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Effective Quantum Field Theories

Jürg Gasser

University of Bern

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- LECs from lattice calculations
- Precision physics
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INTRODUCTION

A comment

About 250 talks at this conference

⇒ I just offer 2 messages which are hopefully of general interest:

- Lattice calculations have come into contact with chiral perturbation theory
- While doing precision physics, we should not forget about Bloch and Nordsieck, and should keep in mind that $m_u \neq m_d$

Still: $250 \times 2 = 500$ messages ...

Effective quantum field theories

An effective quantum field theory

- contains the degrees of freedom to describe physical phenomena below a chosen energy scale
- ignores the degrees of freedom at higher energies

EFTs are the counterpart of the Theory of Everything.

Ecker 2005

Consider here 2 simple examples

Beta decay

Effective quantum field theories are as old QFT:

β -decay of neutron:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$\mathcal{L}_{\text{eff}} = \mathbf{G} \bar{p} \gamma^\mu n \bar{e} \gamma_\mu \nu + \text{h.c.}$$

Fermi 1933,1934

This is an effective, non-renormalizable QFT for the weak interactions at low energies (with due addition of further couplings).

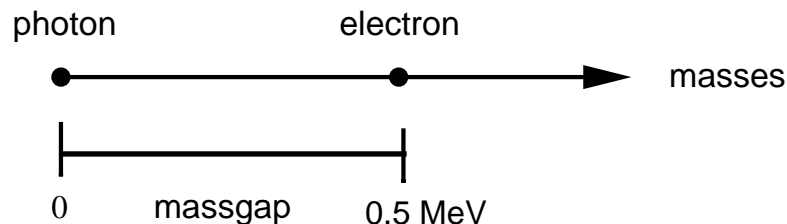
no W, Z

Underlying renormalizable theory:

Weinberg 1967; Salam 1968

Photon–photon interactions in QED

Consider interaction between photons at very low energies



- Effective Lagrangian for photon interactions: Write all terms allowed by symmetry (gauge, Lorentz, P, C, T)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e_1}{m_e^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{e_2}{m_e^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \dots$$

- Amounts to an expansion in powers of ∂_μ/m_e and $F_{\mu\nu}/m_e^2$
- Scale: electron mass
- Low energy constants (LECs) e_i fixed through QED

Euler, Heisenberg 1936

- \mathcal{L}_{eff} is a non–renormalizable QFT

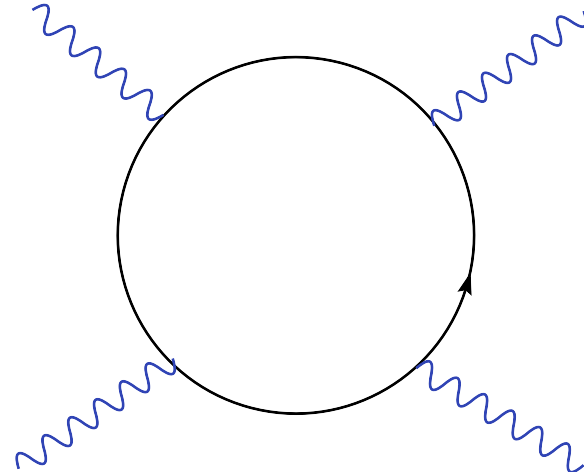
With properly chosen coefficients e_i , the above Lagrangian reproduces the matrix elements

$$n\gamma \rightarrow m\gamma$$

in full QED, to any order in α and in $\text{momenta}/m_e$ (for small momenta). This example contains all the bits and pieces relevant for effective Lagrangian techniques:

- Constructing the Lagrangian \Leftrightarrow symmetries
- Loop calculations \Leftrightarrow unitarity
- Power counting \Leftrightarrow calculations become systematic
- Matching \Leftrightarrow relation to underlying theory
- Validity of expansion

Calculating e_i (matching)



Generates $e_{1,2}$ at leading order in an expansion in α :

$$e_1 = \frac{\alpha^2}{90} + O(\alpha^3)$$

$$e_2 = \frac{7\alpha^2}{360} + O(\alpha^3), \quad \alpha = 1/137.036\dots$$

More efficient version: Schwinger 1951

CHIRAL PERTURBATION THEORY

Carry over the previous construction in QED to the effective
Lagrangian of QCD

Effective field theory of QCD

I skip the “chiral symmetry” discussion and refer you to the talk by Uwe–Jens Wiese.

Systematic determination of the structure of matrix elements, as dictated by chiral symmetry:

$$\mathcal{L}_{\text{QCD}} \xrightarrow{E \ll M_\rho} \mathcal{L}_{\text{eff}}$$

$$\boxed{\mathcal{L}_{\text{eff}}}$$

- expressed in observed hadron fields
- same symmetry as QCD

The transition

$$\mathcal{L}_{\text{QCD}} \implies \mathcal{L}_{\text{eff}}$$

is non–perturbative $\overset{!}{\iff}$ QED

The leading term

- Consider pions only:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \dots$$

$$\mathcal{L}_2 = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + M^2 (U + U^\dagger) \rangle$$

$U \in SU(2)$, contains the pion fields

- $M^2 = (m_u + m_d)B$
- Is derivative expansion
- F, B : low-energy constants (LECs): not fixed by chiral symmetry

\mathcal{L}_2 : Euler–Heisenberg Lagrangian for mesons – is a non renormalizable quantum field theory

Higher order Lagrangians

$$\mathcal{L}_4 = \sum_{i=1}^{10} l_i Q_i ; \quad \mathcal{L}_6 = \sum_{i=1}^{56} c_i P_i ; \quad \dots$$

- LECs l_i, c_i not fixed by symmetry. Local polynomials Q_i, P_i (expressed in meson fields) are known.

J.G., Leutwyler 1985; Bijnens, Colangelo, Ecker 1999

- Calculations with \mathcal{L}_{eff} generate an expansion in powers of quark masses and of external momenta.
Scale: $4\pi F \simeq 1 \text{ GeV}$.

Chiral perturbation theory (ChPT)

Same keywords as in QED, page 8.

For instructive examples, see Hans Bijnens, this morning

Advantages/disadvantages

Pros

Calculating with this Lagrangian and properly chosen LECs, one reproduces S -matrix elements of QCD, at low energy, in a systematic manner

Weinberg 1979; Leutwyler 1994

Contras

Limited energy range of validity

Many LECs

Note, however: LECs are fixed by QCD, see below

Applications

MANY

see SPIRES, hep-search

In particular: Including in the effective Lagrangian also nucleon fields, one enters the rich field of nuclear physics – see talks at this conference.

LECs FROM LATTICE

Evaluation of LECs in ChPT

Of course, LECs cannot be determined as in Euler-Heisenberg Lagrangian: perturbation theory does not apply. Proper method:



Machines

Main machine: **IBM Blue Gene/L at KEK**

- 57.6 Tflops peak (10 racks)
- 0.5TB memory/rack
- 8x8x8(16) torus network
- ~30% performance for Wilson kernel (Doi and Samukawa, IBM Japan)
- Overlap HMC: 10~15% on one rack



Also using

- **Hitachi SR11000 (KEK)**
 - 2.15TFlops/0.5TB memory
- **NEC SX8 (YITP, Kyoto)**
 - 0.77TFlops/0.77TB memory



Hideo Matsufuru, Lattice 2007, 30 July



From Matsufuru, Lattice 2007

Example: l_3

Expansion of the pion mass. Contains **chiral logarithms**:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

$$M^2 = B(m_u + m_d), \quad \bar{l}_3 \equiv \frac{\Lambda_3^2}{M_\pi^2}$$

\bar{l}_3 is UV-finite part of l_3

Crude result, based on $SU(3)$ formulae:

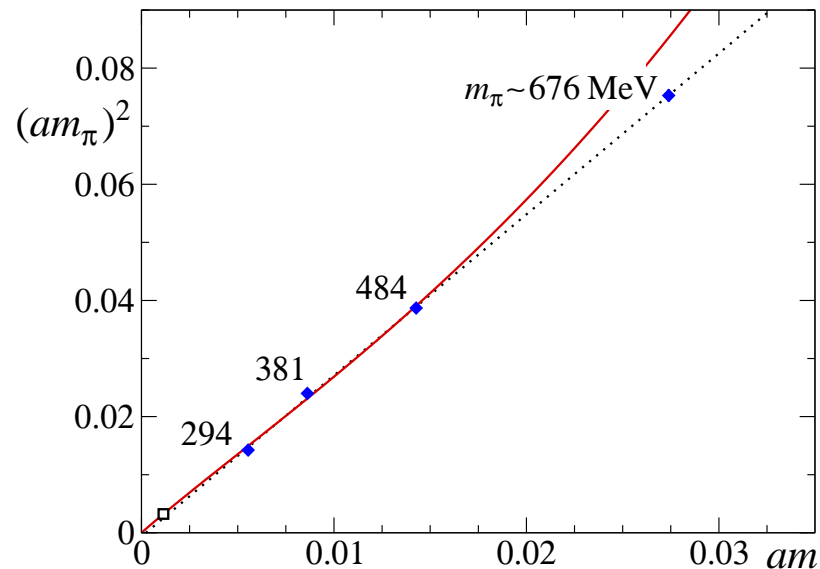
$$0.2 \text{ GeV} \leq \Lambda_3 \leq 2 \text{ GeV}$$

Leutwyler, J.G. 1984

Lattice allows more accurate determination of scale Λ_3 :

Set $m_u = m_d = m$, determine M_π^2 as a function of quark mass m .

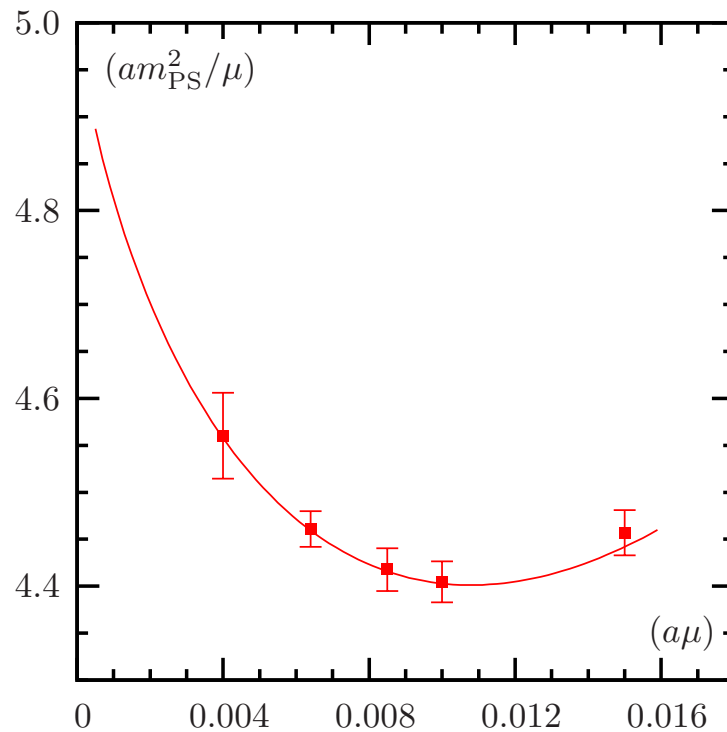
Lattice: M_π^2 versus m



Lüscher, Lattice conference 2005

- Quark masses sufficiently light \Rightarrow ChPT can be used to extrapolate to physical values of m
- Data of very high quality
- Chiral logarithms – are these seen?

Lattice: M_π^2/m versus m



ETM collaboration 2006

$$\bar{l}_3 = 2.9 \pm 2.4 \quad \Leftrightarrow \quad 0.2 \text{ GeV} \leq \Lambda_3 \leq 2 \text{ GeV} \quad \text{Leutwyler, J.G. 1984}$$

$$\bar{l}_3 = 3.44 \pm 0.28 \quad \text{ETM 2007}$$

Lattice: F_π versus m

Similarly, lattice can measure quark mass dependence of pion decay constant

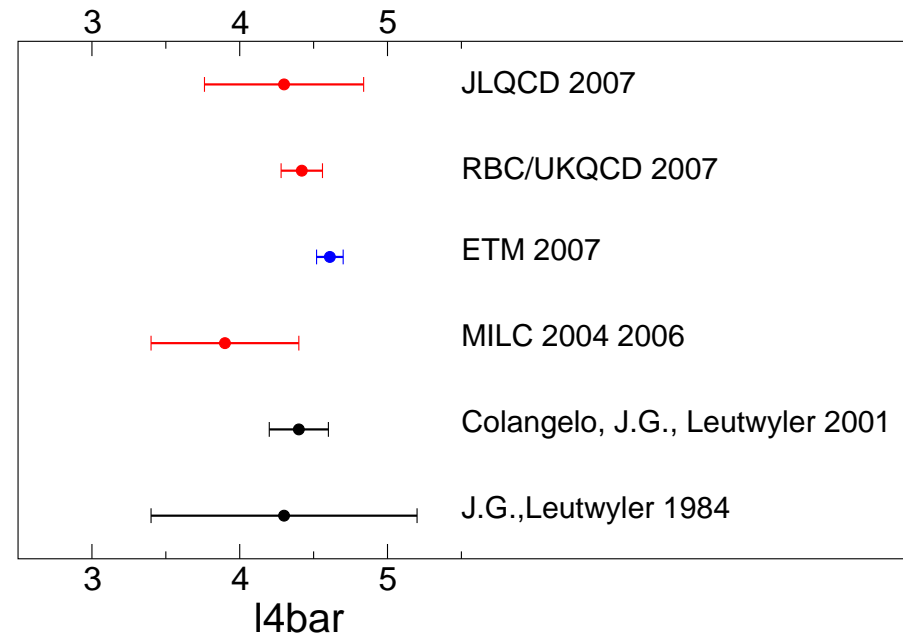
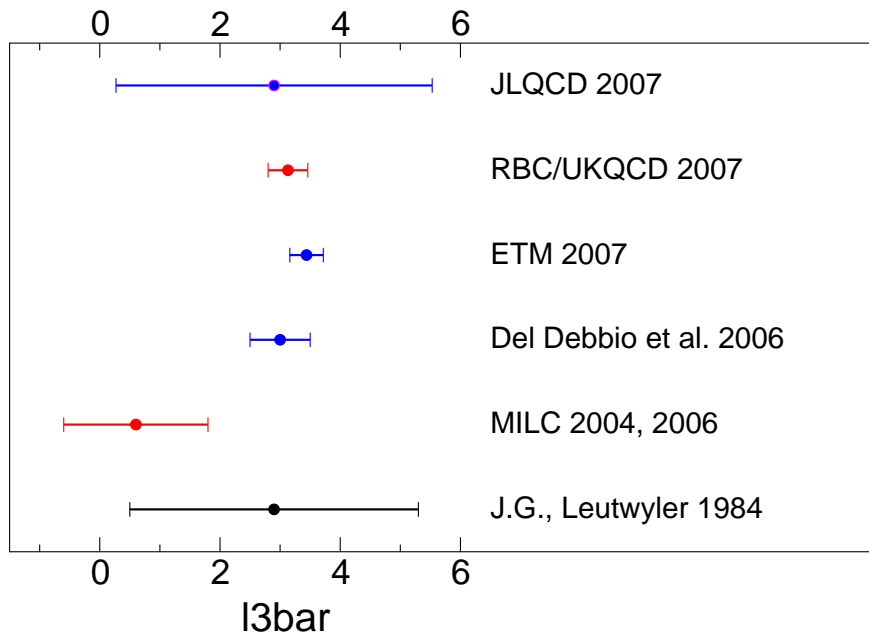
$$F_\pi = F \left\{ 1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\Lambda_4^2} + O(M^4) \right\}$$

$$\bar{l}_4 \equiv \frac{\Lambda_4^2}{M_\pi^2}$$

F : Pion decay constant, in the chiral limit $m_u = m_d = 0$

Several measurements of $\bar{l}_{3,4}$ reported at Lattice 2007

Lattice: Values of $\bar{l}_{3,4}$



message 1

$\bar{l}_{3,4}$ also occur in chiral expansion of $a_{0,2}$

PRECISION PHYSICS

$\pi\pi$ scattering lengths

ChPT + Roy equations + data above 800 MeV \Rightarrow precise predictions for $\pi\pi$ S -wave scattering lengths:

$$a_0 = 0.220 \pm 0.005, \quad a_0 - a_2 = 0.265 \pm 0.004$$

$a_0 \leftarrow$ isospin

Colangelo, J.G., Leutwyler 2001

Interesting for several reasons:

- Given a_0, a_2 , the $\pi\pi$ scattering amplitude can be calculated rather precisely in the low-energy region.

Roy 1971

Ananthanarayan, Colangelo, G., Leutwyler 2001

Descotes, Fuchs, Girlanda, Stern 2002

\Rightarrow Applications in $(g - 2)_\mu$ (pion form factor)

- scattering lengths \Leftrightarrow spontaneous chiral symmetry breaking

Stern and collaborators since long time

Other determinations

	[1]	[2]	[3]
a_0	0.228 ± 0.032	0.224 ± 0.013	0.230 ± 0.015
$-10 \cdot a_2$	0.382 ± 0.038	0.343 ± 0.036	0.480 ± 0.046

S. Descotes-Genon, N. H. Fuchs, L. Girlanda and J. Stern 2002 [1]

R. Kaminski, L. Lesniak and B. Loiseau 2003 [2]

J. R. Pelaez and F. J. Yndurain 2005 [3]

See talk Colangelo at KAON07 for comments.

How can one test the predictions?

- Lattice calculations: energy levels of $\pi\pi$ states depend on volume, expansion coefficients contain $\pi\pi$ scattering lengths

Lüscher 1986

- Pionium lifetime (DIRAC, CERN):
decay rate $\sim (a_0 - a_2)^2$

Uretsky, Palfrey 1961

- $K \rightarrow 3\pi$: cusp effect

Cabibbo 2004

See talk by Rainer Wanke, this morning

- $K^+ \rightarrow \pi^+\pi^-e^+\nu_e$: final state interactions

Shabalin 1963

General comment

Predictions as well as measurements are performed with high precision. Need to specify the framework before a meaningful comparison between theory and experiment can be done.

The paradise world

Predictions for $a_{0,2}$ are made in QCD (6 flavours), with

$$m_u = m_d = m, m_s, \Lambda_{\text{QCD}},$$

chosen such that

$$M_\pi = 139.6 \text{ MeV}, M_K = 493.6 \text{ MeV}, F_\pi = 92.4 \text{ MeV}$$

Precise values of heavy quark masses are irrelevant here.

No photons: $F_{\mu\nu} = 0$

A paradise world

Lattice calculations of $a_{0,2}$ can also be done in this framework ($N_f = 2, 3$).

Lattice: measurement of a_2

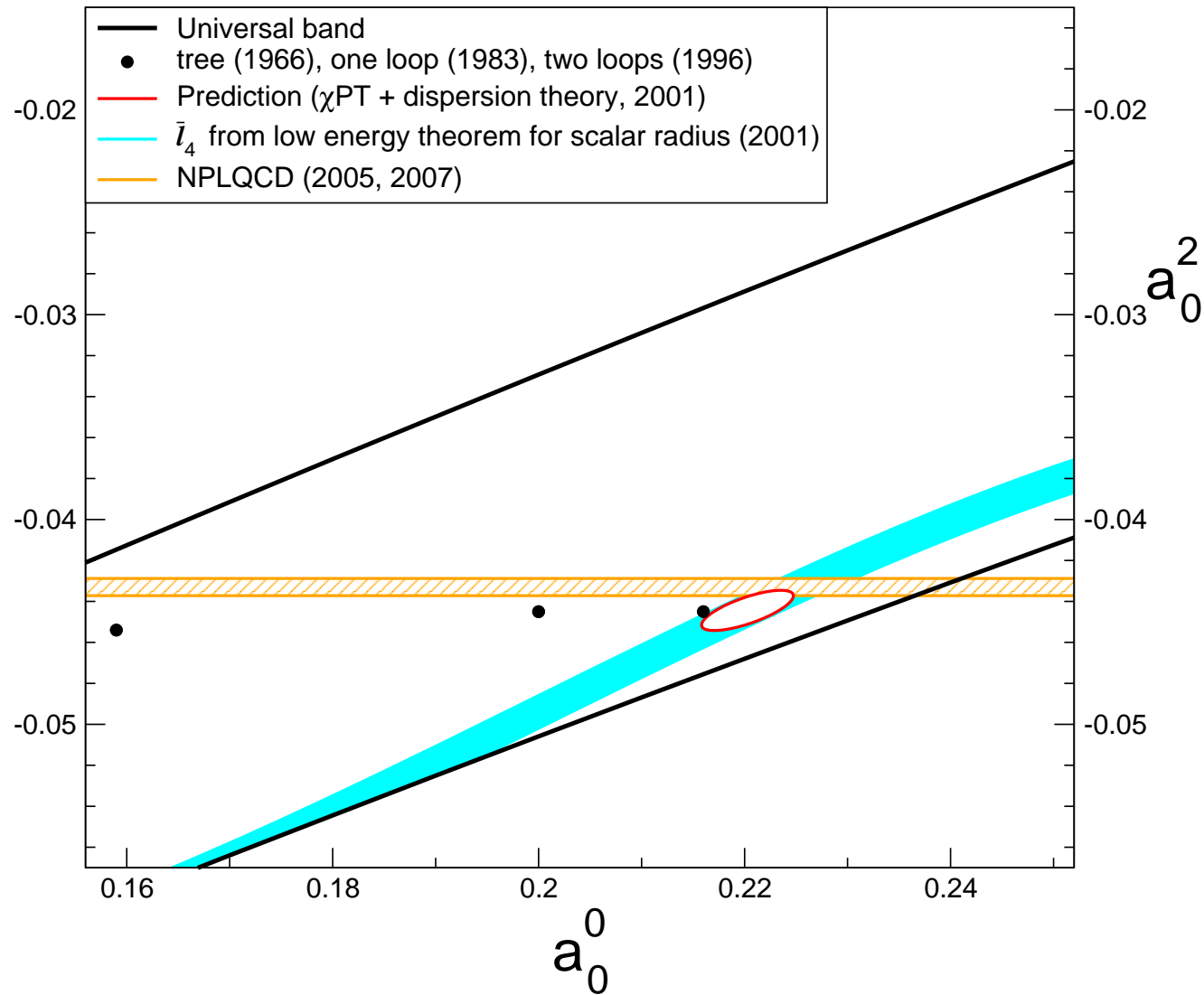


Figure courtesy of H. Leutwyler; $a_0^I = a_I$

Lattice: using $\bar{l}_{3,4}$

The uncertainties in $\bar{l}_{3,4}$ are the main source of uncertainties in the prediction of the scattering lengths. **May use lattice determinations**

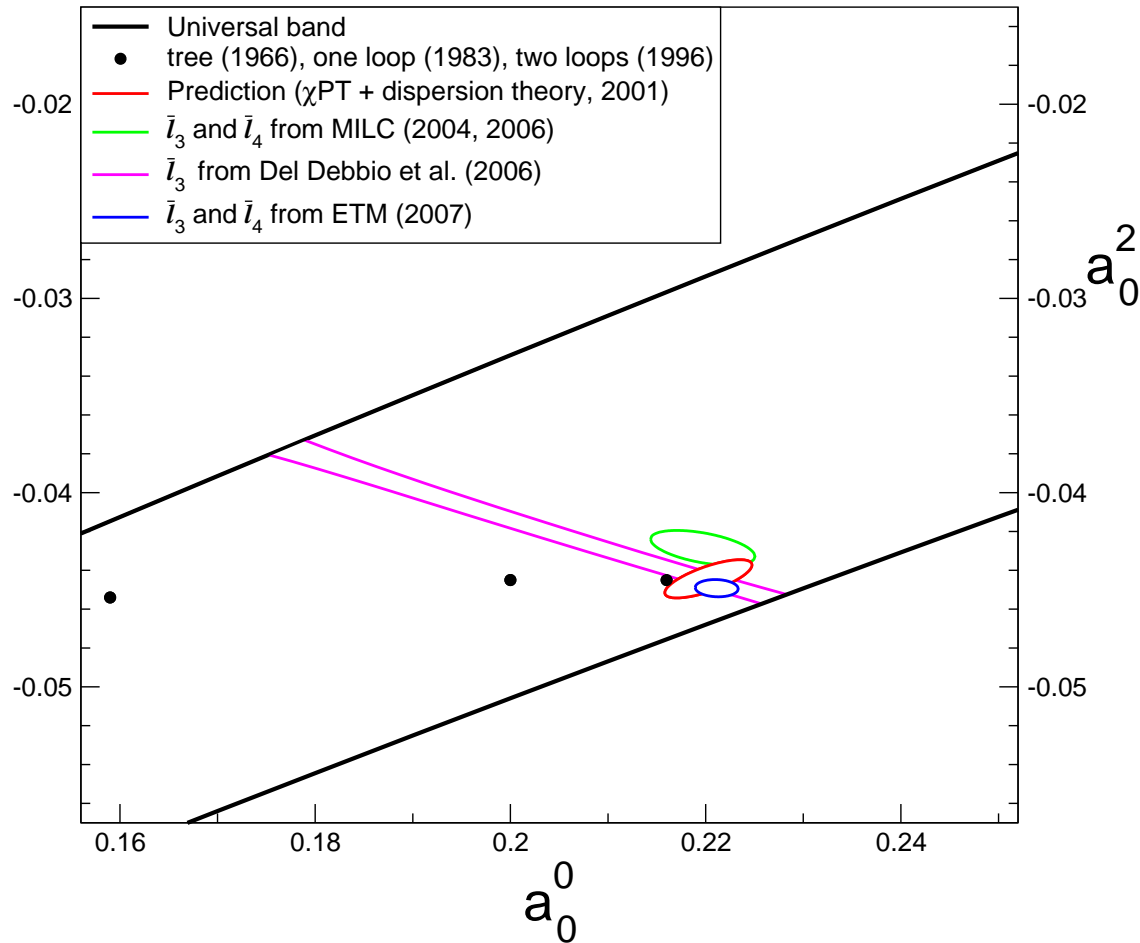


Figure courtesy of H. Leutwyler

Experiments: pionium lifetime, cusps, phases

In the paradise world:

Pionium does not form (no photons)

The kaon is stable (strangeness conservation): $K \not\rightarrow 3\pi$

\Rightarrow A direct comparison with experiment is not possible – have to enlarge the framework

Experiments

Experiments are performed in the real world, described by the Standard Model

$$\alpha \neq 0, m_u \neq m_d$$

Pionium does form

However:

$$K^+ \not\rightarrow \pi^+ \pi^0 \pi^0$$

$$K^+ \not\rightarrow \pi^+ \pi^- e^+ \nu_e$$

ZERO probability that these processes occur in the laboratory.

Bloch, Nordsieck 1937

⇔ message 2 at the beginning

⇒ Need a careful analysis of the situation

HOW?

In some cases, proper method is

NREFT

“Non-relativistic effective field theory”

Pionium

Pionium: bound state of $\pi^+\pi^-$. Unstable: ground state decays into $\pi^0\pi^0, 2\gamma, \dots$

DGBT-formula:

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^* (a_0 - a_2)^2$$

Deser, Goldberger, Baumann, Thirring 1954

How to render this formula exact?

Looks very complicated: a_0, a_2 are the scattering lengths in the paradise world. Pionium is bound state \Rightarrow

- Bethe-Salpeter equation?
- Potential models?

Clue: pions are non-relativistic – use non-relativistic QFT

Caswell, Lepage for QED 1986

Expand decay rate in powers of isospin breaking parameters,

$$\text{count } \alpha, (m_d - m_u)^2 \text{ as } O(\delta)$$

$$\begin{aligned} \Rightarrow \Gamma &= A\delta^{7/2} + B\delta^{9/2} + O(\delta^5) \\ &= \Gamma_{2\pi^0} + O(\delta^5) \end{aligned}$$

Invoke non-relativistic field theory:

$$\text{QCD+QED} \Rightarrow \text{ChPT} \Rightarrow \text{NREFT}$$

Do calculations in NREFT. The pertinent non-relativistic Lagrangian contains unknown LECs. At the end, match these to ChPT:

$$\text{NREFT} \Rightarrow \text{ChPT} \Rightarrow \text{QCD+QED}$$

NREFT: number of massive particles conserved by fiat.

Improved DGBT formula

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^* (a_0 - a_2)^2 (1 + \epsilon),$$
$$\epsilon = (5.8 \pm 1.2) \times 10^{-2}$$

Eiras, Gall, G., Ivanov, Jallouli, Kong, Labelle, Lipartia, Lyubovitskij, Raha, Ravndal, Rusetsky, Schweizer, Sazdjian, Soto, Zemp, ... after 1996

ϵ contains LECs from ChPT. Formula valid in QCD+QED.

Because a_0, a_2 known \Rightarrow

$$\tau = (2.9 \pm 0.1) \times 10^{-15} \text{ s}$$

prediction 2001

whereas

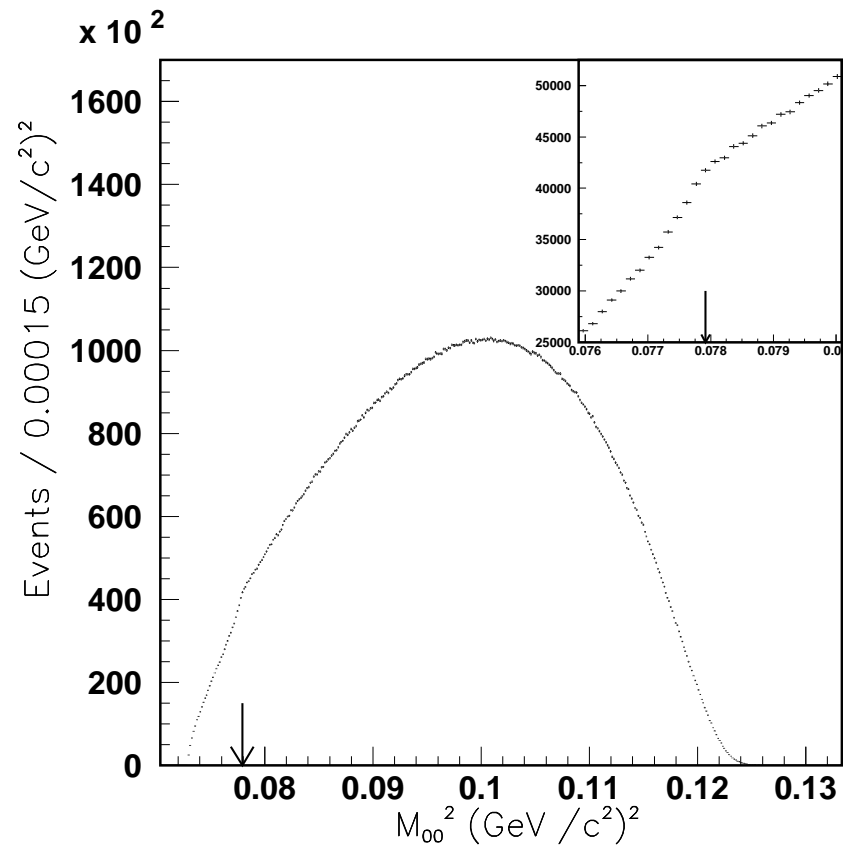
$$\tau = 2.91_{-0.62}^{+0.49} \times 10^{-15} \text{ s}$$

DIRAC 2005

Potential models: relation to QCD not manifest, unless NRQFT is built in. Superseded by NREFT.

Lipartia, Lyubovitskij, Rusetsky 2002

Cusp in $K \rightarrow 3\pi$



Histogram for $K^+ \rightarrow \pi^+ \pi^0 \pi^0$

Taken from NA48-collaboration: hep-ex/0511056

- Strength of cusp proportional to $a_0 - a_2$
- Position of cusp dictated by isospin breaking effect

Cabibbo 2004; Cabibbo, Isidori 2005

These authors use S -matrix theory to derive form of transition amplitude.

Alternative framework: NREFT. Advantages:

- Holomorphic properties, unitarity, cluster decomposition built in
Colangelo, J.G., Kubis, Rusetsky 2006
- Radiative corrections can be performed straightforwardly [QFT].
Bissegger, Fuhrer, J.G., Kubis, Rusetsky, work in progress

No radiative corrections:

Amplitudes algebraically not identical, their analytic properties differ.
Effect is numerically small.

NA48 collaboration, private communication

COMMENT ON K_{e4} decays

Bloch and Nordsieck

$$m_u \neq m_d$$

Radiative corrections in K_{e4}

- Real/virtual photons:
NA48: PHOTOS

Barberio, Was 1991,...

Sommerfeld factor (Summing vertex corrections)

E865: Neveu, Scherk (1968): scalar QED, ϕ^4

Remark: PHOTOS does not know about matrix element
 $K \rightarrow \pi\pi e\nu\gamma \Rightarrow$ check is mandatory

M. Knecht, L. Mercolli, work in progress

After having taken these corrections into account:

Recent results

Preliminary results from NA48 analyses

$$a_0 = 0.256 \pm 0.008_{\text{stat}} \pm 0.007_{\text{syst}} \pm 0.018_{\text{th}}$$

$$a_0 = 0.256 \pm 0.011$$

(stat.+syst. error added)

Moriond 2007

Montpellier 2006

B. Bloch-Devaux

These results are in conflict with the prediction

$$a_0 = 0.220 \pm 0.005$$

Colangelo, Gasser, Leutwyler 2000

Phases

The decay amplitude contains the axial current matrix element

$$\langle \pi^+ \pi^- | A_\mu | K^+ \rangle = \frac{1}{iM_K} [P_\mu F + Q_\mu G + L_\mu R]$$

with

$$F = f_s \exp i\delta_0^0 + f_p \exp i\delta_1^1 + D - \text{waves}$$

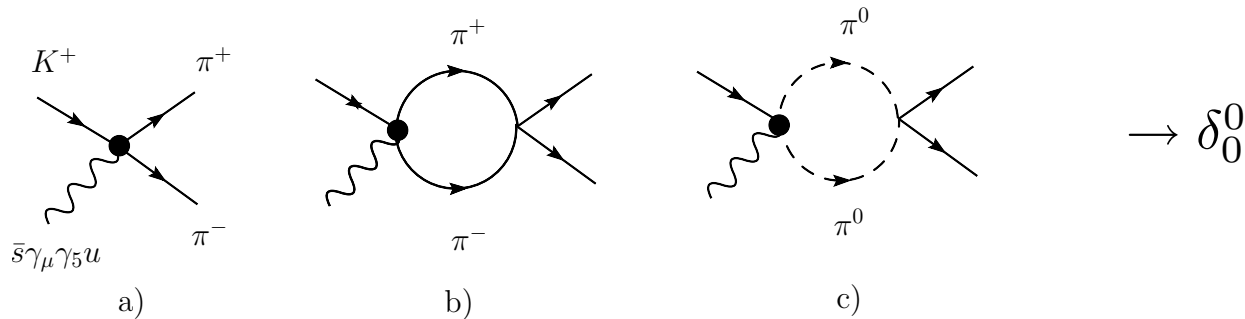
This is Watson's theorem, true in the isospin symmetry limit.

Allows one to measure the phase difference $\delta_0^0 - \delta_1^1$ and thus to determine the scattering lengths.

Which phase is measured?

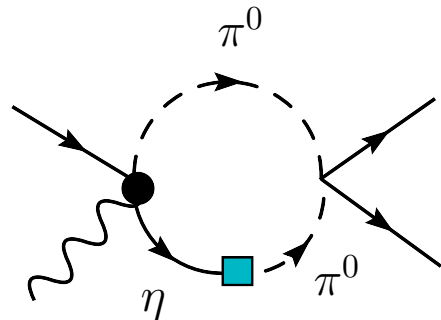
$$\underline{m_u = m_d; e = 0}$$

Lowest order:



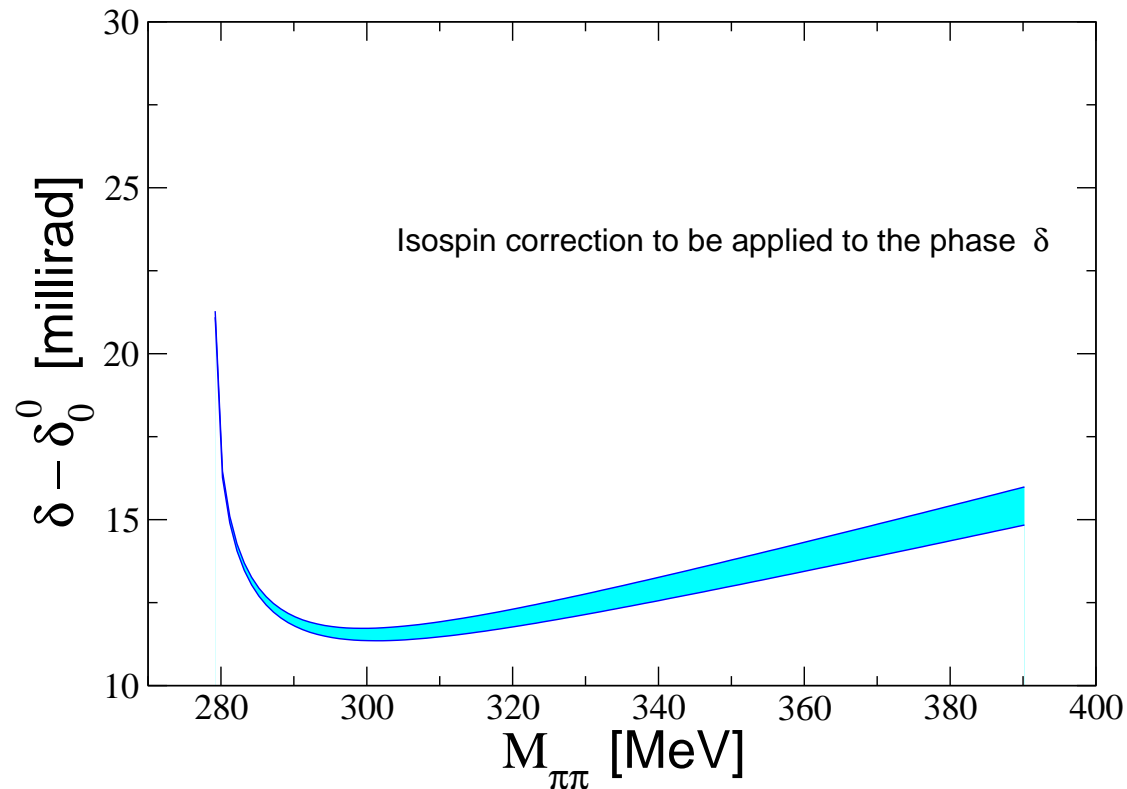
$$\underline{m_u \neq m_d; e \neq 0}$$

Analytic structure of diagrams change. Additional diagram:



Phase is changed: $\delta_0^0 \rightarrow \delta$. Experiment measures δ . **Effect is large.**

The missing piece



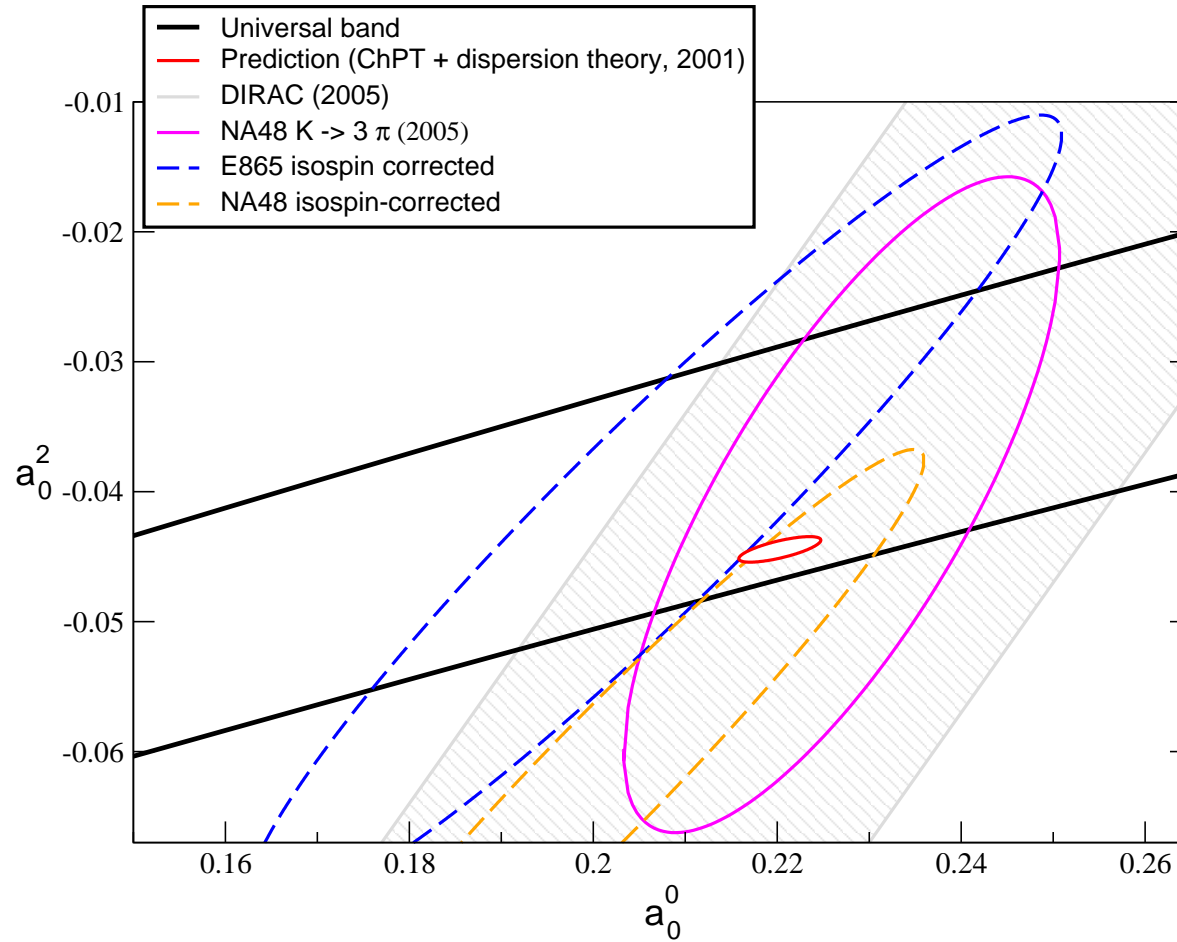
This part must be subtracted from the measured phase before a comparison with the prediction can be made.

Note: the correction displayed stems from a one-loop calculation.

Colangelo, J.G., Rusetsky, work in progress

see also Gevorkyan et al., 2007

Predictions vs. data



SUMMARY

- Lattice calculations have reached a precision that allows one to match the low–energy effective Lagrangian of QCD to the underlying theory, at least in some sectors.
- Some care is needed when translating experimental data into the paradise world. In particular, one should keep in mind that $m_u \neq m_d$ in the real world, and should not forget about Bloch and Nordsieck.

