$\pi\pi$ SCATTERING LENGTHS FROM MEASUREMENTS OF K_{e4} AND $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ DECAYS AT NA48/2

Dmitry Madigozhin [a] (for NA48/2 Collaboration)
Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

Abstract. The NA48/2 experiment at the CERN SPS has collected a large sample of charged kaon decays. From the 2003 and most of 2004 statistics, the $\pi\pi$ scattering lengths a_0^0 and a_0^2 have been extracted through the interpretation of an anomaly (cusp) in the $\pi^0\pi^0$ effective mass distribution of the decay $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$. Using 2003 data, the form factors of K_{e4} decays have been obtained, allowing the independent measurement of the same $\pi\pi$ scattering parameters.

1 Introduction

In description of $\pi\pi$ scattering at low energies, the S-wave scattering lengths (multiplied by $m_{\pi^{\pm}}$) for states with isospin $I = 0$ (a_0^0) and isospin $I = 2$ (a_0^2) are the main parameters of amplitude calculation. In Chiral Perturbation Theory (ChPT) the a_0^0 is related to the size of chiral condensate and is predicted to be $a_0^0 = 0.220 \pm 0.005$ and $a_0^2 = -0.0444 \pm 0.0010$ [1].

The earlier K_{e4} decay studies [2, 3] provided a measurement of a_0^0 with up to 10% precision. The DIRAC collaboration has presented the $(a_0^0 - a_0^2)$ value extracted from the pionium lifetime measurement [4].

The main goal of NA48/2 experiment was the search for the CP-violating asymmetries in decays $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ and $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ [5]. But the large data samples and high quality of measurements allow many other studies. Here we present the preliminary results on the $\pi\pi$ scattering lengths measurement from NA48/2 data on $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ and $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu(K_{e4})$ decays.

2 Beam line and Detector

The NA48/2 setup consisted of the upgraded version of NA48 detector [6] with the new beam line that provided a simultaneous and collinear K^+ and K^- beams. Both beams where generated in collision of primary 400 GeV proton beam with the single Berillium target. Secondary charged particles where selected by the beam line to form the narrow momentum spectra centered near 60 GeV with $RMS \approx 3$ GeV. The negatively and positively charged beams than where superimposed and passed through vacuum decay volume and beam pipe in the center of detectors.

The most important components of the detector for the discussed measurements where a magnetic spectrometer, Liquid Krypton electromagnetic calorimeter and the muon veto counter. The magnetic spectrometer consisted of four drift chambers and a magnet, that provided a transverse mo-

[a] e-mail: madigo@sunse.jinr.ru

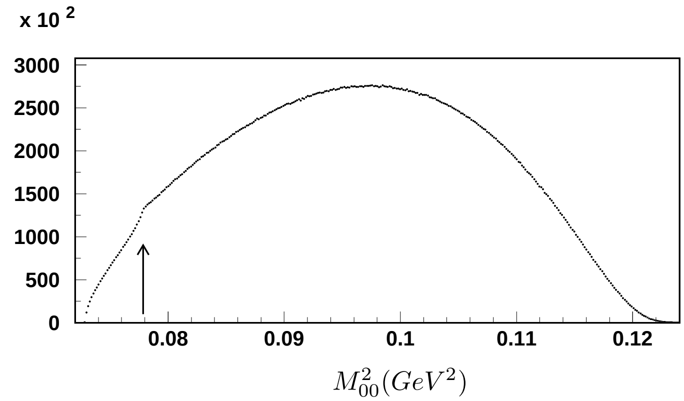

Figure 1: Distribution of the $\pi^0\pi^0$ invariant mass squared in $K^\pm \to \pi^\pm\pi^0\pi^0$ decays for NA48/2 experimental data collected in 2003 and 2004

mentum kick of 120 MeV/c. The momentum resolution of the spectrometer could be expressed as $\sigma(P)/P = (1.0 \oplus 0.044P[Gev/c])\%$. Its resolution for transversal position of hits was about $150\mu m$. The electromagnetic calorimeter depth was $27X_0$ of liquid Krypton. Its energy resolution is given by $\sigma(E)/E = 3.2\%/\sqrt{E} \oplus 9\%/E \oplus 0.42\%$, where E is a photon energy in GeV.

Moun veto counter consisted of three planes of plastic scintillators with 80 cm iron walls upstream each of the planes.

3 Cusp Effect

During the analysis of 2003 data, a sharp change of slope in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays has been observed in the point $M_{00} = 2m_+$, where m_+ is the $\pi^\pm$ mass [7]. A first interpretation of this effect in terms of $\pi\pi$ scattering in final state has been made by N.Cabibbo [8], and than a second-order calculation [9] have provided us with a formula for the experimental data fit. The decay matrix element M was assumed to be

$$M \propto 1 + \frac{g_0}{2}u + \frac{h'}{2}u^2 + M_1(a_0^0, a_0^2, u), \quad (1)$$

where $u = (s_3 - s_0)/m_+^2$. M_1 term represents the $K^\pm \to \pi^\pm\pi^+\pi^-$ decay with the $\pi^+\pi^- \to \pi^0\pi^0$ rescattering as well as the relevant next-order diagrams. The isospin symmetry breaking correction was applied in the formulae connecting a_0^0 and a_0^2 to the five $\pi\pi \to \pi\pi$ rescattering amplitudes with different charges of pions (see, for example, [10]).

The study of cusp effect now is based on 2003 and majority of 2004 data (Fig. 1). Improved selection is implemented to diminish the sensitivity of the result to the quality of Monte Carlo simulation, while the simulation was tuned to

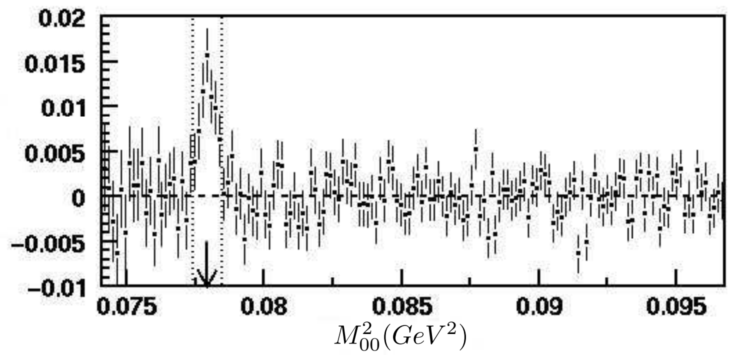

Figure 2: Deviation of the experimental data (collected by NA48/2 in 2003 and 2004) from the fit by 1 result ($(data - fit)/fit$) for the M_{00}^2 distribution.

the experimentally measured variations of beam geometry, detector efficiency and resolutions. In total about 59.6×10^6 $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ events where selected using the electromagnetic calorimeter data to reconstruct the photons from π^0 decays and the magnetic spectrometer for charged pion tracks measurement.

A matrix element (1) was used to fit the experimental one-dimensional u distribution taking into account the acceptance and resolution effects calculated by means of Monte Carlo simulation. The deviation of the data spectrum from the fit result is shown on the Fig. 2. Seven bins (0.00015 GeV each, see Fig. 2) around the cusp point were excluded from the fit in order to reduce the sensitivity to electromagnetic corrections. The excess of events in the region around the cusp can be interpreted as a poinium signature, yielding a rate of pionium formation about two times higher than the theoretical prediction for pionium production [11]. But the recent calculation of electromagnetic effects for the $\pi^+\pi^-$ scattering [12] has explained about a half of excess as an additional contribution of electromagnetic interaction that doesn't lead to formation of a bound state.

When we have added to (1) a term $\frac{k'}{2}\frac{s_1-s_2}{m_+^2}$ to describe a possible weak dependence of matrix element on another Lorentz invariant, we got by separate two-dimensional fit the result $k' = -0.0097 \pm 0.0003_{stat.} \pm 0.0008_{syst}$. Its non-zero value is taken into account in frameworks of one-dimensional fit and lead to some small change of another slopes, but the scattering lengths is not sensitive to k' of this size. The results of the fit for the matrix element slope parameters are the following: $g_0 = 0.649 \pm 0.003_{stat.} \pm 0.004_{syst}$, $h' = -0.048 \pm 0.007_{stat.} \pm 0.005_{syst}$. The scattering parameters measured values are: $a_0^0 - a_0^2 = 0.261 \pm 0.006_{stat.} \pm 0.003_{syst} \pm 0.001_{external}$, $a_0^2 = -0.037 \pm 0.013_{stat.} \pm 0.009_{syst} \pm 0.002_{external}$. External errors are related to the uncertainties of PDG information used for the fit, but it doesn't include the additional 5%

uncertainty of the theoretical formula (1) itself (currently, this is a dominant source of uncertainty).

4 K_{e4} decay

The kinematics of $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ decay is described by the five Cabibbo-Maksymowicz variables [13]: invariant mass squared of a dipion $S_{\pi} = M^2_{\pi\pi}$, invariant mass squared of dilepton $S_{\pi} = M^2_{e\nu}$ and angles $\theta_{\pi}, \theta_{e}, \phi$. θ_{π} is the angle between π^{-} and dilepton momenta in the rest frame of dipion, θ_{e} is the angle between ν and dipion momenta in the rest frame of dilepton. ϕ is the angle between the plane of dilepton and $e^{\pm}$ momenta and the plane of dipion and π^{+} momenta in the kaon rest frame.

The matrix element is defined by means of axial form factors F,G and a vector form factor H. A partial wave expansion of the form factors may be restricted to s and p waves: $F = F_s e^{i\delta_s} + F_p e^{i\delta_p} cos\theta_{\pi}$, $G = G_p e^{i\delta_p}$, $H = H_p e^{i\delta_p}$ (only phase shift $\delta = \delta_s - \delta_p$ is observable).

From the 2003 data, about 670000 K_{e4} decays have been selected. Reconstructed events are distributed in $10 \times 5 \times 5 \times 5 \times 12$ iso-populated bins in the $(M_{\pi\pi}, M_{e\nu}, cos(\theta_{\pi}), cos(\theta_{e}), \phi)$ space. Ten independent fits (one per $M_{\pi\pi}$ bin) of five parameters (F_p, G_p, H_p, δ, and a normalization constant, that absorbs F_s) where performed in four dimensional space using the acceptance and resolution information from Monte Carlo simulation. The value of the phase difference δ was extracted from the measured asymmetry of ϕ distribution as a function of $M_{\pi\pi}$. The result of δ measurement is shown on the Fig. 3 together with results from previous experiments [2, 3].

The phase shift measurements can be related to the $\pi\pi$ scattering lengths using the analytical properties and crossing symmetry of amplitudes (Roy equations [15]). One can use the Universal Band approach [16, 17] to extract a_0^0 alone. At the center line of the Universal Band (1-parameter fit), NA48/2 phase measurements translate as $a_0^0 = 0.256 \pm 0.006_{stat} \pm 0.002_{syst} \pm 0.018_{theor}$, which implies $a_0^2 = -0.0312 \pm 0.0011_{stat} \pm 0.0004_{syst} \pm 0.013_{theor}$. In the case of the fit where both a_0^0 and a_0^2 are free parameters, the result is $a_0^0 = 0.233 \pm 0.016_{stat} \pm 0.007_{syst}, a_0^2 = -0.0471 \pm 0.011_{stat} \pm 0.004_{syst}$ (the correlation is 96.7%). Finally, resent work [18] suggests that isospin symmetry breaking effects, neglected so far in the K_{e4} phase shift analysis, would lead, when taken into account, to decrease of a_0^0 by ≈ 0.022, and a_0^2 - by about 0.004, leading to good compatibility between the K_{e4} and cusp analyses results for pion scattering lengths.

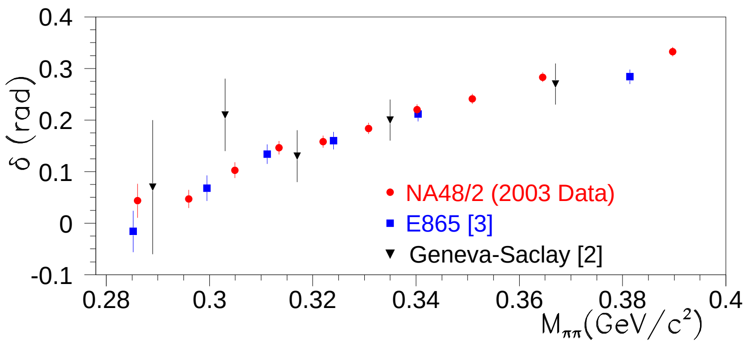

Figure 3: Phase shift δ measurements from K_{e4} experiments

5 Conclusion

Two independent results of $\pi\pi$ scattering lengths measurement, obtained by NA48/2 experiment, are compatible between each other and are in agreement with current predictions of ChPT, if the isospin symmetry breaking effects are taken into account in both analyses of experimental data.

References

[1] G.Colangelo, J.Gasser, H.Leutwyler, *Nucl.Phys.* **B** 603, 125 (2001).
[2] L.Rosselet *et al*, *Phys.Rev.* **D** 15, 574 (1977).
[3] S.Pislak *et al*, *Phys.Lett.* 87, 221801 (2001).
[4] B.Adeva *et al*, *Phys.Lett.* **B** 619, 50 (2005).
[5] J.R.Batley *et al*, *Eur.Phys.J.* **C** 52, 875 (2007).
[6] V.Fanty *et al*, *Nucl.Instrum.Methods* **A** 574, 433 (2007).
[7] J.R.Batley *et al*, *Phys.Lett.* **B** 633, 173 (2006).
[8] N.Cabibbo, *Phys.Rev.Lett.* 93, 121801 (2004).
[9] N.Cabibbo and G.Isidori, *JHEP* 503, 21 (2005).
[10] K.Knecht and R.Urech, *Nucl.Phys.* **B** 519, 329 (1998).
[11] Z.K.Silagadze, *JEPT Lett* 60, 689 (1994).
[12] S.R.Gevorkyan, A.V.Tarasov, O.O.Voskresenskaya. *Phys.Lett.* **B** 649, 159 (2007).
[13] N.Cabibbo and A. Maksymowicz, *Phys. Rev.* 168, 1926 (1968).
[14] G.Amoros and J.Bijnens, *J.Phys G* 25, 1607 (1999).
[15] S.Roy, *Phys. Lett. B* 36, 353 (1971).
[16] B.Ananthanarayan *et al*, *Phys.Rep.* 353, 207 (2001).
[17] S.Descotes, N.Fuchs, L.Girlanda, J.Stern, *Eur.Phys.J. C* 24, 469 (2002).
[18] J.Gasser, Proc. of KAON 2007 Int. Conf., Frascati, May 21-25, (2007).