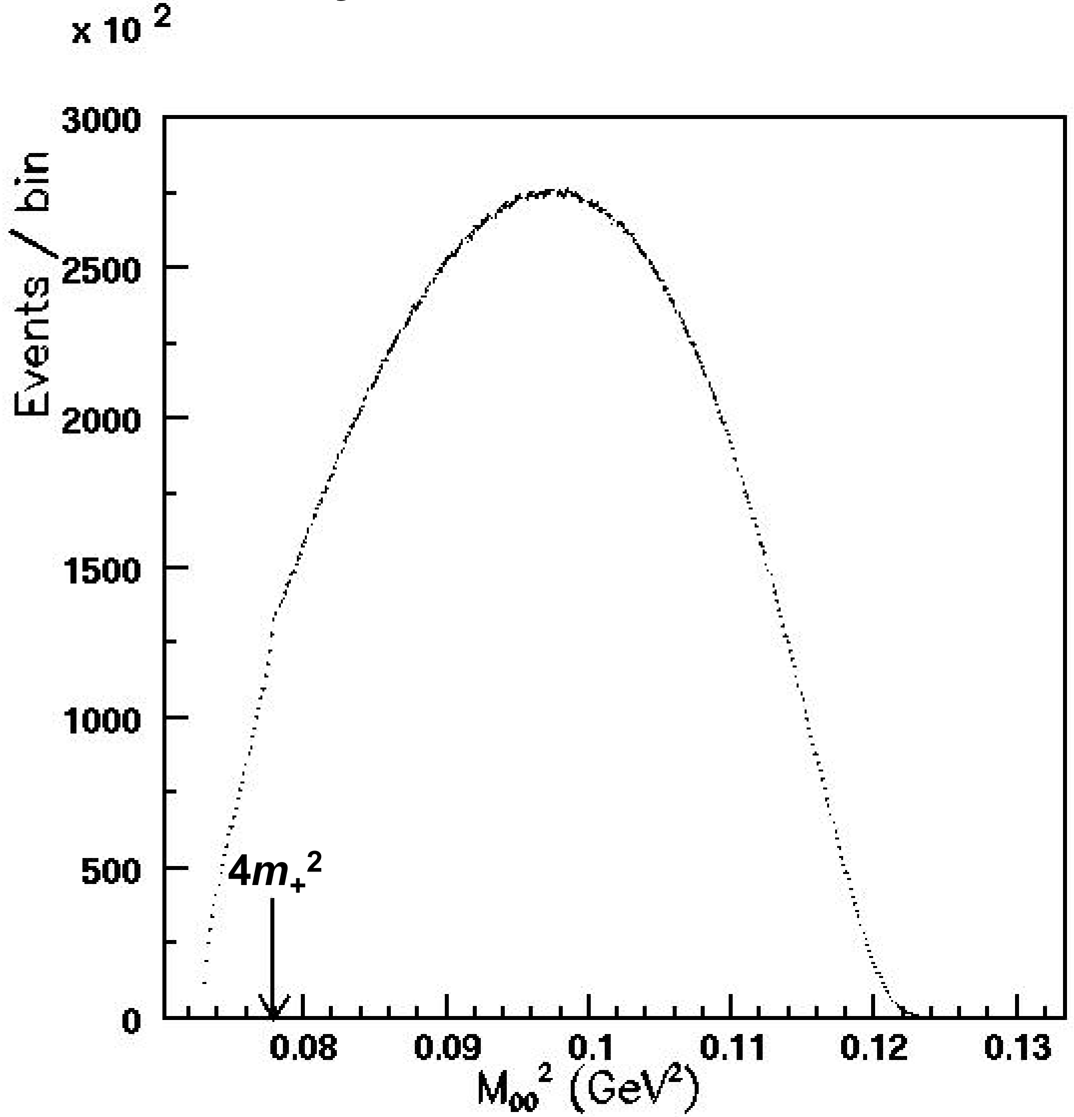
**Initial motivation for this study:**

**the detection of $\pi^{+}\pi^{-}$ atom ("pionium") formation in $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ decays, followed by pionium annihilation to $\pi^{\circ}\pi^{\circ}$**

**$M_{00}$ resolution $\sigma$ = 0.56 MeV at $M_{00} = 4m_{+}$**

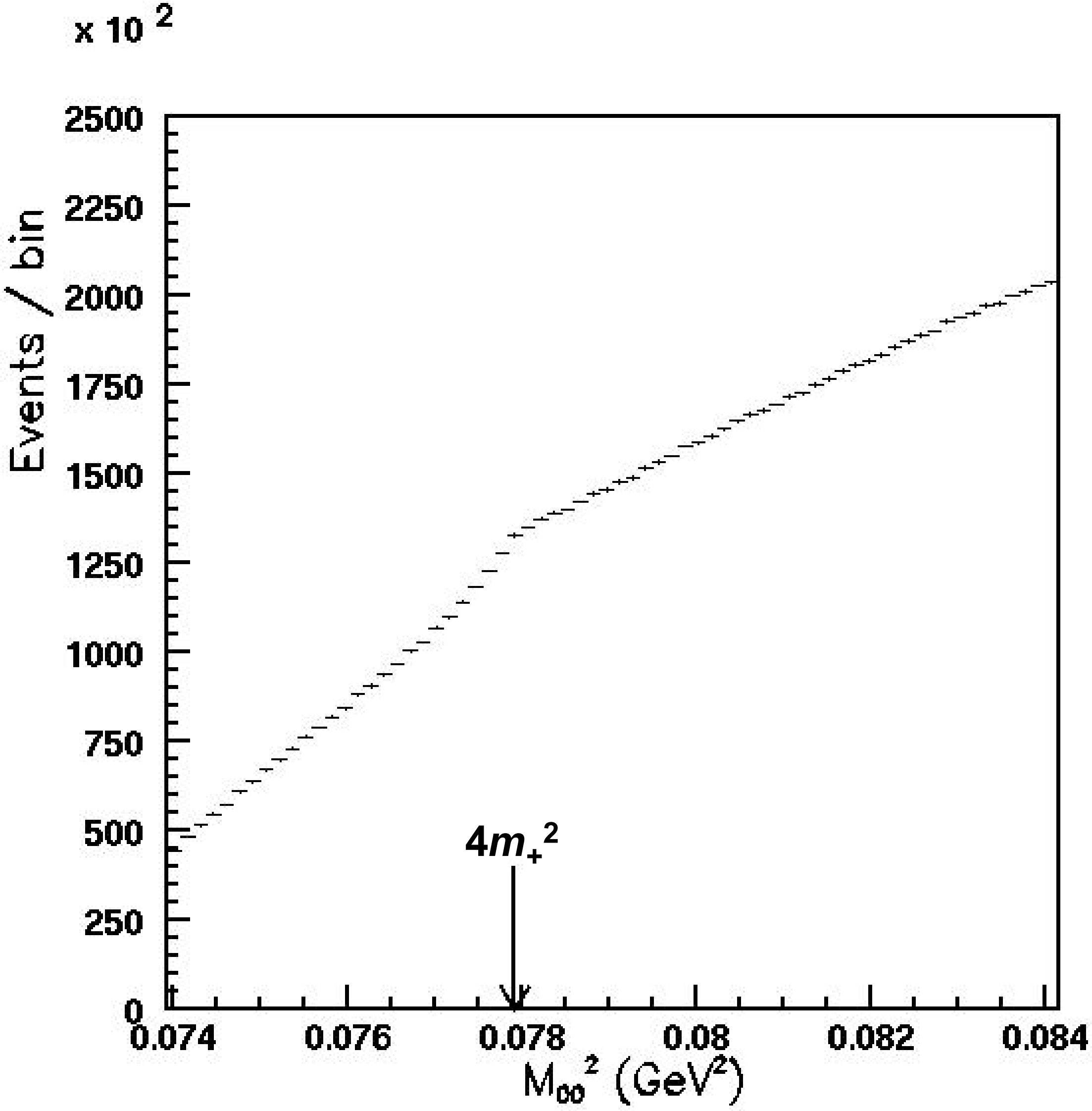
# NA48/2 (PRELIMINARY)

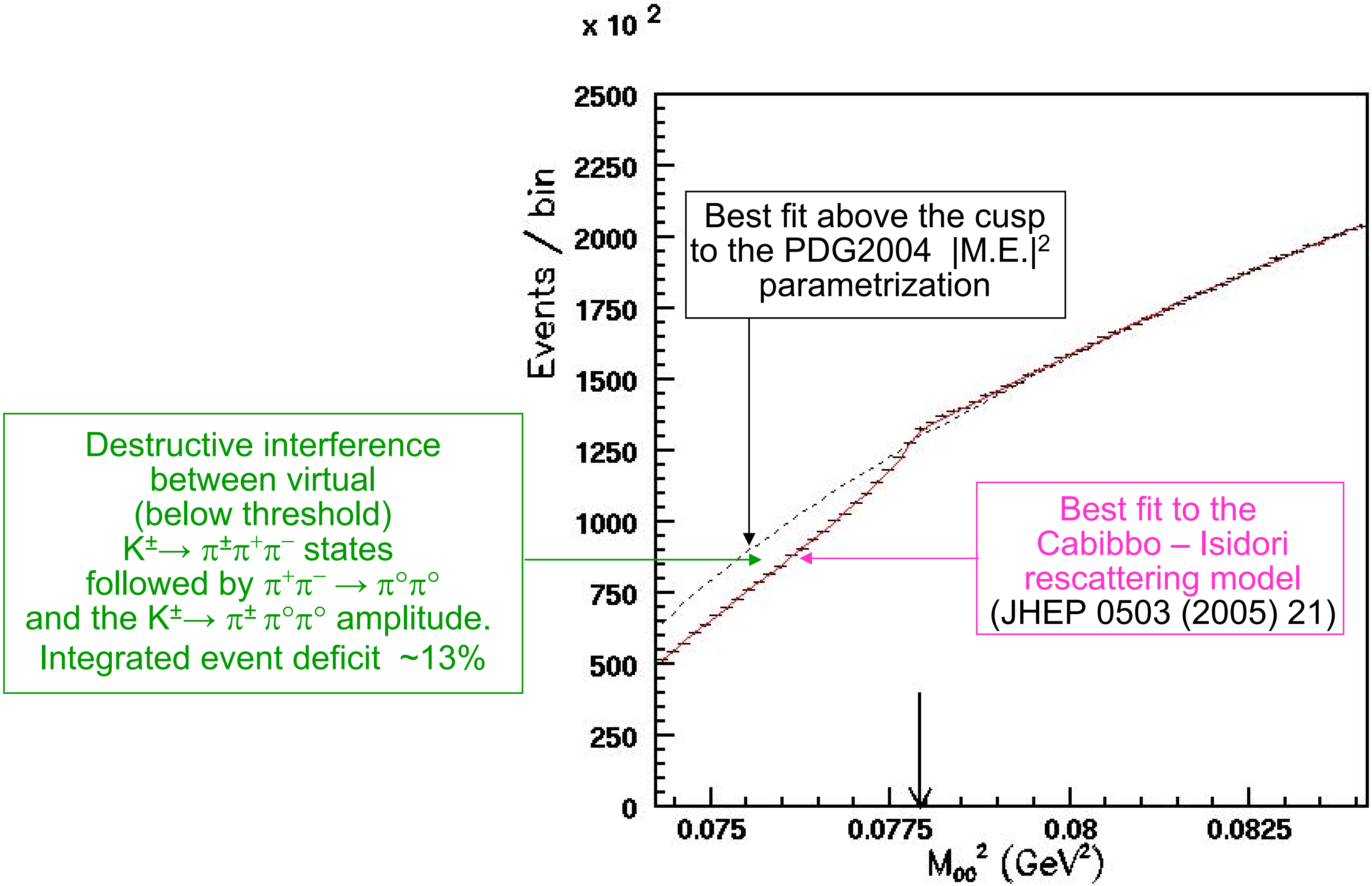
**59,624,170 fully reconstructed $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ events**




$m_{+} \equiv m(\pi^{+})$

# ZOOM ON THE CUSP REGION






**The $\pi^+\pi^-$ charge exchange amplitude near threshold is proportional to the difference of $\pi\pi$ scattering lengths $a_0 - a_2$**

➡ **a new method to measure $a_0 - a_2$ and its sign**

N. Cabibbo, PRL 93 (2004) 12181

## Fit results:

$$(a_0 - a_2)m_+ = 0.261 \pm 0.006 \pm 0.003 \pm 0.0013 \pm 0.013$$

(stat.) (syst.) (ext.) (theor.)

$$a_2 m_+ = -0.037 \pm 0.013 \pm 0.009 \pm 0.002$$

(the sensitivity to $a_2$ comes from higher-order terms)

**External uncertainty:**

from the uncertainty on the ratio of $K^+ \to \pi^+\pi^+\pi^-$ and $K^+ \to \pi^+\pi^\circ\pi^\circ$ decay widths:

$$\frac{\Gamma(K^+ \to \pi^+\pi^+\pi^-)}{\Gamma(K^+ \to \pi^+\pi^0\pi^0)} = 3.182 \pm 0.047$$ **(PDG 2006)**

giving $$\frac{A(K^+ \to \pi^+\pi^+\pi^-)}{A(K^+ \to \pi^+\pi^0\pi^0)} = 1.975 \pm 0.015$$ at the Dalitz plot centres ($u = v = 0$)

(exact isospin symmetry predicts 2)

**Theoretical uncertainty on $(a_0 - a_2)m_+$:** ± 5%

(estimated effect from neglecting higher order diagrams and radiative corrections)

## Fit with analiticity and chiral symmetry constraint between $a_0$ and $a_2$ (Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008)

$$(a_0 - a_2)m_+ = 0.263 \pm 0.003 \pm 0.0014 \pm 0.0013 \pm 0.013$$

(stat.) (syst.) (ext.) (theor.)

## GOOD AGREEMENT WITH THE RESULT OF THE DIRAC EXPERIMENT :

Pionium mean lifetime $\tau_{1s} = (2.91^{+0.24}_{-0.43}) \times 10^{-15}$ s

$$\Longrightarrow |a_0 - a_2| m_+ = 0.264^{+0.020}_{-0.011}$$

**Central values = old values !**

**NA48/2 :** $(a_0 - a_2)m_+ = 0.261 \pm 0.006 \pm 0.003 \pm 0.0013 \pm 0.013$

stat. syst. ext. theor.

**Very little theoretical uncertainty in the prediction of the pionium lifetime because the interaction responsible for $\pi^+\pi^- \rightarrow \pi^\circ\pi^\circ$ is made effectively "weak" by the large pionium radius:**

$$R_{pionium} \approx R_\infty \frac{2m_e}{m_+} \approx 3.9 \times 10^{-11}\, cm$$

($R_\infty$ : Bohr radius for $M_{nucleus} = \infty$ )

$R_{pionium}$ >> strong interaction radius ( $\sim 10^{-13}$ *cm*)

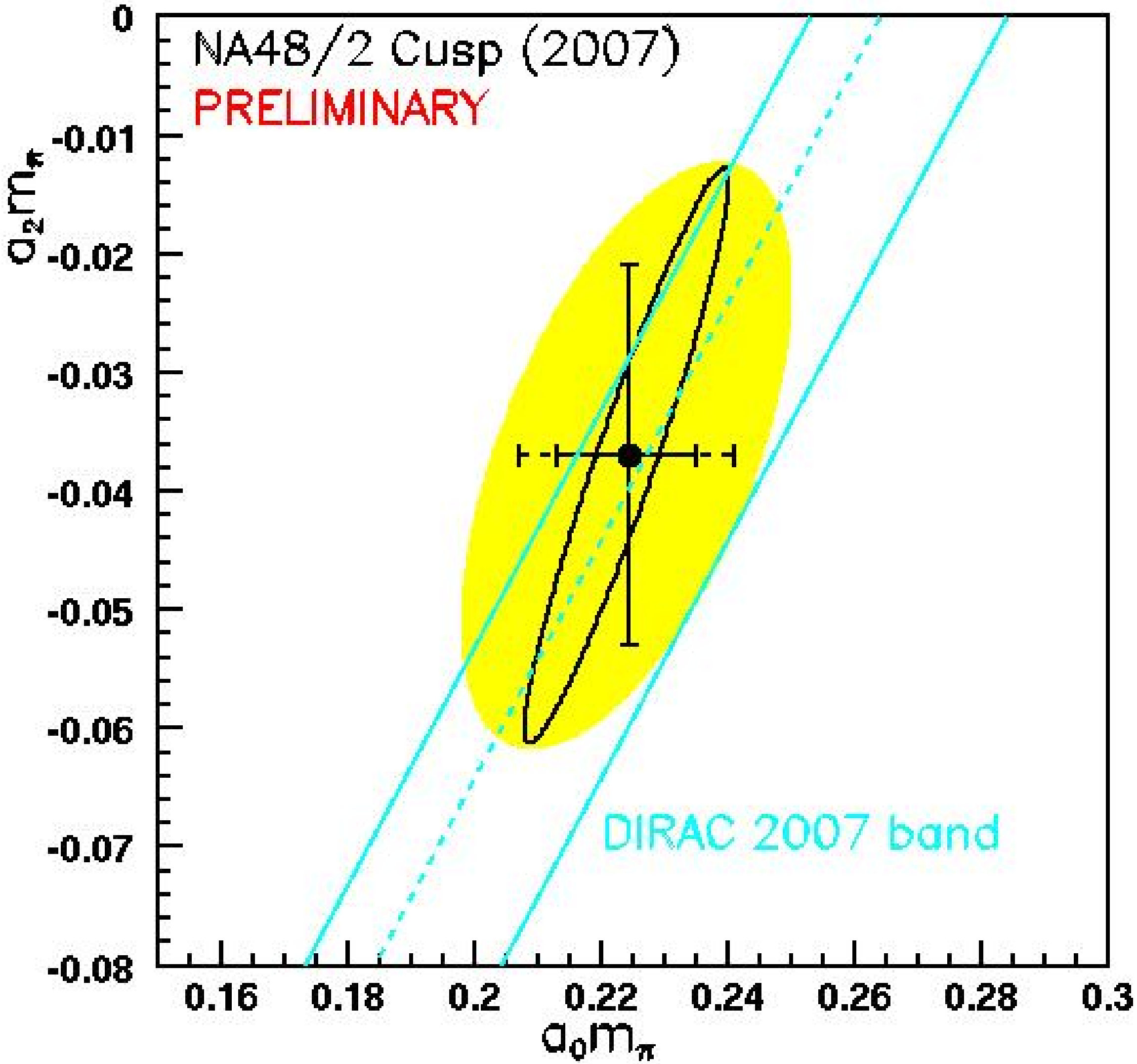
$\Rightarrow$ very little overlap of the $\pi^+\pi^-$ atomic wave function with the strong interaction volume

**NA48/2 (PRELIMINARY): from (a0 – a2) and a2 extract a0**

(must take into account the statistical error correlation coefficient ≈ –0.92)

$a_0 m_+ = 0.224 \pm 0.008 \pm 0.006 \pm 0.003 \pm 0.013$

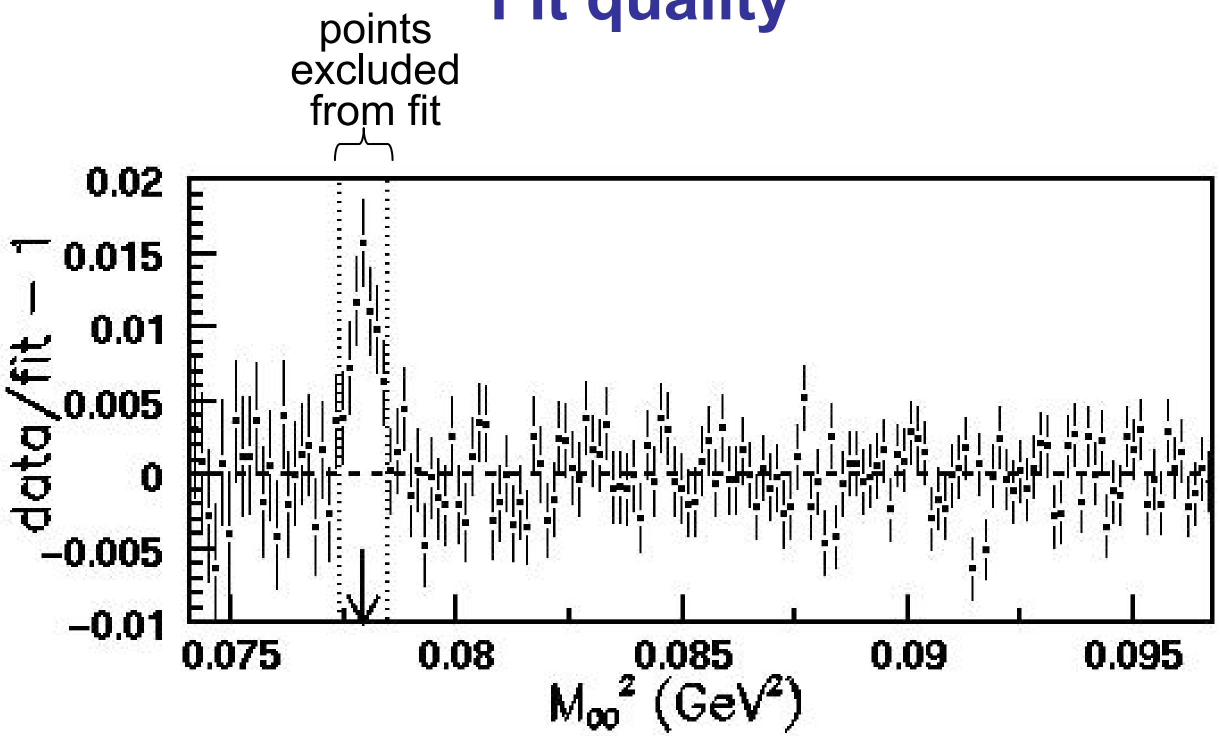
stat. svst. ext. theor.




The yellow area represents theoretical uncertainty (assumed Gaussian)

The dashed bars represent the theoretical uncertainty

# Fit quality




**Observe an excess of events in the $M_{00}^2$ interval excluded from fit.**
**Formation of $\pi^+\pi^-$ atoms (pionium) decaying to $\pi^\circ\pi^\circ$ ?**

**Rate = (1.82 ± 0.21) x $10^{-5}$ per $K^\pm \to \pi^\pm\pi^+\pi^-$ decay**

**= 2.28 ± 0.26 times the predicted pionium formation rate**
( = 0.8 x $10^{-5}$ , Silagadze 1994)

(adding a $\delta$ – function at $M_{00} = 2m_+$ to the theoretical spectrum before taking into account detector effects, and using its integral as a free parameter in the fit)

**Direct count of the event excess in the bins excluded from the fit:**
**Rate = 2.58 ± 0.31 times the predicted pionium formation rate**

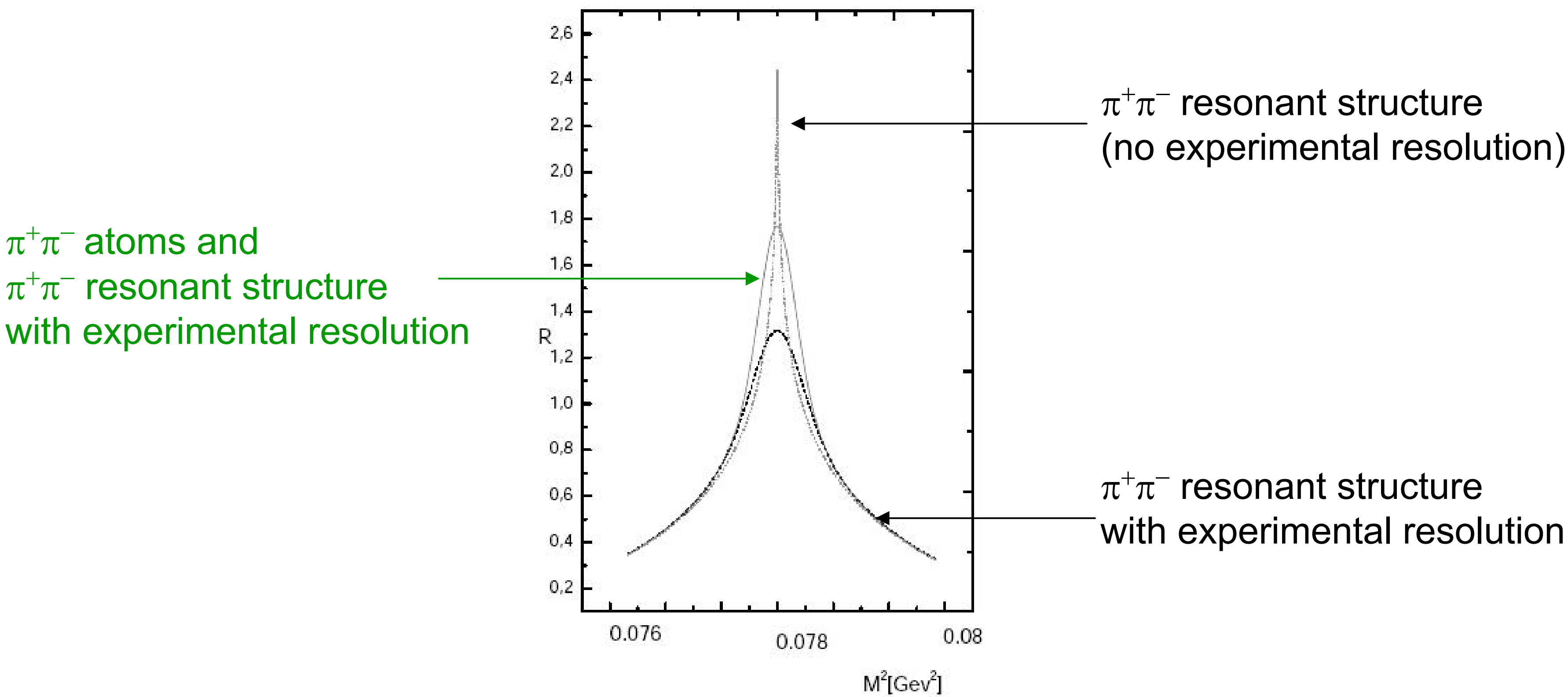
(960 ± 320 events more than the pionium best fit )

**IS THE EVENT EXCESS ONLY DUE TO PIONIUM ATOMS?**

**Electromagnetic corrections to final state interactions in $K \to 3\pi$ decays**
(Gevorkian, Tarasov, Voskresenskaya, hep-ph / 0612129)

**Two contributions from $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ decay to the $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$ cusp region:**

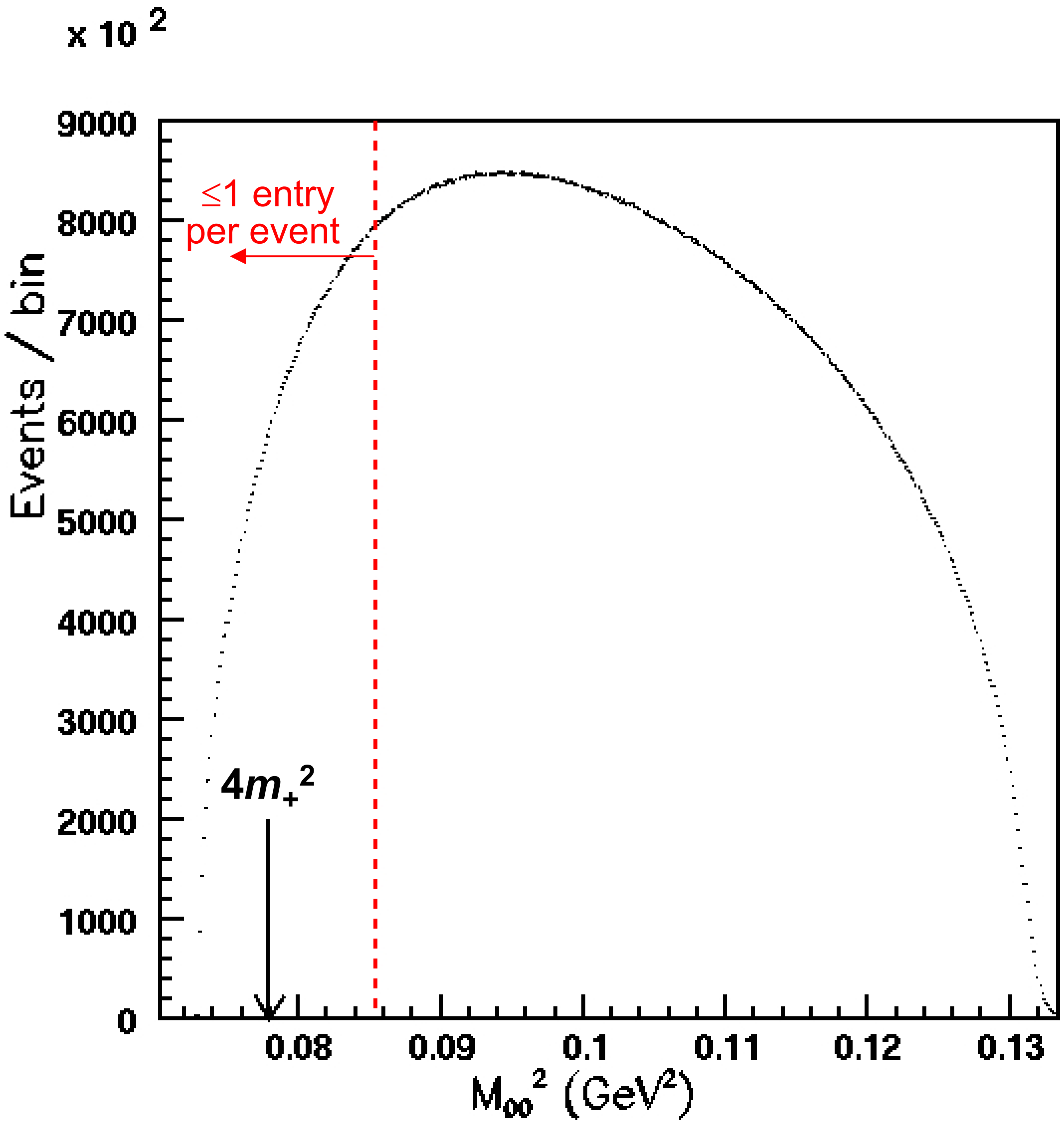
- **Pionium formation : $\pi^{+}\pi^{-}$ atom $\to \pi^{\circ}\pi^{\circ}$ (negligible width)**
- **Additional $\pi^{+}\pi^{-}$ unbound states with resonance structure $\to \pi^{\circ}\pi^{\circ}$**



# $K_L \to \pi^\circ\pi^\circ\pi^\circ$

## 87,950,601 events (NA48)

(3 entries per event)




**CUSP STRUCTURE MUCH LESS VISIBLE THAN IN $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ DECAY**

# $K^+ \to \pi^+\pi^\circ\pi^\circ$ , $K_L \to \pi^\circ\pi^\circ\pi^\circ$ comparison

two possible $\pi^+\pi^-$ pairs

$$\mathcal{R}(K^+) \approx \frac{2\mathcal{M}_{++-}\mathcal{M}_{+00}}{(\mathcal{M}_{+00})^2} = \frac{2\mathcal{M}_{++-}}{\mathcal{M}_{+00}}$$

$\mathcal{M}_{++-}$ : $K^+ \to \pi^+ \pi^+ \pi^-$ matrix element

$\mathcal{M}_{+00}$ : $K^+ \to \pi^+ \pi^\circ \pi^\circ$ matrix element

$$\mathcal{R}(K_L) \approx \frac{\mathcal{M}_{+-0}\mathcal{M}_{000}}{(\mathcal{M}_{000})^2} = \frac{\mathcal{M}_{+-0}}{\mathcal{M}_{000}}$$

$\mathcal{M}_{+-0}$ : $K_L \to \pi^+ \pi^- \pi^\circ$ matrix element

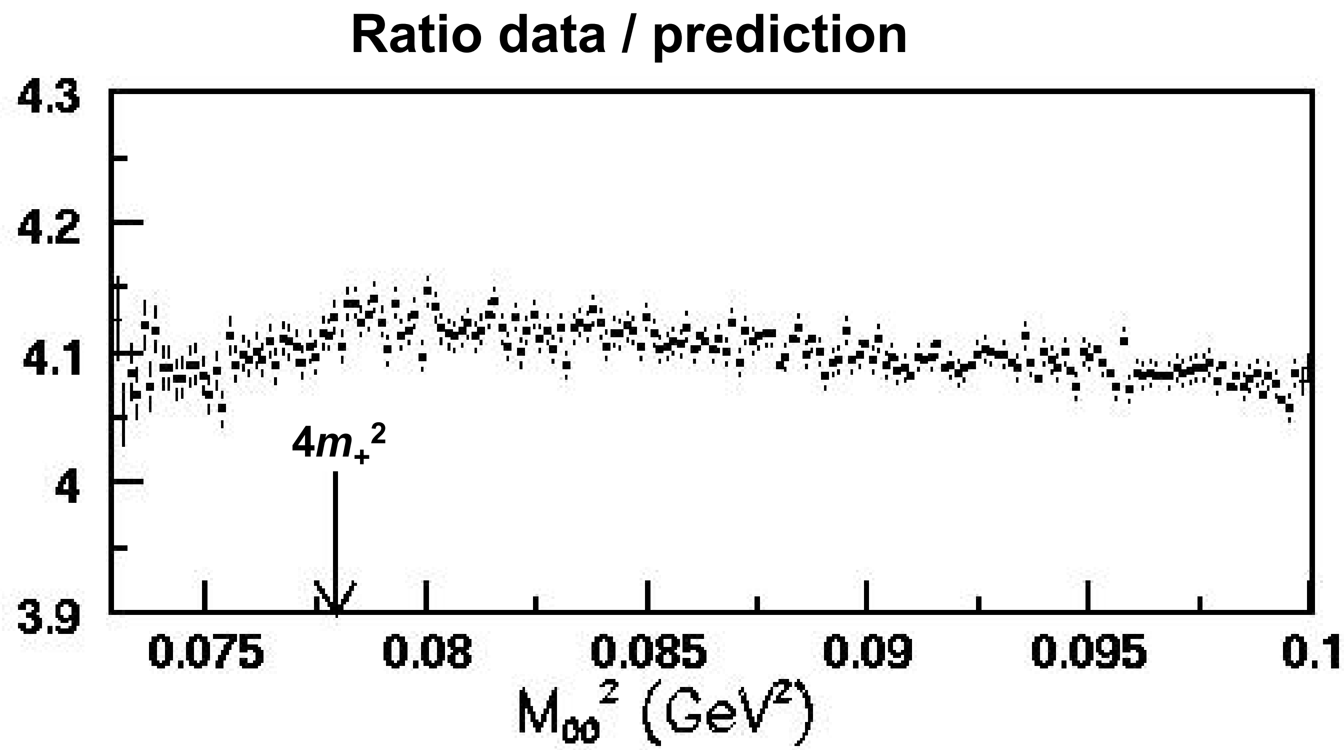
$\mathcal{M}_{000}$ : $K_L \to \pi^\circ \pi^\circ \pi^\circ$ matrix element

**Calculate matrix elements at cusp point ($M_{\pi\pi} = 2m_+$) from <u>measured</u> partial width ratios and slope parameters:**

$$\mathcal{R}(K^+) \approx 6.1\ ;\ \ \mathcal{R}(K_L) \approx 0.47 \quad \Longrightarrow \quad \frac{\mathcal{R}(K+)}{\mathcal{R}(K_L)} \approx 13$$

**Cusp "visibility" is ~ 13 times higher in $K^+ \to \pi^+\pi^\circ\pi^\circ$ decays than in $K_L \to \pi^\circ\pi^\circ\pi^\circ$ decays**

## Compare data with predicted distribution <u>without</u> $\pi^+\pi^-$ charge exchange (obtained from Monte Carlo simulation)




➡ **Evidence for a change of slope in the data near the expected cusp point**

# Rescattering in $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ decay

**First attempt to perform a combined fit of $M(\pi\pi)$ distributions from $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decays using the Bern – Bonn non-relativistic effective field theory**

(Colangelo, Gasser, Kubis, Rusetsky, Phys. Lett. B 638 (2006) 187)

**(VERY PRELIMINARY – only statistical errors are shown)**

**Fit with 8 free parameters:**

- **1 normalization constant, 2 slope parameters ($g_0$, $h_0$) for $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$ $M_{00}^2$ distribution (2004 data) ; $k_0$ = 0.0096 is kept fixed ;**
- **1 normalization constant common to the $M_{+-}^2$ and $M_{\pm\pm}^2$ distributions, 3 slope parameters (g, h, k) for $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ (2003 data);**
- **$(a_0 - a_2)m_+$ ; $a_2 m_+ = -0.044$ (fixed in the fit)**

**Fit intervals:**

- **$0.074094 < M_{00}^2 < 0.11744$ GeV$^2$**
- **$0.080694 < M_{+-}^2 , M_{\pm\pm}^2 < 0.119844$ GeV$^2$**

Fit results:

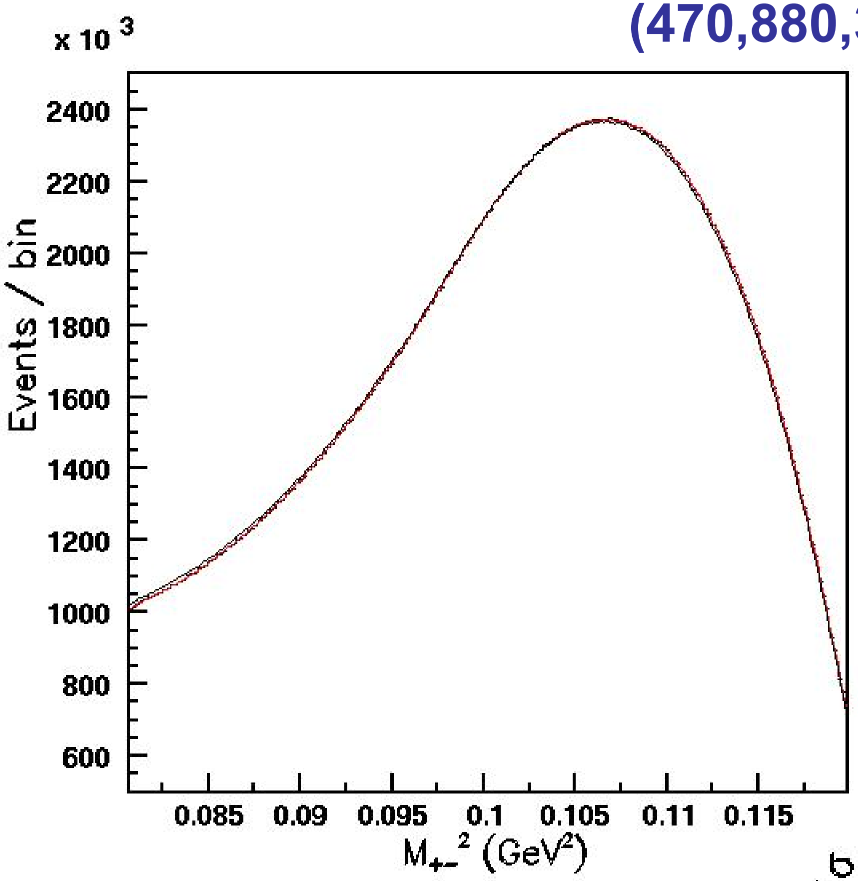
$\chi^2 = 757.1 / 757$ d.f.

$g_0 = 0.6191 \pm 0.0008$ ; $h_0 = -0.0497 \pm 0.0006$

$g = -0.1837 \pm 0.0006$ ; $h = 0.0004 \pm 0.0003$ ; $k = -0.0059 \pm 0.0001$

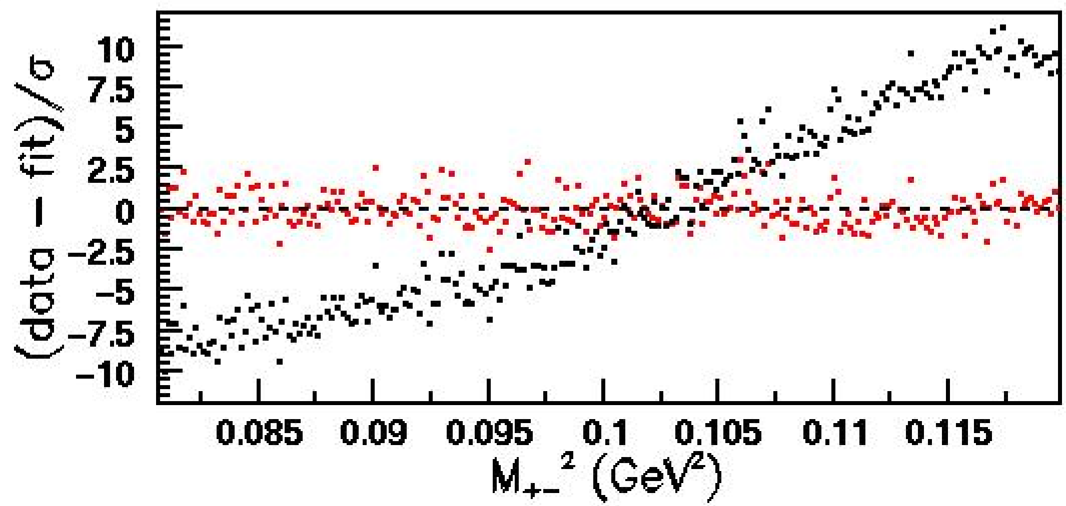
$(a_0 - a_2)m_+ = 0.266 \pm 0.003$

# $M_{+-}^2$ distribution from $K^\pm \to \pi^\pm\pi^+\pi^-$ decay (470,880,352 entries)

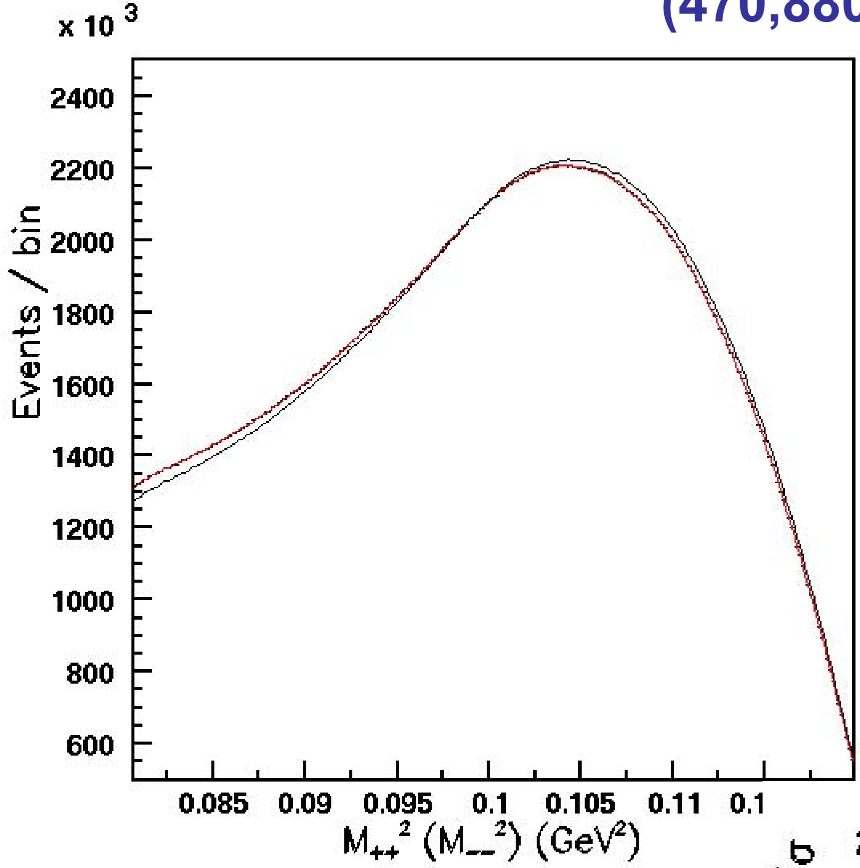



Combined best fit with rescattering

Best fit to $K^\pm \to \pi^\pm\pi^+\pi^-$ only; no rescattering; slope parameter values as from fit with rescattering. $\chi^2$ = 9573 / 260 d.f.

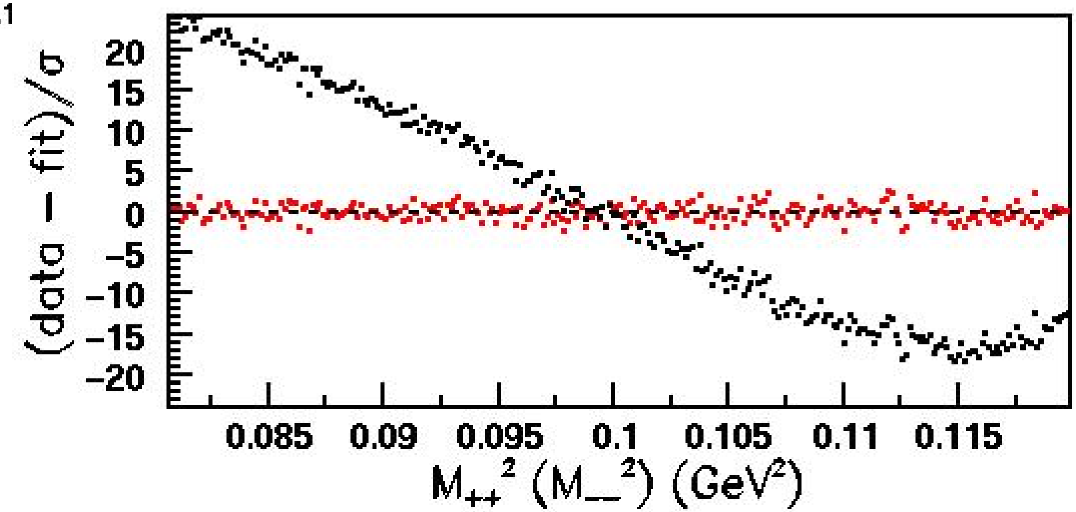


# $M_{\pm\pm}^2$ distribution from $K^\pm \to \pi^\pm\pi^+\pi^-$ decay (470,880,352 events)




Combined best fit with rescattering

Best fit to $K^\pm \to \pi^\pm\pi^+\pi^-$ only; no rescattering; slope parameter values as from fit with rescattering. $\chi^2$ = 47106 / 260 d.f.

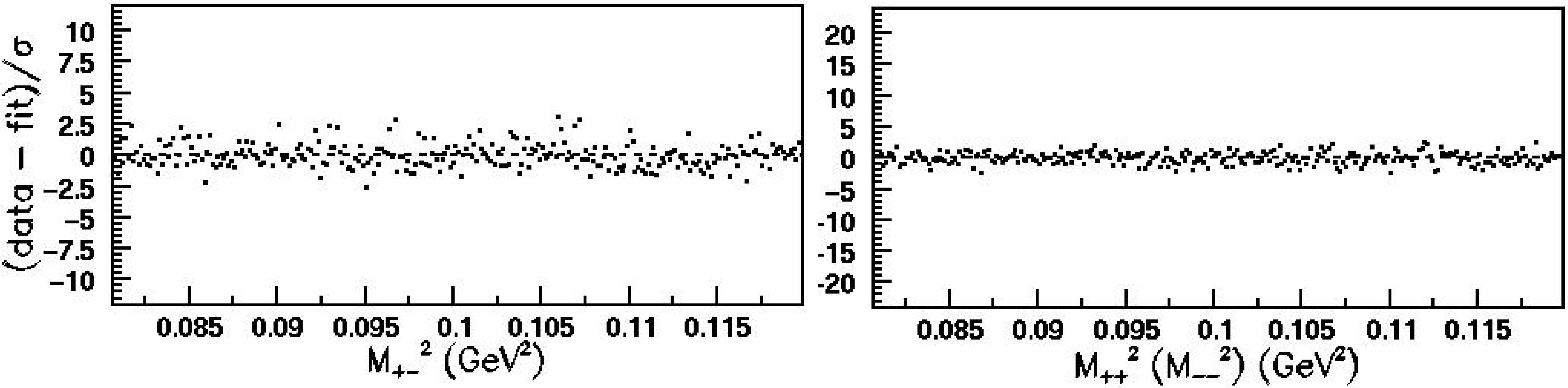


## Fit to $K^\pm \to \pi^\pm\pi^+\pi^-$ only

## No rescattering;

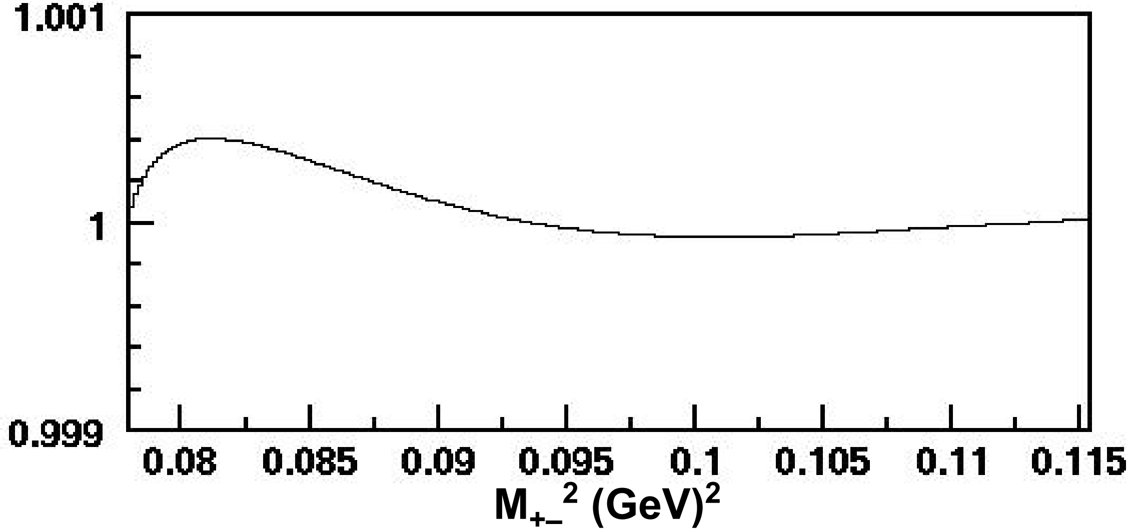
## slope parameters allowed to vary

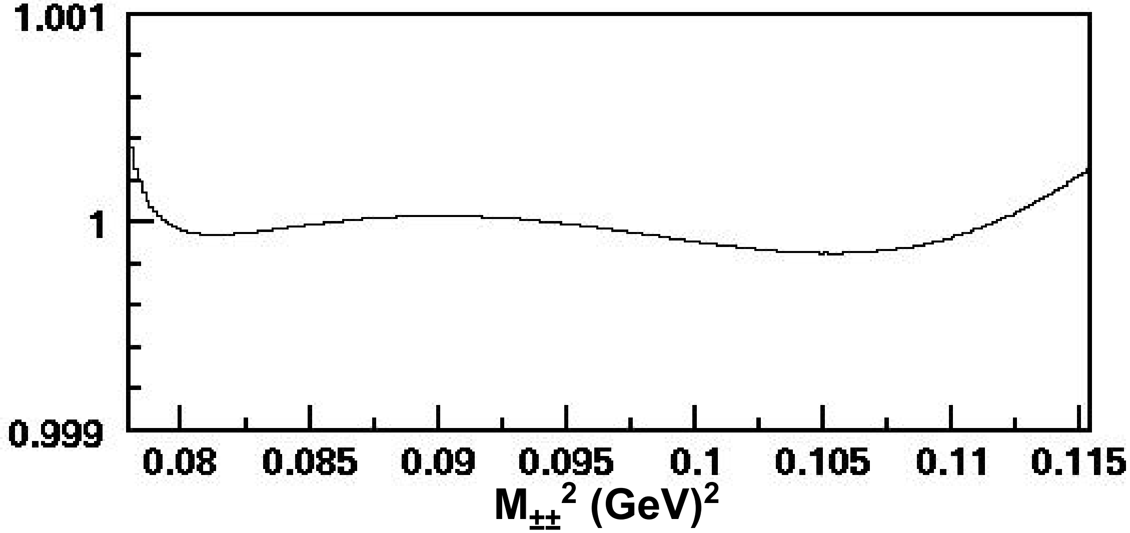
## $\chi^2$ = 516.9 / 517 d.f.




- **Rescattering effects in $K^\pm \to \pi^\pm\pi^+\pi^-$ decay can be "faked" by different slope parameter values:**
  $g$ = –0.2101 ± 0.0001
  $h$ = 0.0088 ± 0.0003
  $k$ = –0.0039 ± 0.0001
- **It is very difficult (impossible?) to obtain evidence for $\pi\pi$ rescattering from $K^\pm \to \pi^\pm\pi^+\pi^-$ decay alone**

$$\text{Ratio} \quad \frac{\text{Best fit with rescattering}}{\text{Best fit without rescattering}}$$



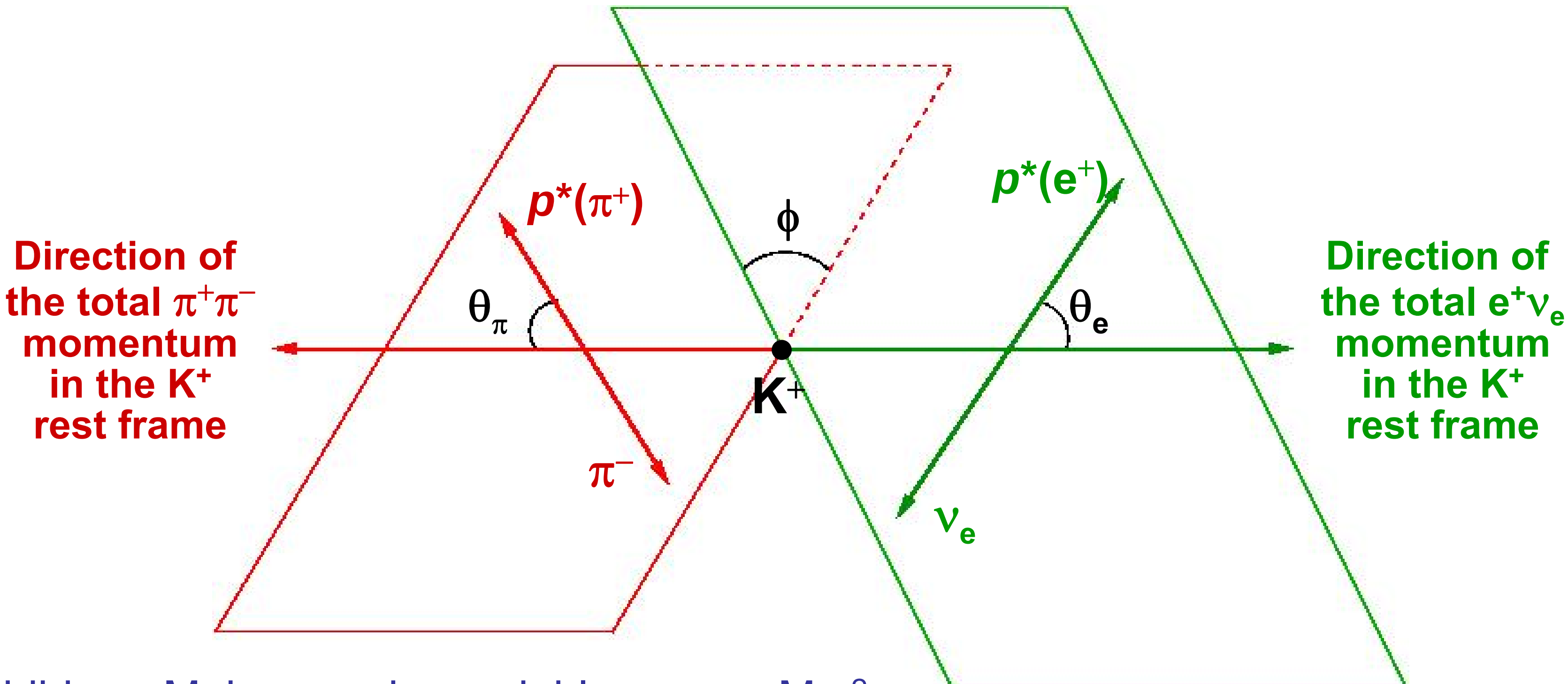
# A message to the Particle Data Group

**The parametrization of the |matrix element|$^2$ for $K^\pm \to \pi^\pm\pi^+\pi^-$, $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$, $K_L \to \pi^+\pi^-\pi^\circ$, and $K_L \to \pi^\circ\pi^\circ\pi^\circ$ by a series expansion of two Dalitz variables (see PDG 2006, 660-1, 681) should be abandoned:**

- **It cannot be used to include other amplitudes interfering with the main, weak decay amplitude;**
- **It does not describe the cusp structure observed in $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ by NA48/2;**
- **The slope parameter values (coefficients of the series expansions) depend on whether $\pi\pi$ scattering in the final state is taken into account.**

# $\pi\pi$ scattering in $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\overset{(-)}{\nu}$ decay

A rare decay [ B.R. = $(4.09 \pm 0.09) \times 10^{-5}$ ] described by five independent variables




Cabibbo – Maksymowicz variables : $s_{\pi} \equiv M_{\pi\pi}^{2}$

$s_{e} \equiv M_{e\nu}^{2}$

$\theta_{e}$

$\theta_{\pi}$

$\phi$

**For $K^{+} \Rightarrow K^{-}$**

**$\phi \Rightarrow \pi + \phi$**

**$\theta_{e} \Rightarrow \pi - \theta_{e}$**

**The Ke4 decay amplitude depends on two complex phases identified with**

- $\delta_0$ : **the $\pi\pi$ scattering phase shift in the I = 0 , $\ell$ = 0 state (*s* – wave)**
- $\delta_1$ : **the $\pi\pi$ scattering phase shift in the I = 1 , $\ell$ = 1 state (*p* – wave)**

**The Ke4 decay rate depends on the phase shift difference $\delta = \delta_0 - \delta_1$**

**$\delta$ is an increasing function of $M_{\pi\pi}$ ; $\delta \rightarrow 0$ for $M_{\pi\pi} \rightarrow 2m_+$**

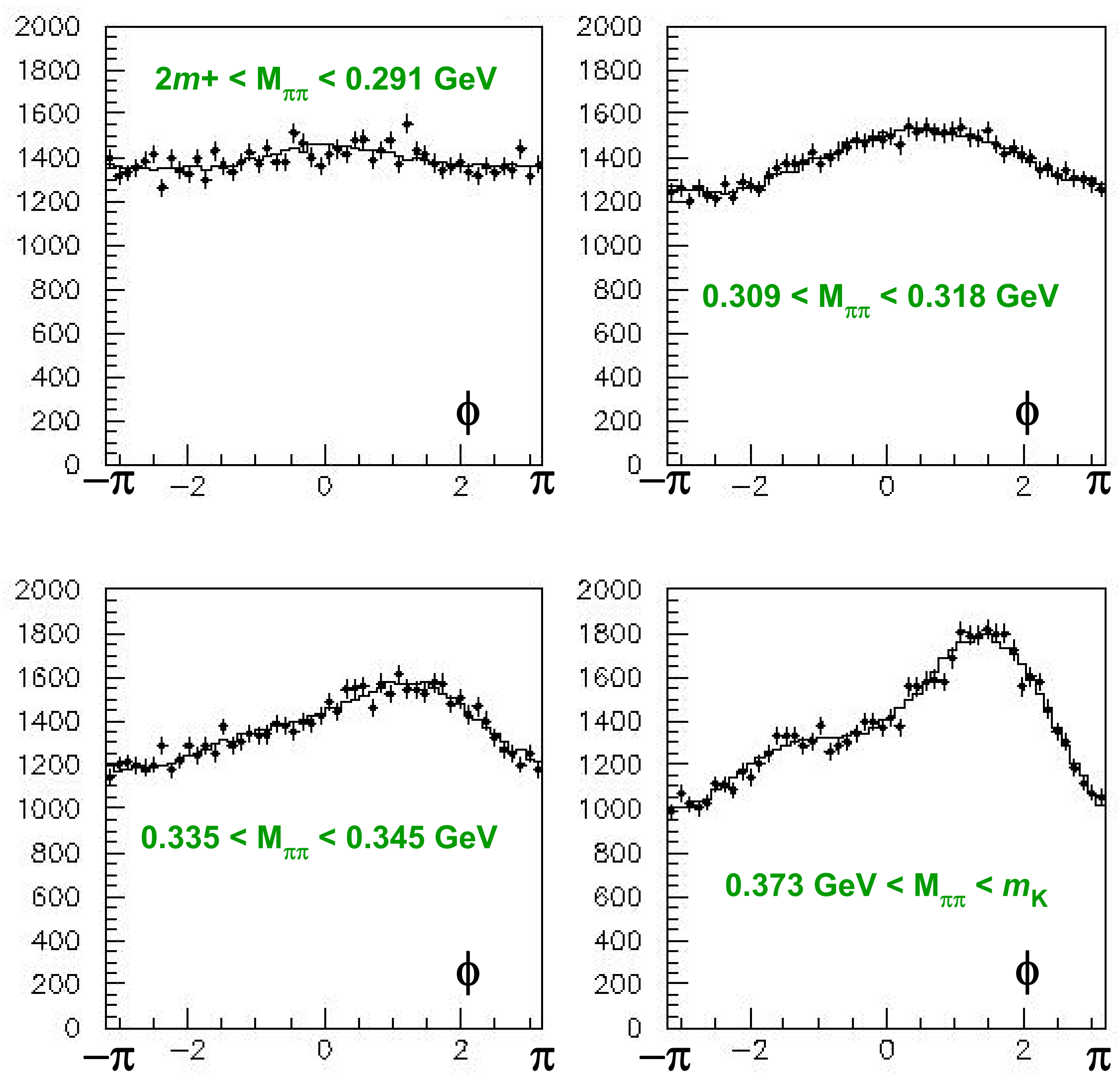
( from scattering theory : $\delta(k) \approx ak$ at very low centre-of-mass momentum *k* ;
*a* is the scattering length ; $a \neq 0$ for *s* – waves only )

**$\delta \neq 0$ in the Ke4 decay amplitude ⟹ asymmetric distribution of the charged lepton direction with respect to the plane defined by the two pions ($\phi$ distribution)**
(Shabalin 1963)

**The asymmetry of the $\phi$ distribution increases with $M_{\pi\pi}$**

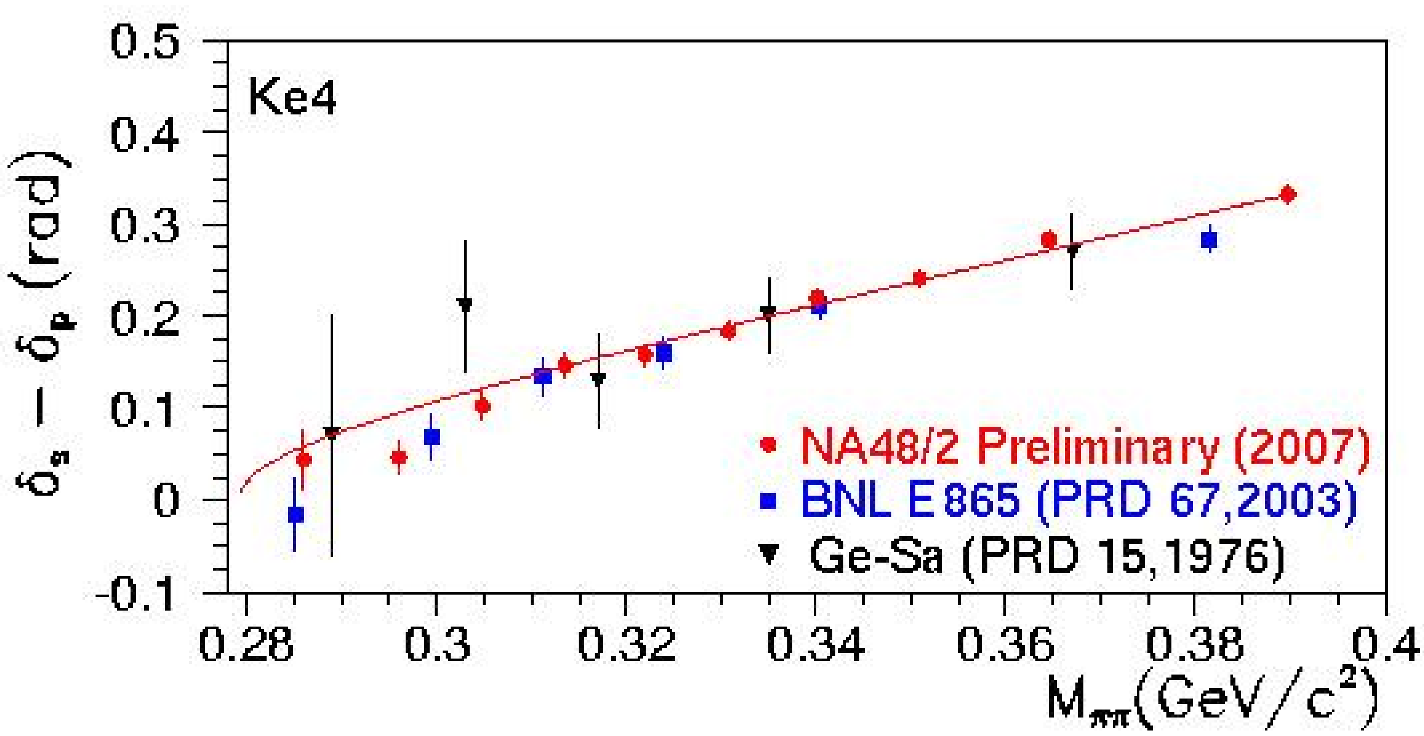
# NA48 / 2 (Preliminary)

# $\phi$ distributions for four $M_\pi$ bins



# $\delta$ versus $M_{\pi\pi}$

**(no isospin – breaking corrections)**



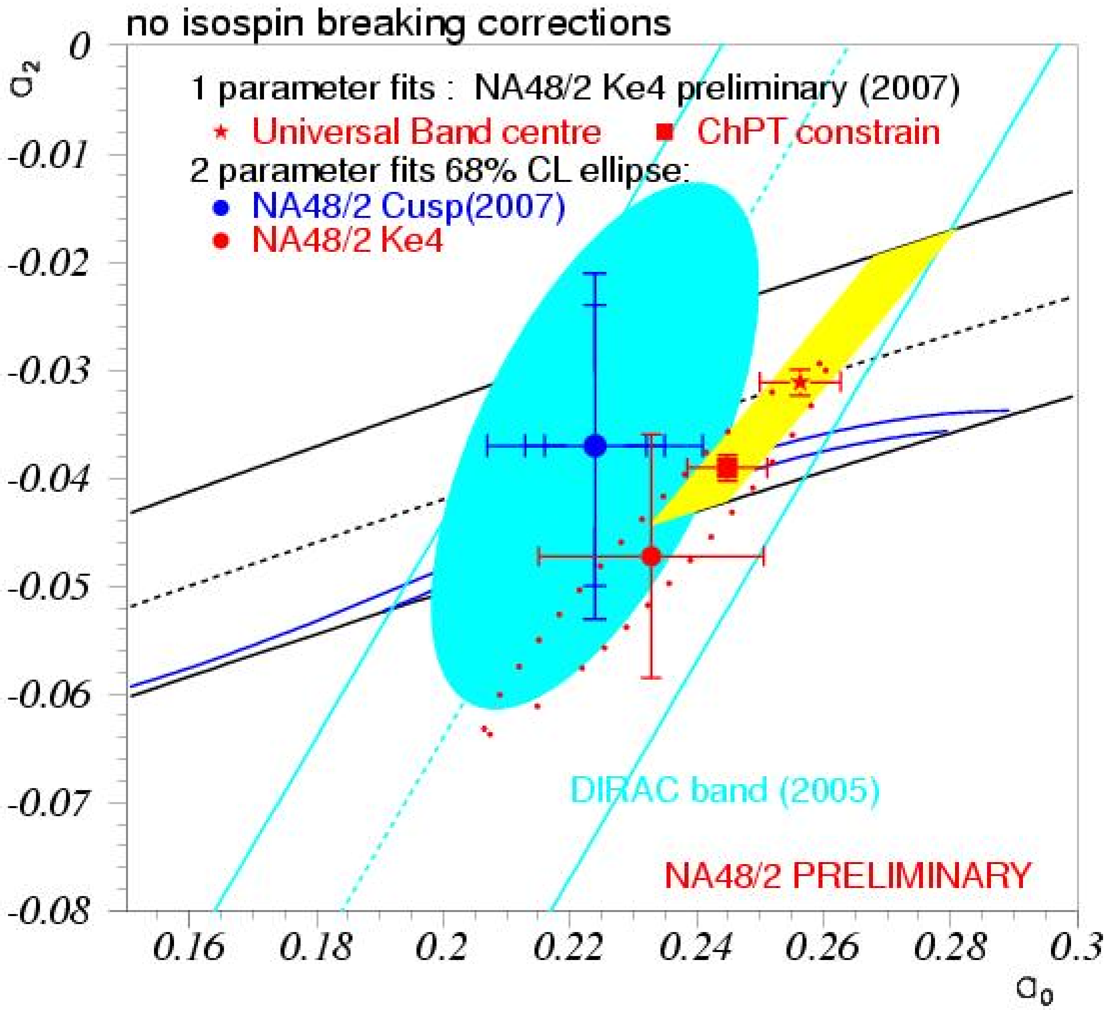

Geneva – Saclay : ~ 30,000 events , $p_{K^+}$ = 2.8 GeV/c

BNL E865 : 406,103 events (with ~ 4.4% background), $p_{K^+}$ = 6 GeV/c

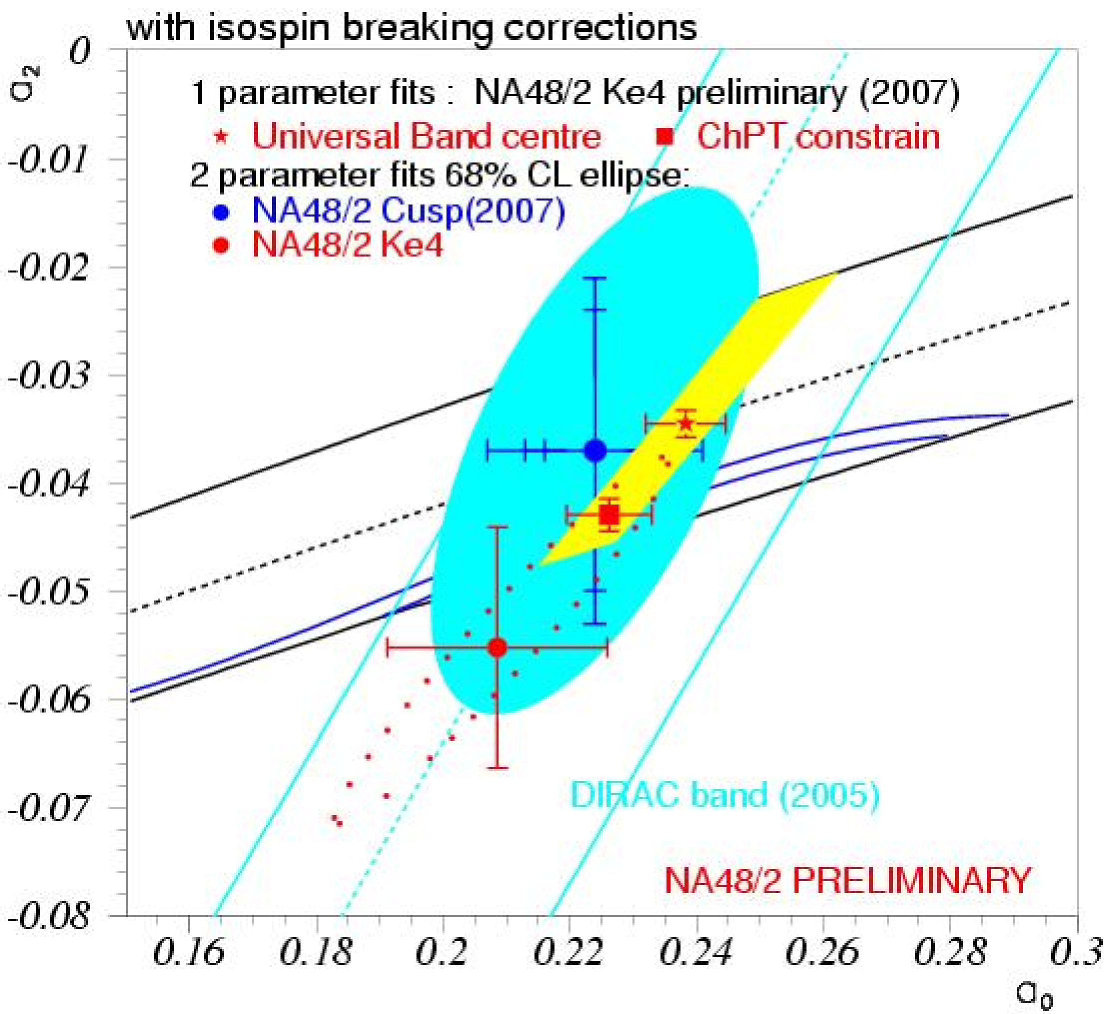
NA48/2 : 677,510 events (with ~0.5% background), $p_{K^\pm}$ = 6 GeV/c

**NOTE:** the isospin – breaking corrections reduce $\delta$ by 0.01 – 0.012 (see the presentation by J. Gasser)

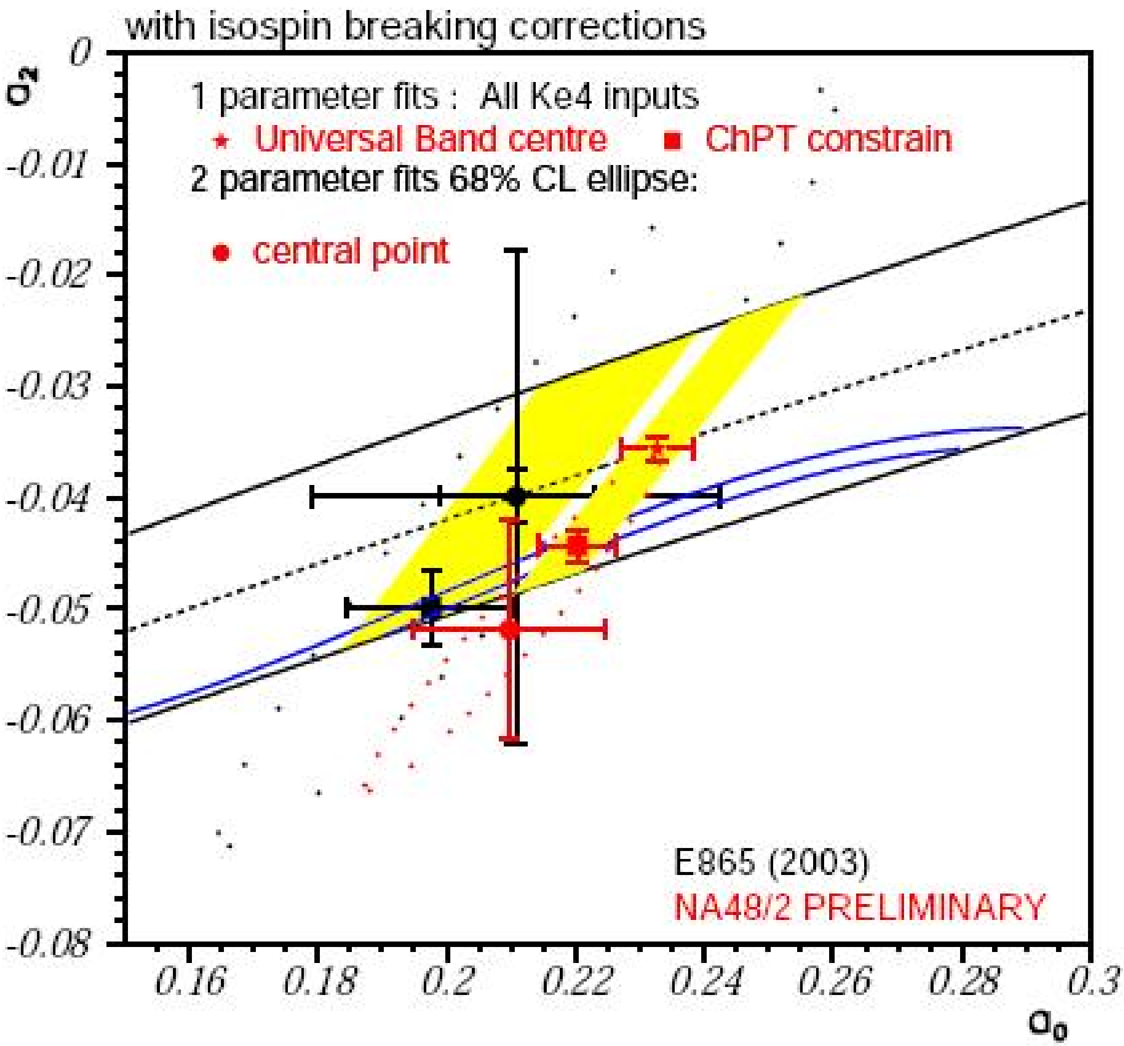
# NA48/2 Cusp – Ke4 comparison



# NA48/2 Cusp – Ke4 comparison



# Ke4: NA48/2 – BNL E865 comparison




**Marginal agreement for constrained fits**

# CONCLUSIONS

- **Cusp in $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decay**

  Little improvement is expected from the experimental side.
  The determination of $a_0$ and $a_2$ can become more precise only if the theoretical models to fit the data are improved (electromagnetic effects, higher order terms)
  Using the Bern – Bonn non-relativistic effective field theory to simultaneously fit the $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ and $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ data has just started.
  A direct measurement of $A_{++-} / A_{+00}$ from the NA48/2 data is very difficult, but (perhaps) not impossible. Will try.

- **Cusp in $K_L \rightarrow \pi^{\circ}\pi^{\circ}\pi^{\circ}$ decay**

  Visible, but the effect is not large. Probably only a consistency check, not competitive with $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ to extract a precise value of $a_0 - a_2$.

- **Ke4**

  Expect further improvement from NA48/2 by the addition of the 2004 data ( $\Rightarrow$ total number of events > $10^6$).
  A new development of the last few weeks: isospin – breaking corrections.
  Better agreement between NA48/2 Ke4 scattering lengths and those measured from the cusp.
  Agreement between NA48/2 and E865 is marginal and should be understood