Precision measurement of ππ scattering lengths in Ke4 decays by NA48/2

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KAON 09, EPOCHAL Tsukuba, 9-12 June 2009

2009 KAON INTERNATIONAL CONFERENCE
KAON09
June 09(Tue) ▸ 12(Fri), 2009 EPOCHAL TSUKUBA

On behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

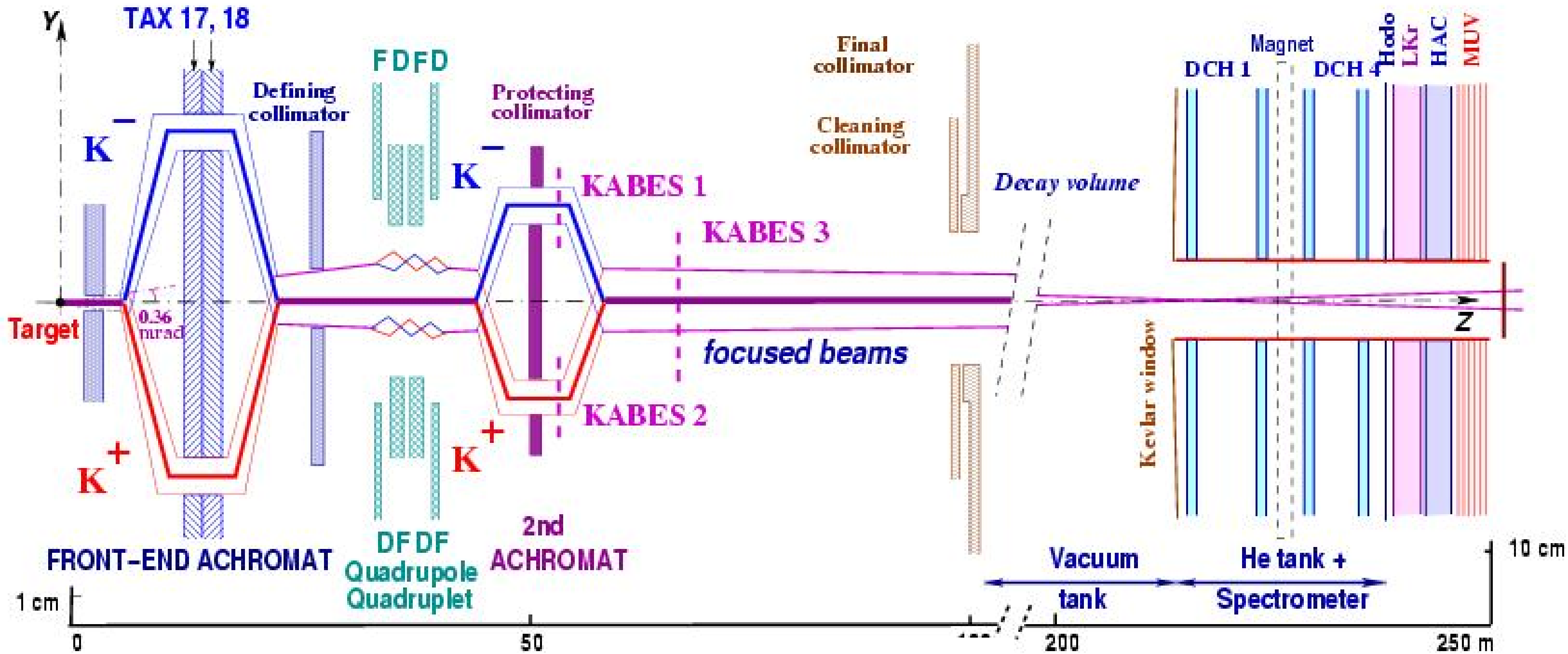
outline

- Brief introduction to NA48/2
- QCD tests from study of Kaon decays @ NA48/2
- Ke4 decays ($K^{\pm} \rightarrow \pi^{+}\pi^{-} e^{\pm} \nu$) :

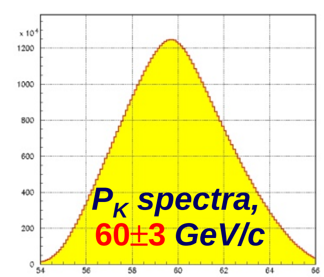
  Form Factors and $\pi\pi$ scattering lengths
- Combining cusp and Ke4 results
- Summary

The NA48/2 experiment at the CERN-SPS : primarily designed for CP violating charge asymmetries studies in K3π decays

2003 run: ~ 50 days + 2004 run: ~ 60 days

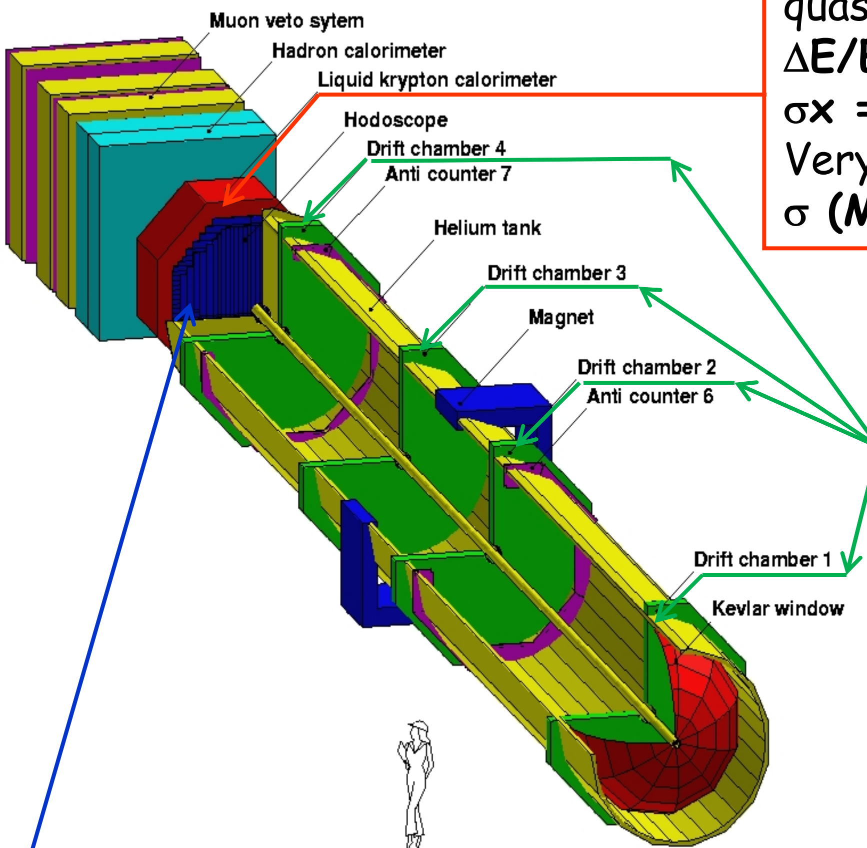


Simultaneous K+ and K- beams: large charge symmetrization of experimental conditions



Beams coincide within ~1mm all along the 114m decay volume flux ratio K+/K- ~1.8

The NA48/2 experiment: detector and performances



LKr electromagnetic calorimeter :
quasi-homogenous and high granularity
$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)
$\sigma x = \sigma y \sim 1.5$ mm for E=10 GeV
Very good resolution for neutrals ($\pi^0 \to \gamma\gamma$)
$\sigma (M\pi\pi^0\pi^0) = 1.4$ MeV/c²

+

Magnetic spectrometer :
4 high-resolution DCH's + dipole magnet
$\Delta p/p = (1.0 \oplus 0.044\, p)\%$ (p in GeV/c)
Very good resolution for charged invariant masses: $\sigma (M3\pi^{\pm}) = 1.7$ MeV/c²

E/p ratio used for e /π discrimination

Hodoscope for charged fast trigger
$\sigma t = 150$ ps

Kaon decays : what can be learned on QCD @ Low Energy ?

in press

Hadronic decay modes into 3 pions: (D.Madigojine 's talk)

- large Br's : $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$ (1.7 %) and $K^{\pm} \rightarrow \pi^+\pi^-\pi^{\pm}$ (5.6 %),

60 Millions events now analyzed (PRL B633 (2006) partial sample)

- three pions : $\pi^0\pi^0$ system + nearby hadron
- accessible $M_{\pi\pi}$ range from $\pi^0\pi^0$ threshold to (M_K-M_π)

preliminary

Semileptonic decay mode Ke4: (this talk)

- small Br's : $K^{\pm} \rightarrow \pi^+\pi^- e^{\pm} \nu$ (4.1 10^{-5}),

1.1 Million events now analyzed (EPJC 54 (2008) partial sample)

- only two $\pi^+\pi^-$ pions, very clean environment
- accessible $M_{\pi\pi}$ range from $\pi^+\pi^-$ threshold to $(M_K-M_e) \cong M_K$

Two different but complementary approaches to $\pi\pi$ scattering near threshold to extract s-wave scattering lengths (a0, a2) for Isospin I = 0 and I = 2

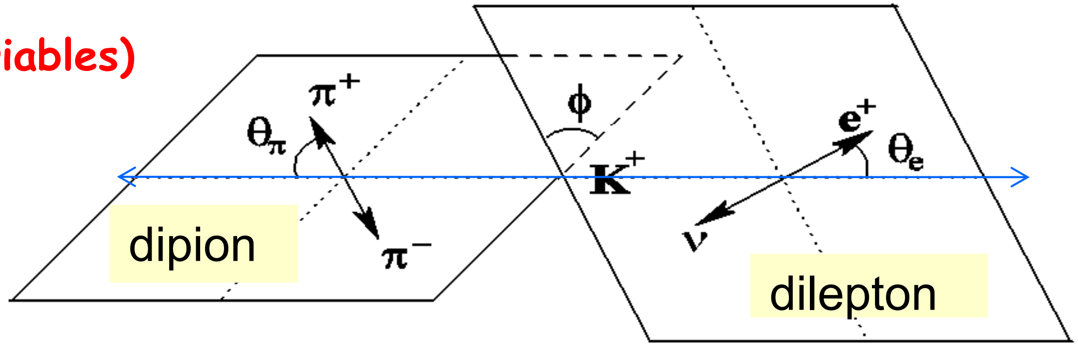
Ke4 decays : formalism

Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$),

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



Partial Wave expansion of the amplitude into s and p waves (Pais-Treiman 1968) + Watson theorem (T-invariance) for δ_ℓ^I

$\delta_0^0 \equiv \delta_s$ and $\delta_1^1 \equiv \delta_p$

F, G = 2 Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi$

$G = G_p e^{i\delta g}$

H = 1 Vector Form Factor

$H = H_p e^{i\delta h}$

Map the distributions of the Ca.Ma. variables in the **five-dimensional space** with 4 Form factors and one phase shift , assuming identical phases for the p-wave Form Factors F_p, G_p, H_p:

The fit parameters are :

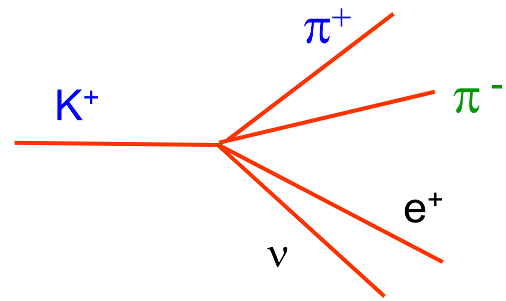
F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

(F_s , F_p , G_p, , H_p are real)

F, G, H are complex and dimensionless

Ke4 decays: event selection and background rejection

Signal ($\pi^+\pi^- e^{\pm} \nu$) topology:
- 3 charged tracks and a good vertex
- two opposite sign pions,
- 1 electron (E(LKr) /p ~ 1),
- some missing energy and p_T (ν)
- reconstruct PK (missing ν hypothesis)



Background main sources :

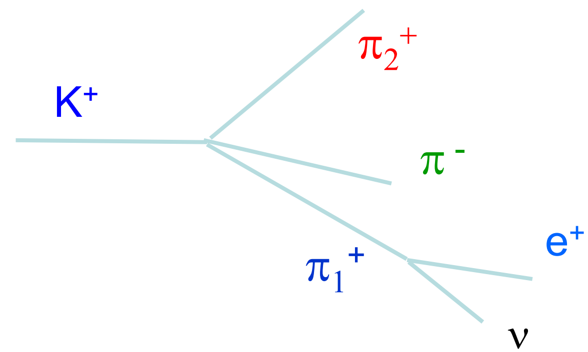
$K^+ \rightarrow \pi^- \quad \pi_2^+ \quad \pi_1^+$ (dominant)

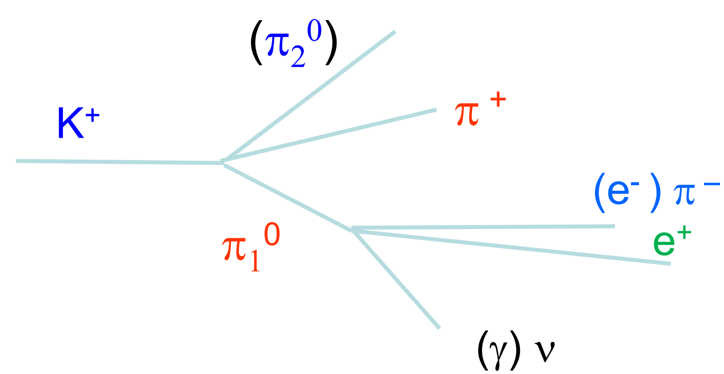
↳ $e^- \nu$ or mis-ident e^-

$K^+ \rightarrow \pi^0 (\pi^0) \pi^+$

↳($e^+e^-\gamma$)

↳ mis-ident π^+ and γ (s) undetected





Ke4 decays: background rejection

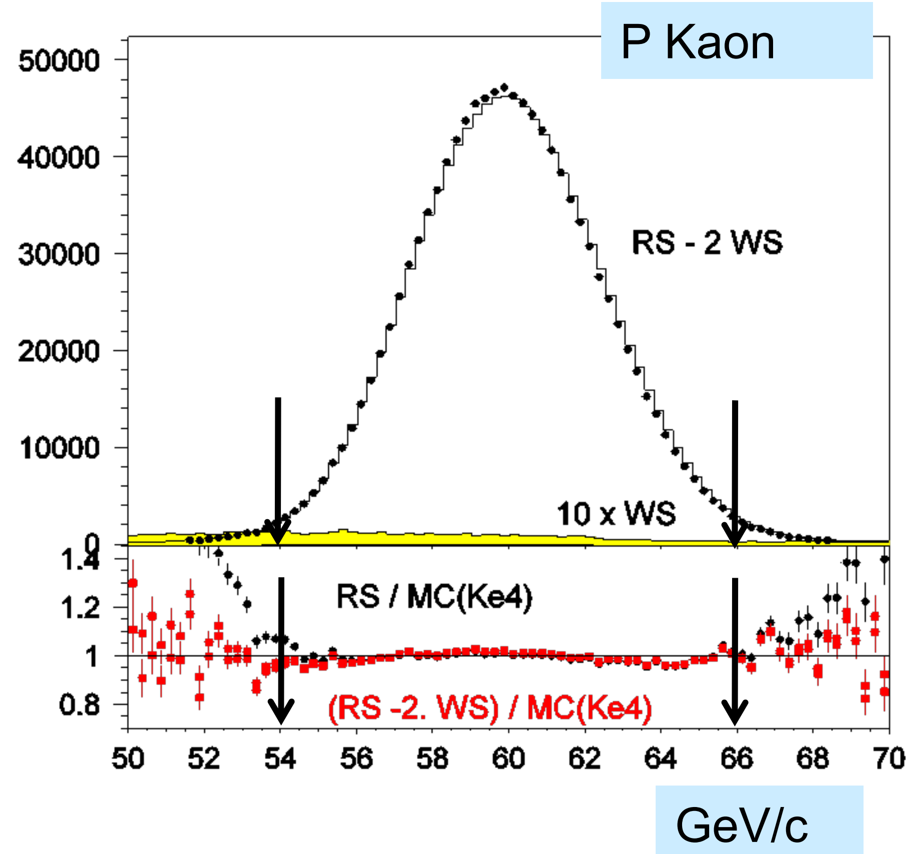
Control sample from data
(assuming $\Delta S=\Delta Q$ valid)
$K^{\pm} \rightarrow \pi^{\pm} \pi^{\pm} e^{\mp} \nu$ "Wrong Sign" events

- total charge (±1) as "Right Sign" events
- electron charge opposite to total charge
- same sign pions

Ratio (RS/WS) events:

2/1 if coming from $K3\pi$ (dominant)

1/1 if coming from $K2\pi$ (π^0)



Total background level can be kept at ~ 2 x 0.3 % relative level estimated from WS events rate and checked from MC simulation

Ke4 decays : fitting procedure

Total (2003+2004) 1.13 Million Ke4 decays

Using iso-populated boxes in the 5-dimension space of the Ca.Ma. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ) one defines a grid of

10x5x5x5x12=15000 variable size boxes.

In each $M_{\pi\pi}$ "slice" (1500 boxes), a set of 4 fit parameters is found which minimizes the difference between the data and predicted populations

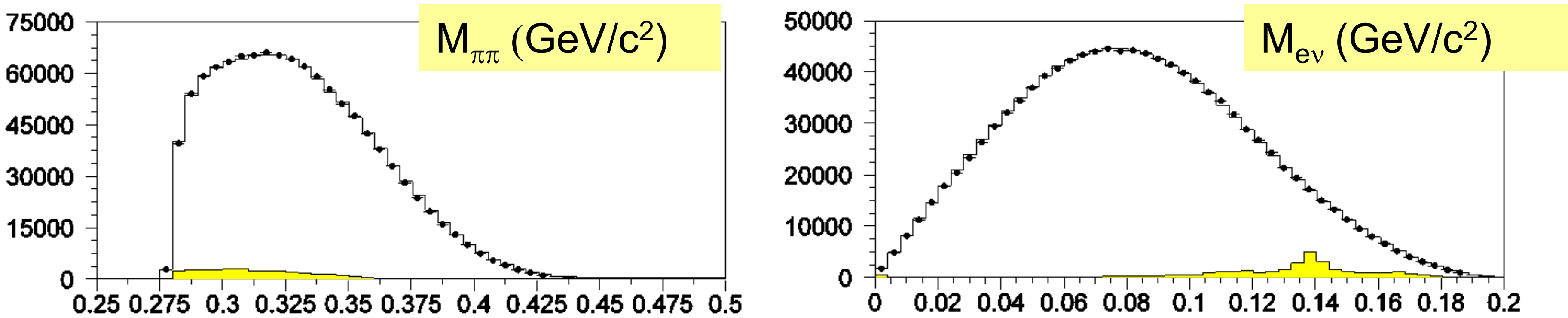
The normalisation F_s^2 is obtained in each bin/slice by the ratio $x_{slice} = \Sigma_{j \text{ in slice}} Nj/\Sigma_{j \text{ in slice}} MCj$

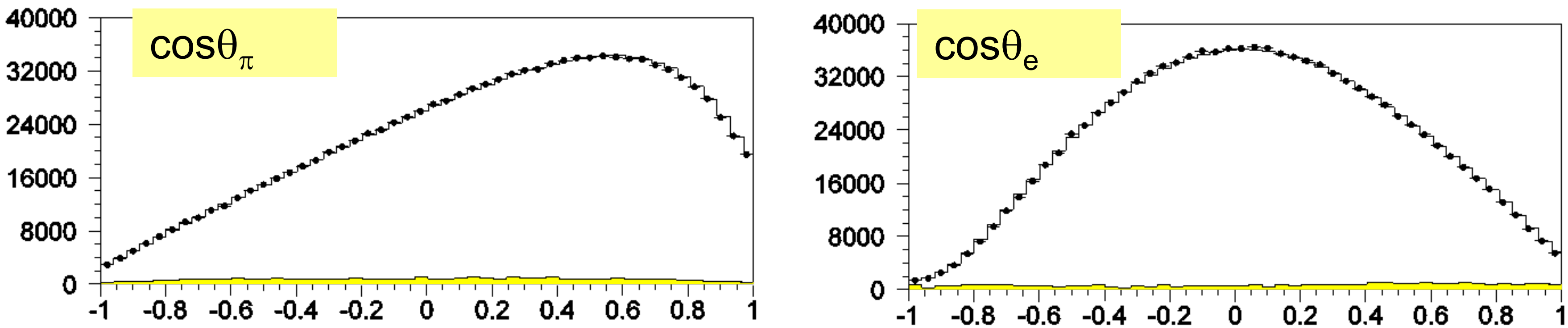
Data sample	MC sample
K^+ sample (726 400 events) 48 events/box	K^+ MC (17.4 Million events) 1160 events/box
K^- sample (404 400 events) 27 events/box	K^- MC (9.7 Million events) 650 events/box

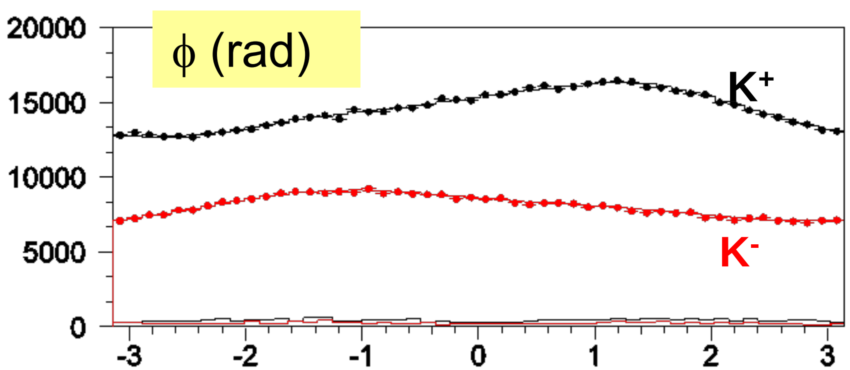
K^+ and K^- samples fitted separately in 10 independent $M_{\pi\pi}$ bins/slices, then combined in each slice according to their statistical error.

No assumption is made on the shape of the variation of the phase δ (and FF) from one $M_{\pi\pi}$ slice to the next (i.e. "model independent" analysis)

Ke4 decays : Data/MC comparison after fit







= Data

histogram = Simulation after fit

Yellow hist = WS Background (x 10 to be visible)

CP symmetry :

(K^+) ϕ distribution is opposite of (K^-) ϕ distribution

Ke4 Form Factors : fit results

Series expansion with q^2 ($q^2 = S_\pi/4m_\pi^2 - 1$) and $S_e/4m_\pi^2$ used to describe the FF variations, in the isospin symmetry limit (Amoros Bijnens 1999)

$$F_s^2 = f_s^2(1 + f_s'/f_s\, q^2 + f_s''/f_s\, q^4 + f_e'/f_s\, S_e/4m_\pi^2)^2$$

Correlation	f_s''/f_s	f_e'/f_s
f_s'/f_s	-0.95	0.08
f_s''/f_s		0.02

$$G_p/f_s = g_p/f_s + g_p'/f_s\, q^2$$

Correlation -0.91

Preliminary (2003+2004)

	value ± stat ± syst
f_s'/f_s	0.152 ± 0.007 ± 0.005
f_s''/f_s	-0.073 ± 0.007 ± 0.006
☆ f_e'/f_s	0.068 ± 0.006 ± 0.007
☆ f_p/f_s constant	-0.048 ± 0.003 ± 0.004
g_p/f_s	0.868 ± 0.010 ± 0.010
g_p'/f_s	0.089 ± 0.017 ± 0.013
h_p/f_s constant	-0.398 ±0.015 ± 0.008

systematics

- mostly from background + acceptance control
- ~ same size as statistical error or smaller

☆ first evidence by NA48

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)

$\pi\pi$ phases at threshold can be predicted from data above 0.8 GeV using **Roy equations** (unitarity, analyticity and crossing symmetries) and 2 subtraction constants **a_0 and a_2**

Numerical solutions have been developed (ACGL, DFGS) valid in the **Isospin symmetry limit (Universal Band in the [a_2,a_0] plane)**, but broken in the experimental world.

factorization of electromagnetic and mass effects :

Gamow factor x PHOTOS generator × Isospin corrections

Radiative effects (except mass effects) included in the simulation,

Gamow factor : "classical" Coulomb attraction between the 2 charged pions

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

(-> effect on event selection + possible bias on reconstructed quantities)

Mass effects:

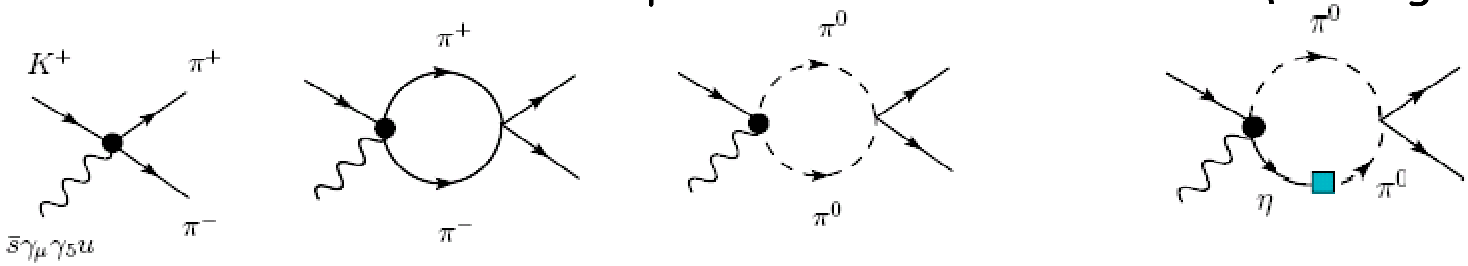
- recently computed as a correction to the measurements
- even larger than current experimental precision !

(CGR EPJ C59 (2009) 777,
DK preliminary June 2008 in progress)

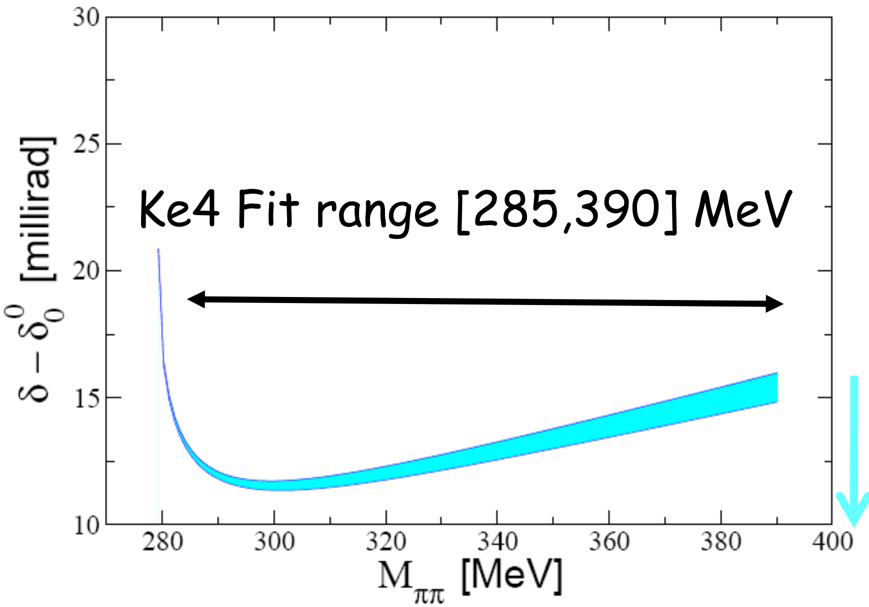
Ke4 charged decays : isospin corrections to δ

CGR EPJ C59 (2009) 777 formulation developed in close contact with NA48

tree | one loop | π^0–η mixing



δ_0^0 →
$$\delta = \frac{1}{32\pi F^2}\left\{(4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2)\left(1+\frac{3}{2R}\right)\sigma_0\right\}$$



$$\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2,$$

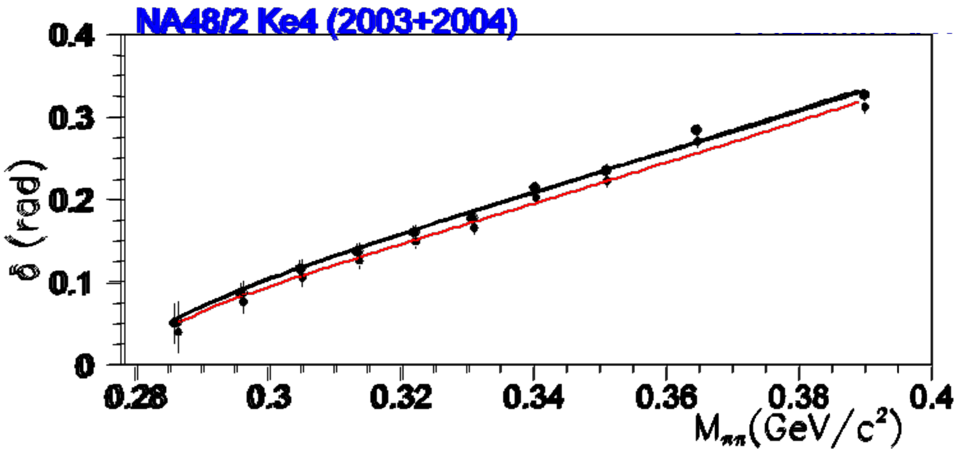
$$\sigma = \sqrt{1 - \frac{4M_\pi^2}{s}},$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

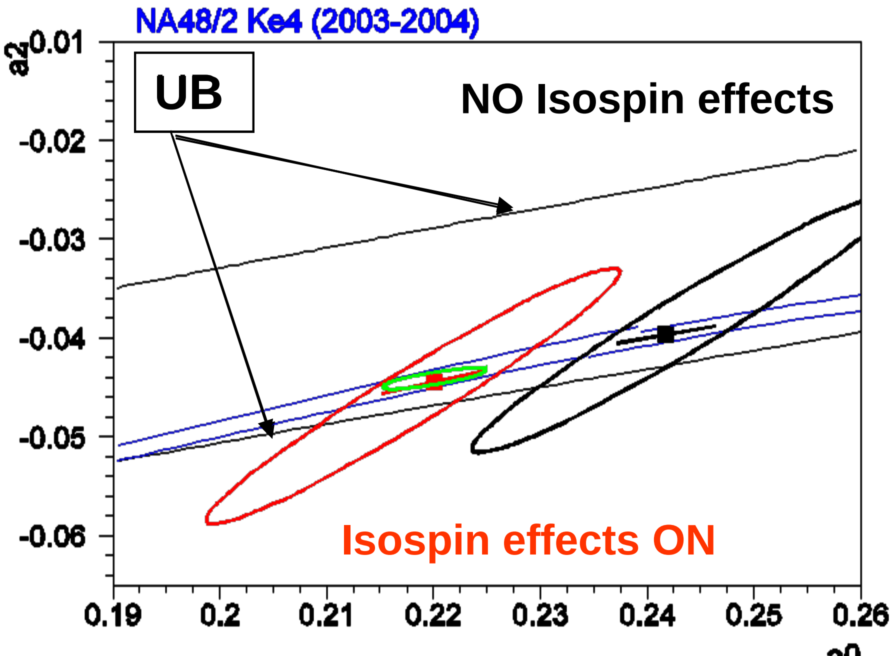
Correction is ~ 10-15 mrad

Exp. stat precision (δ) is ~ 7-8 mrad

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)



a tiny effect from theory…. a big change in now precise experimental measurement !



This induces a large **change** on (a_0,a_2) values

from a 2p fit	from a 1p fit
Δa_0 = -0.025, Δa_2 = -0.007	Δa_0 = -0.022
error stat syst	stat syst
$\sigma(a_0)$: ± 0.0128 ± 0.0050	± 0.005 ± 0.002
$\sigma(a_2)$: ± 0.0084 ± 0.0034	

Ellipses are 68% CL contours in 2p fits

Ke4 decays: comparison with theoretical predictions

Preliminary (2003+2004)

THEORY prediction

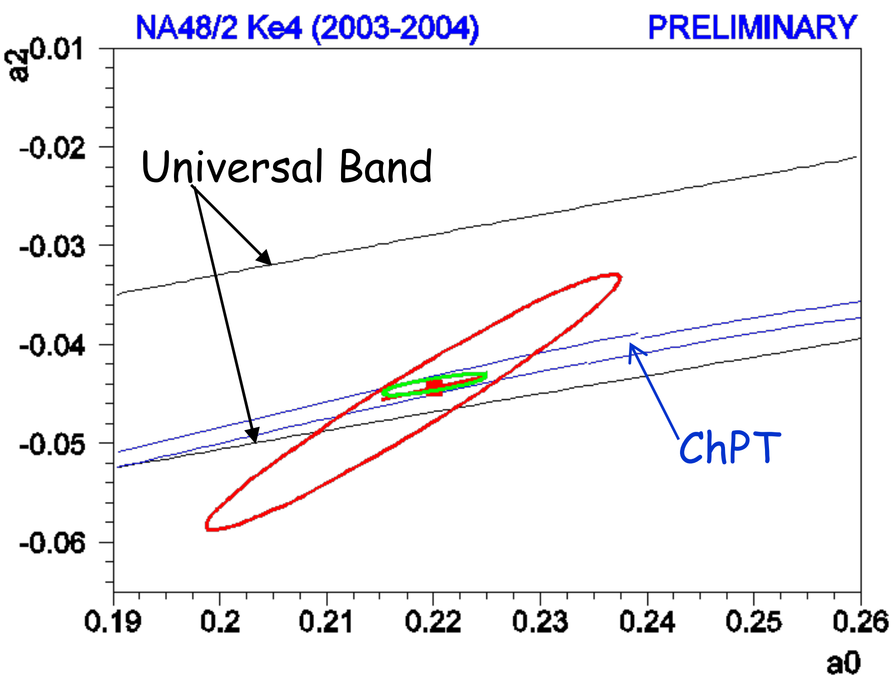
Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001),PRL86(2001)):

a_0 = 0.220 ± 0.005

a_2 = -0.0444 ± 0.0008

Experimental measurement

a_0 ChPT 1p fit	0.2206 ± 0.0049 stat ± 0.0018 syst ± 0.0064 theo *
a_0 free a_2 free 2p fit	0.2220 ± 0.0128 stat ± 0.0050 syst ± 0.0037 theo* -0.0432 ± 0.0086 stat ± 0.0034 syst ± 0.0028 theo* Correlation 96.7%



*Theory error evaluated from control of the isospin corrections & inputs to Roy equation numerical solutions (CGR EPJ C59 (2009)777)

Comparison of Ke4 phase shift experimental measurements

Apply Isospin corrections (10-15 mrad) to all published points :

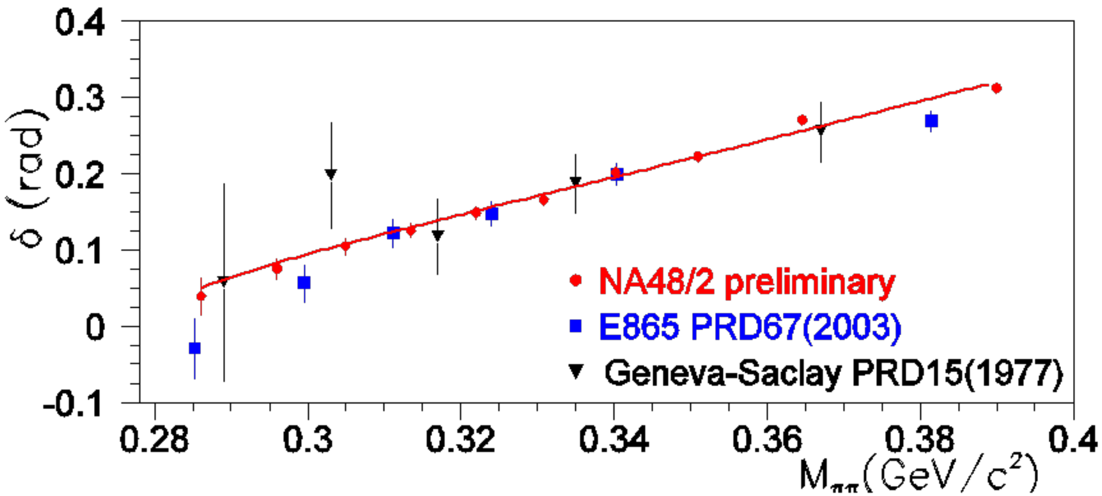
S118 (Geneva-Saclay): typical error 40-50 mrad

E865: typical error 15-20 mrad

Correction small wrt experimental error but coherent shift downwards for all data points

NA48/2 typical error 7-8 mrad

improved precision due to both
- larger statistics ~3 x E865
- larger acceptance at high $\pi\pi$ mass



- All Phase points corrected for isospin mass effects
- Independent experiments
- Errors = stat + syst

Line from a 2p fit to NA48 data alone

Fit to all data points (21 points) : dominated by NA48/2 measurements

2p fit:

$a_0 = 0.2199 \pm 0.0125_{exp} \pm 0.0037_{theo}$

$a_2 = -0.0430 \pm 0.0083_{exp} \pm 0.0028_{theo}$

1p fit :

$a_0 = 0.2168 \pm 0.0048_{exp} \pm 0.0064_{theo}$

(theory error common to all expts)

Cusp and Ke4 scattering lengths results

•Two statistically independent measurements by NA48/2

60 Millions K3π decays ⇔ 1.13 Million Ke4 decays

• systematic uncertainties from different origins : mostly independent

calorimeter + trigger ⇔ background + electron identification

• different theoretical inputs :

(1- + 2-loop) re-scattering models ⇔ Roy equations + isospin mass effects

• Large overlap in the (a_0,a_2) plane or (a_0-a_2,a_2) plane with different correlations suggests to combine the two results in a single one with improved uncertainties

• similar combination can be done for the results using the same ChPT constraint

Combining cusp and Ke4 results : method

Ke4:		Δstat	Δsyst	Δtheo		Correlation
a_0 =	0.2220	± 0.0128	± 0.0050	± 0.0037	x1	ρ_{12} = 0.967 (stat only)
a_2 =	-0.0432	± 0.0086	± 0.0034	± 0.0028	x2	(0.969 with all errors)

Cusp:		Δstat	Δsyst+ext	Δtheo		
a_0-a_2 =	0.2571	± 0.0048	± 0.0029	± 0.0088	x3	ρ_{34} = -0.839 (stat only)
a_2 =	-0.0241	± 0.0129	± 0.0096	± 0.0149	x4	ρ_{34} = -0.879 (all errors)

- Four measurements : (x1, x2,x3,x4)
- Two fitted parameters : (a_0, a_2) or (a_0-a_2, a_2)
- Error matrix is block-diagonal if no correlated error cusp-Ke4

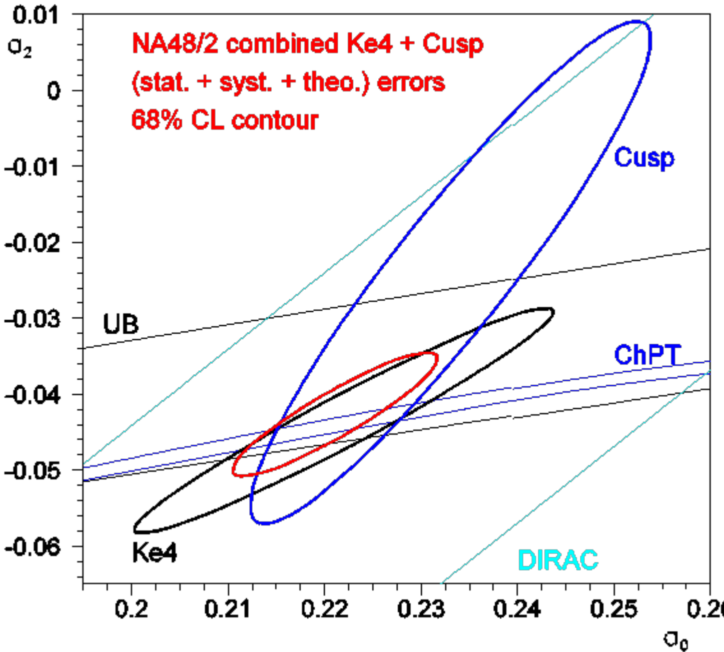
$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 & 0 \\ 0 & 0 & \sigma_{33} & \sigma_{34} \\ 0 & 0 & \sigma_{43} & \sigma_{44} \end{pmatrix}$$

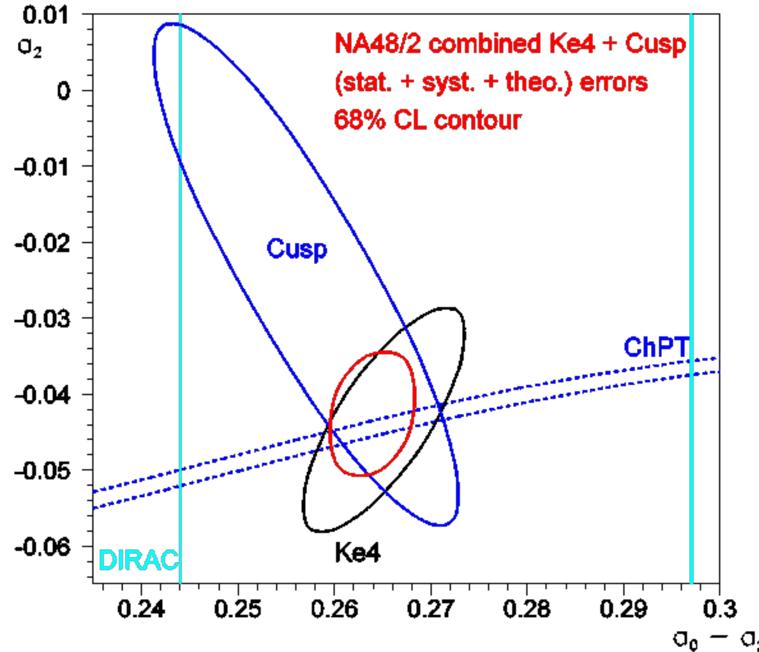
where $\sigma_{ii} = \Delta x_i^2$

and $\sigma_{ij} = \sigma_{ji} = \rho_{ij} \Delta x_i \Delta x_j$

Minimize $\chi^2 = \sum_{i,j=1}^{4} (x_i - y_i(a))^T (V_{ij})^{-1} (x_j - y_j(a))$ where V_{ij} is the covariance matrix

Combined results from cusp and Ke4





stat syst theo

a_0 = 0.2210 ± 0.0047 ± 0.0015 ± 0.0049
a_2 = - 0.0429 ± 0.0044 ± 0.0016 ± 0.0030
***Correlation* 0.912**
Total errors Δa_0: ± 0.0070 (3% rel. precision)
Δa_2: ± 0.0055 (13% rel. precision)

stat syst theo

$a_0 - a_2$ = 0.2639 ± 0.0020 ± 0.0004 ± 0.0021
a_2 = - 0.0429 ± 0.0044 ± 0.0016 ± 0.0030
***Correlation* 0.277**
Total errors $\Delta (a_0 - a_2)$: ± 0.0030 (1% rel. precision)
Δa_2: ± 0.0055 (13% rel. precision)

Including the ChPT constraint: stat syst theo

a_2 = -0.0444 ± 0.007 ± 0.005 ± 0.0012
a_0 = 0.2196 ± 0.0027 ± 0.0021 ± 0.0048 or $a_0 - a_2$ = 0.2640 ± 0.0020 ± 0.0017 ± 0.0035
Total error Δa_0 : ± 0.0059 Δa_2 : ± 0.0015 $\Delta (a_0 - a_2)$: ± 0.0044

Comparison with other experimental measurements

Preliminary (2003+2004)

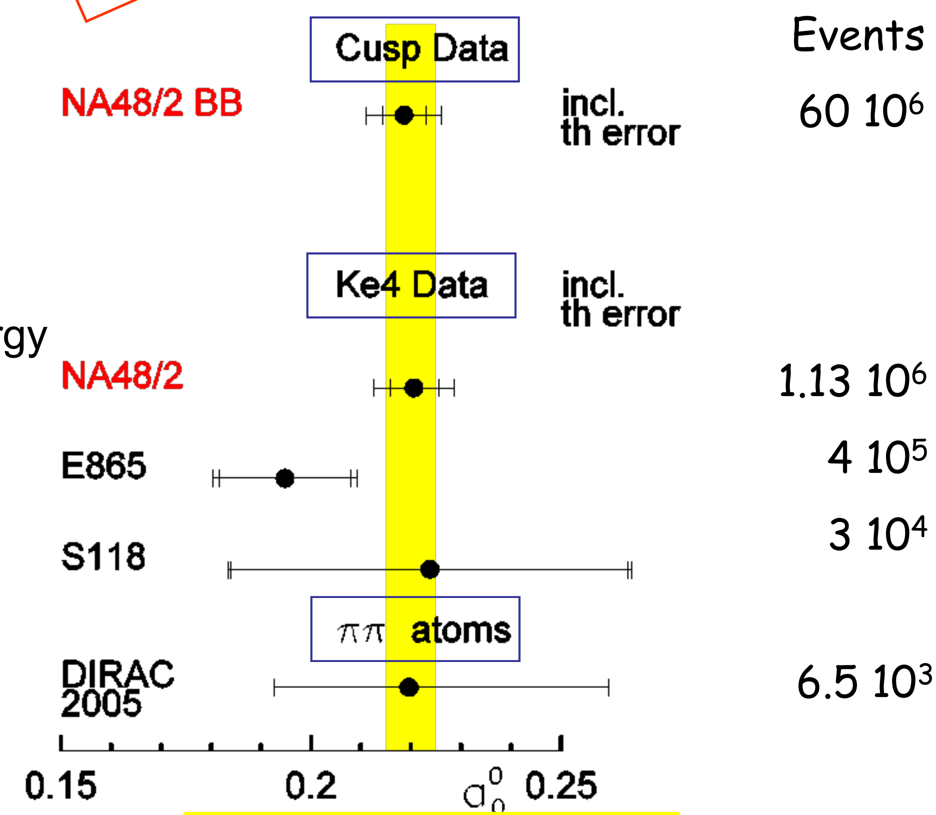
Cusp : (a_0-a_2) ChPT fit with 2 models, BB is the most complete in terms of radiative corrections

Ke4 : apply isospin corrections to published phase points of all experiments and perform a_0 ChPT fit

Note : E865 result dominated by highest energy data point, otherwise compatible

ππ atoms DIRAC: |a_0-a_2| errors from PLB619 (2005), use ChPT constraint (only 40% Data analyzed)

a0 ChPT band prediction



Yellow band: best ChPT prediction

a_0 (ChPT) = 0.220 ± 0.005

NA48/2 experimental precision and most precise theory prediction now at the same level!

Summary

NA48/2 has analyzed ~1.13 M Ke4 events recorded in (2003+2004)

- Ke4 Form Factors measured with an improved precision
- Scattering lengths, extracted in Ke4 and K3π Cusp analyses, give a consistent experimental picture and an impressive agreement with predictions from ChPT, both for free (a_0, a_2) and constrained fits :

ChPT best prediction $a_0 = 0.220 \pm 0.005$, $a_2 = -0.0444 \pm 0.0008$, $a_0 - a_2 = 0.264 \pm 0.004$

(ChPT cusp) $a_0 - a_2 = 0.2633 \pm 0.0024_{stat} \pm 0.0014_{syst} \pm 0.0019_{ext} \pm 0.0053_{th}$

(ChPT Ke4) $a_0 = 0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{th}$

(ChPT combined) $a_0 = 0.2196 \pm 0.0027_{stat} \pm 0.0021_{syst} \pm 0.0048_{th}$

$a_2 = -0.0444 \pm 0.0007_{stat} \pm 0.0005_{syst} \pm 0.0012_{th}$

$a_0 - a_2 = 0.2640 \pm 0.0020_{stat} \pm 0.0017_{syst} \pm 0.0035_{th}$

- The achieved experimental precision (stat + syst) of combined result for a_0 and $a_0 - a_2$ is now smaller than the theoretical precision .

The collaboration with many theory groups was/still is invaluable in understanding how to extract scattering lengths from NA48 Data. Thanks to all groups in Bern, Bonn, Dubna, Orsay, Madrid, Marseille, Rome .. !

Two final publications (Cusp and Ke4) in progress

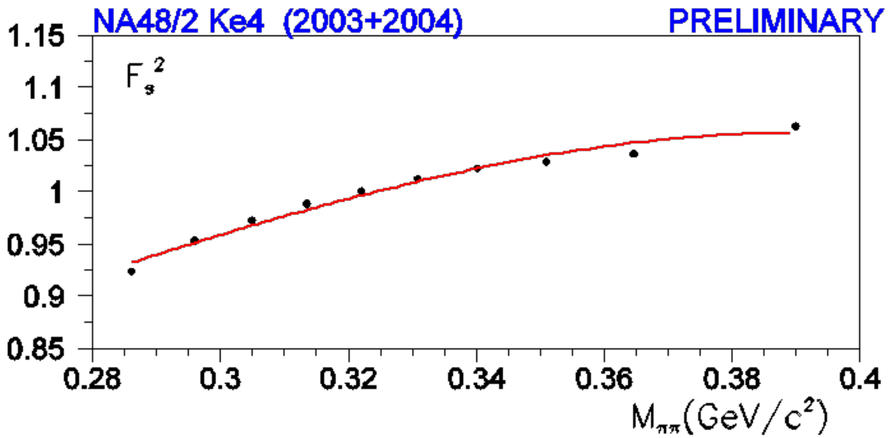
spares

Ke4 Form Factors : getting F_s

- variations of the normalization with q^2, q^4 and $S_e/4m_\pi^2$ are investigated by a 4-parameter fit in the plane $[M_{\pi\pi}, M_{e\nu}]$
- Introducing a slope with S_e improves the fit χ^2 (211./83 → 94./82)

$$F_s^2 = f_s^2(1 + f_s'/f_s\, q^2 + f_s''/f_s\, q^4 + f_e'/f_s\, S_e/4m_\pi^2)^2$$

$$q^2 = (S_\pi/4m_\pi^2 - 1)$$

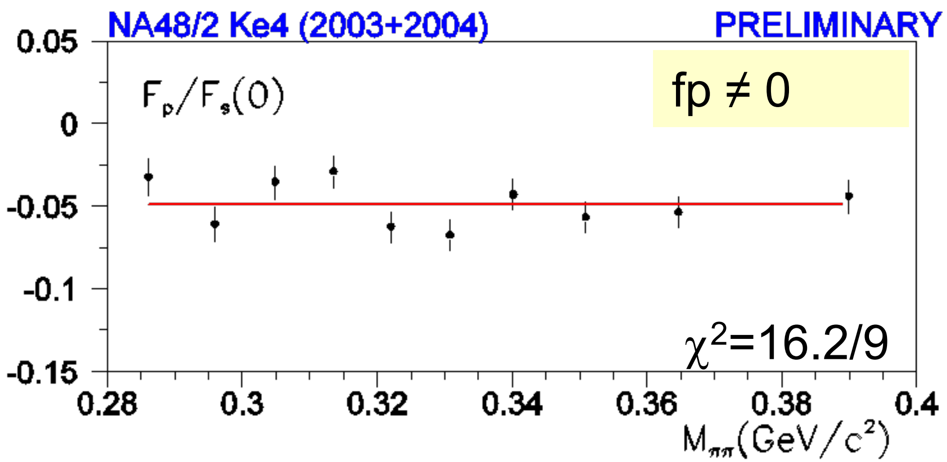


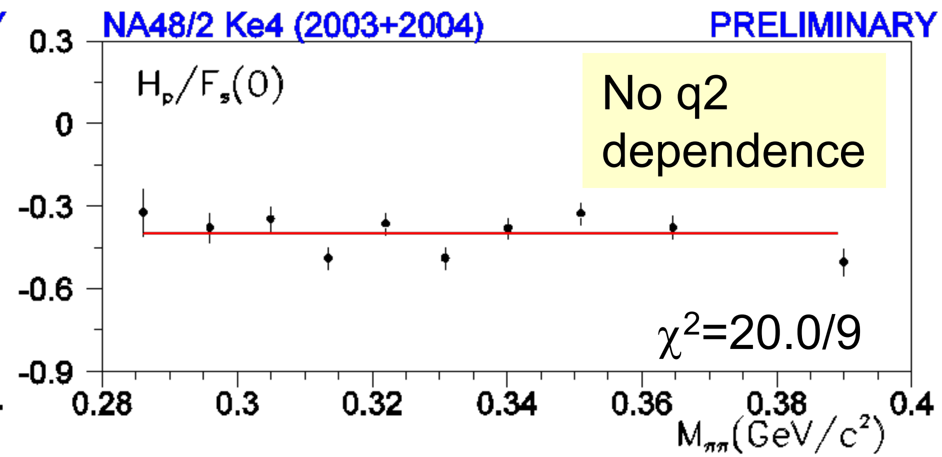
Correlations:

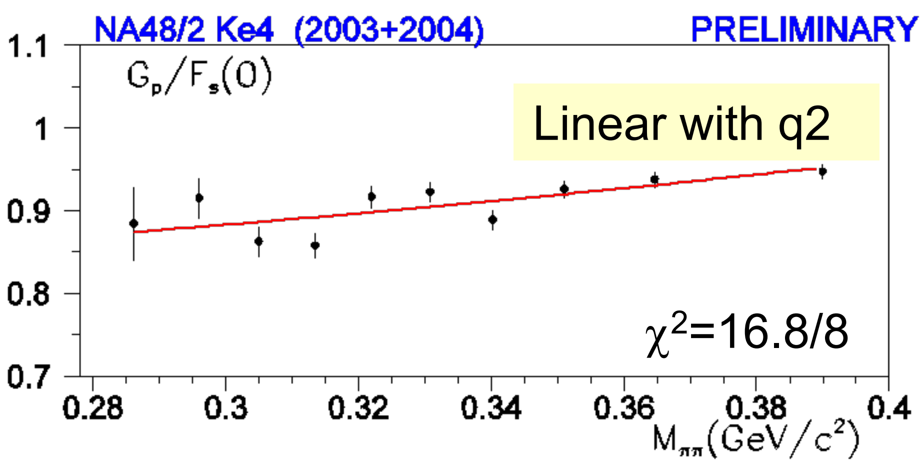
	f''_s/f_s	f'_e/fs
f'_s/fs	-0.95	0.08
f''_s/fs		0.02

Other parameterizations could be easily tried

Ke4 Form Factors: getting Fp, Gp, Hp







$G_p/f_s = g_p/f_s + g'_p/f_s\ q^2$

Correlation

	g'_p/f_s
g_p/f_s	-0.91

Common errors to Cusp and Ke4 results ?

- Statistical errors from cusp and Ke4 measurements are independent
- Systematic errors : if (partly) correlated, contribute to the off-diagonal terms of the error matrix but very few common items
- Theoretical uncertainties come from different sources

Cusp units 10^{-4}	a_0-a_2	a_2
Acceptance (z)	4	20
Acceptance (v)	1	8
PK spectrum	13	32
Trigger eff.	10	39
LKr resol	9	29
LKr nonlinear.	12	67
LKr shower	10	18
MC (time)	5	1
k0 control	2	6
Total	**25**	**94**

Ke4 units 10^{-4}	a_0	a_2
Acceptance (z) +acceptance (r) + PK spectrum (~ 2/3)	20	13
Trigger eff. (but not the same)	6	4
Fit method	9	5
Bkg shape	31	18
e-ident	14	19
Radiative corrections	16	10
Bkg level	19	10
Se control	13	7
Total	**50**	**34**

Ke4 analysis : theoretical uncertainties

Work done in collaboration with Colangelo + Gasser

Ingredients to extract (a_0,a_2) from phase δ

- Roy equations : two formulations ACGL(Bern)/ DFGS(Orsay)

Orsay can vary the solutions with phases @ matching point (0.8 GeV)

Reference : $\delta_0^0 = 82.3^\circ$ and $\delta_1^1 = 108.9^\circ$

Vary : δ_0 by ± 3.4 °

Vary : δ_1 by ± 2.0 °

- Isospin corrections

Vary R = 37 ± 5

Vary $F\pi$ = (86.2 ± 0.5) MeV

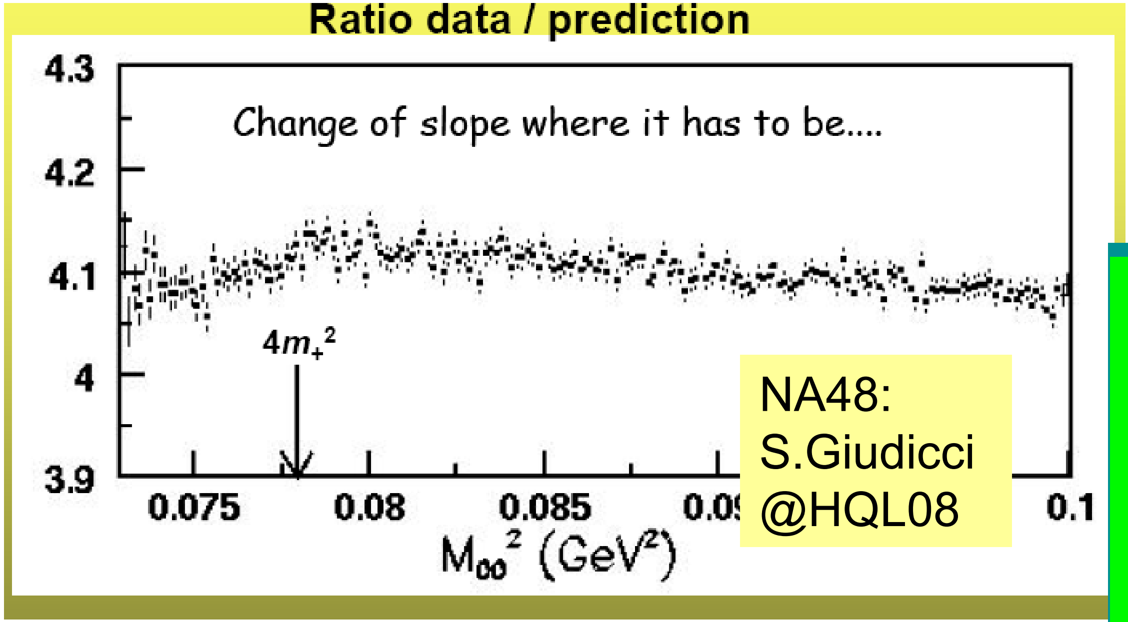
Neglected Higher orders

- CHPT constraint

Vary by a_2 ± 0.0008

	2p-fit		1p-fit
	a_0	a_2	a_0
Roy equation solutions			
|BERN-ORSAY|	0.0000	0.0006	0.0013
$\delta_0^0 \pm 3.4^\circ$	0.0010	0.0027	0.0043
$\delta_1^1 \pm 2.0^\circ$	0.0000	0.0002	0.0003
Isospin corrections			
$R \pm$ 5.	0.0005	0.0000	0.0008
$F_\pi \pm 0.5$MeV	0.0003	0.0001	0.0003
Higher Orders	0.0035	0.0005	0.0042
ChPT constraint ±0.0008	–	–	0.0017
quadratic sum	0.0037	0.0028	0.0064

The cusp in $K_L \to 3\pi^0$ decays: work in progress



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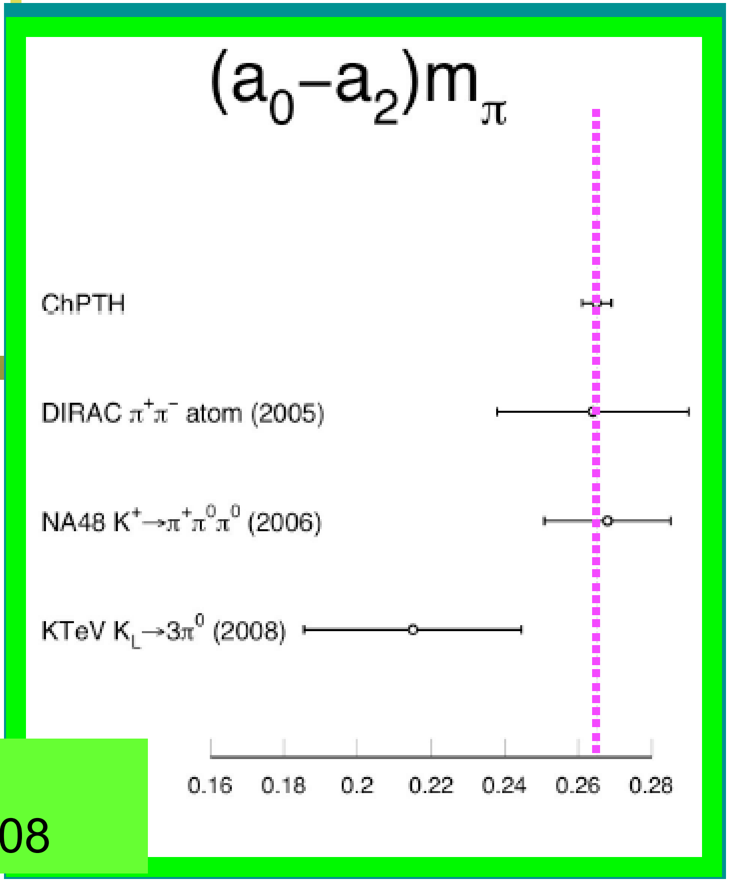

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