

Recent **NA48/2** results on **Ke4** and **K3 π** decays and determination of the **$\pi\pi$** scattering lengths

Andrea Bizzeti

I.N.F.N. Sezione di Firenze
and University of Modena and Reggio Emilia

On behalf of the **NA48/2** collaboration:

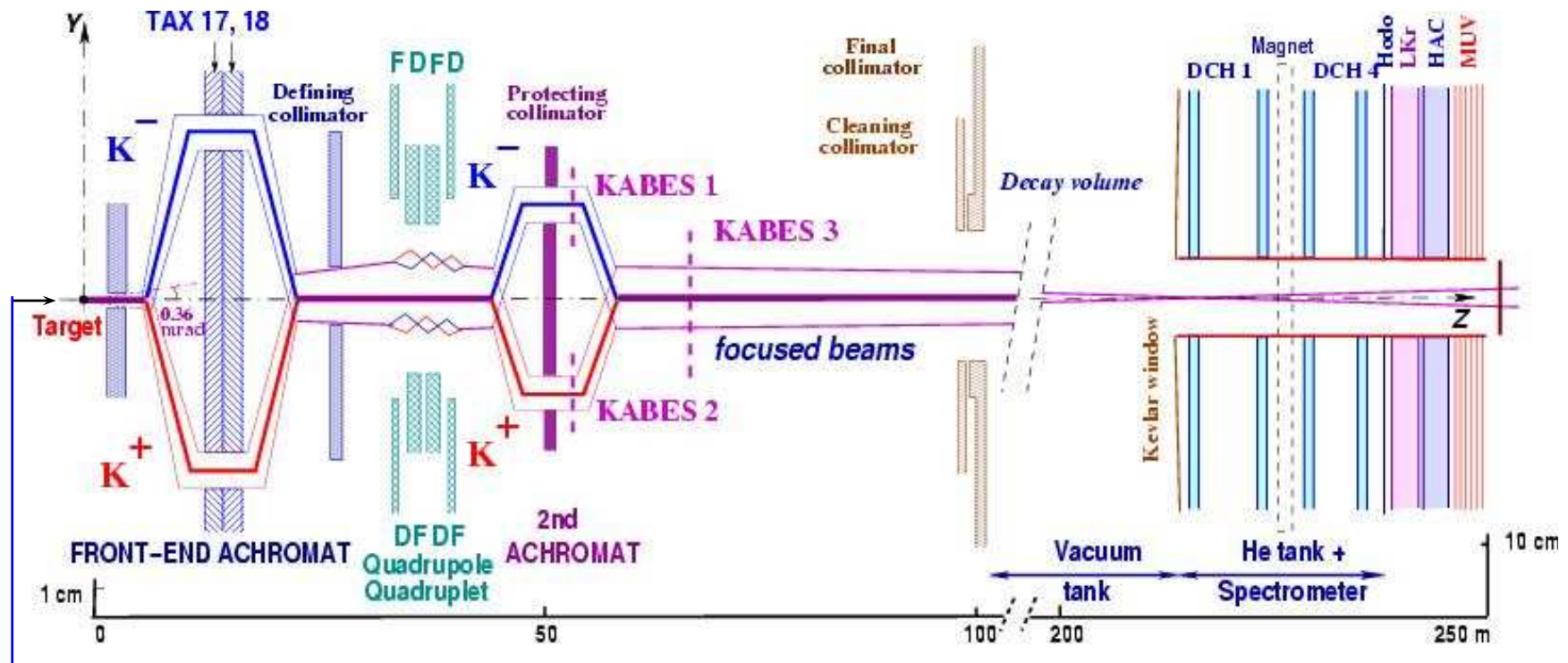
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz,
Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna



Outline

- The CERN NA48/2 experiment: setup and data
- Theory: ChPT, $\pi\pi$ S-wave scattering lengths, cusp(s):
 \Rightarrow see A. Rusetsky's talk, Plenary Session 1
- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$: new measurement on the “cusp”
and extraction of the $\pi\pi$ scattering lengths
- $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$: measurement of form factors
and pion scattering lengths

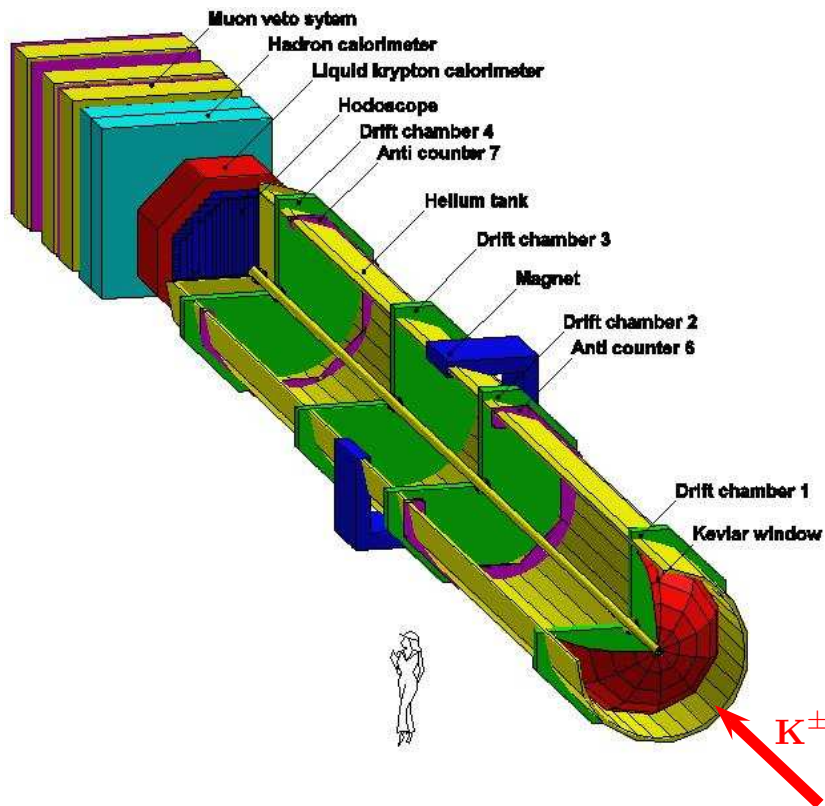
The NA48/2 experiment



400 GeV/c protons
from CERN SPS

Simultaneous K^+/K^- beams: $P = 60 \pm 3$ GeV/c
 $K^+/K^- \simeq 1.8$

The NA48 detectors



Liquid Krypton e.m. calorimeter:

$$\frac{\sigma_E}{E} = \frac{3.2\%}{\sqrt{E/1\text{GeV}}} \oplus \frac{9\%}{E/1\text{GeV}} \oplus 0.42\%$$

Magnetic Spectrometer:

$$\frac{\sigma_P}{P} \simeq 1\% \oplus 0.044\% \frac{P}{(1\text{GeV}/c)}$$

Charged Hodoscope: $\sigma_t = 150 \text{ ps}$

Hadron calorimeter, muon and photon vetos

- 1997-2001: NA48 K_L, K_S (ϵ'/ϵ)
- 2002: NA48/1 K_S
- 2003-2004: **NA48/2** K^\pm (Δg)
- 2007: NA62 K^\pm ($K_{e2}/K_{\mu2}$)
- 2011: NA62 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Main goal of NA48/2: search for CP violation in $K^\pm \rightarrow 3\pi$ Dalitz plots.

2003+2004 data: $4 \cdot 10^9 K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$; $1 \cdot 10^8 K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$; $1 \cdot 10^6 K_{e4}$.

$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ event selection

- require 1 track + 4 e.m. clusters
- cluster pairing ($\pi^0 \rightarrow \gamma\gamma$):
 - consider all (3) combinations
 - reconstruct each $\pi^0 \rightarrow \gamma\gamma$ vertex using π^0 mass
 - choose 2-vtx combination with closest vertices
- K decay vertex = average of π^0 vertices
- calculate $M_{00} \equiv M(\pi^0 \pi^0)$ and $M_{\pm 00} \equiv M(\pi^\pm \pi^0 \pi^0)$ invariant masses

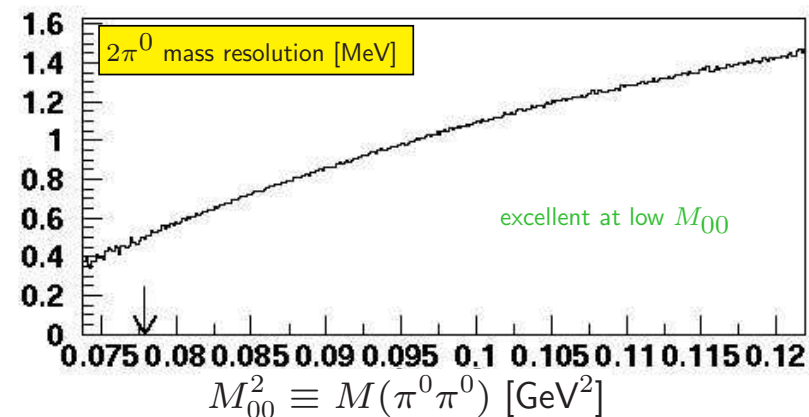
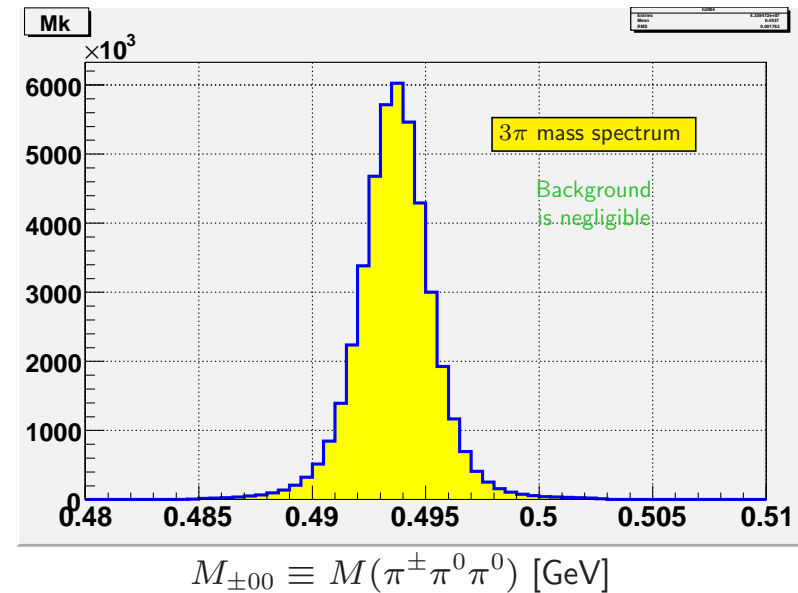
$\sigma(M_{00})$ optimized for low M_{00} values:

$$\sigma(M_{00}) = 0.56 \text{ MeV at } M_{00} = 2m_{\pi^+}$$

First cusp result (2003 data, $2.3 \cdot 10^7$ events):

→ published on PLB 633 (2006) 173

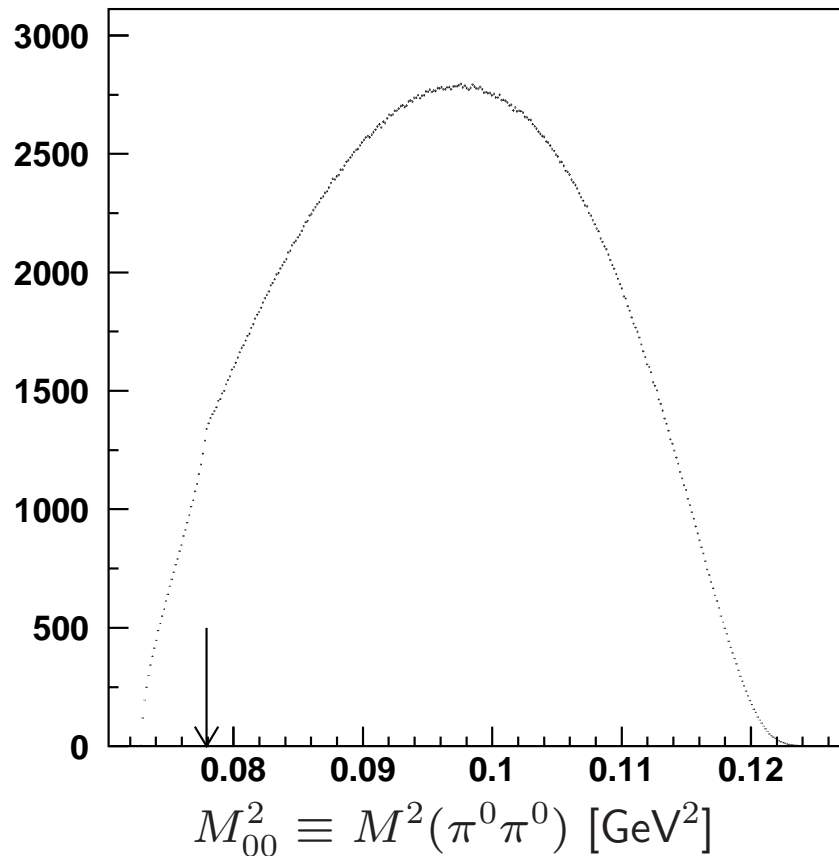
This analysis (2003 + 2004 data): $6 \cdot 10^7$ events



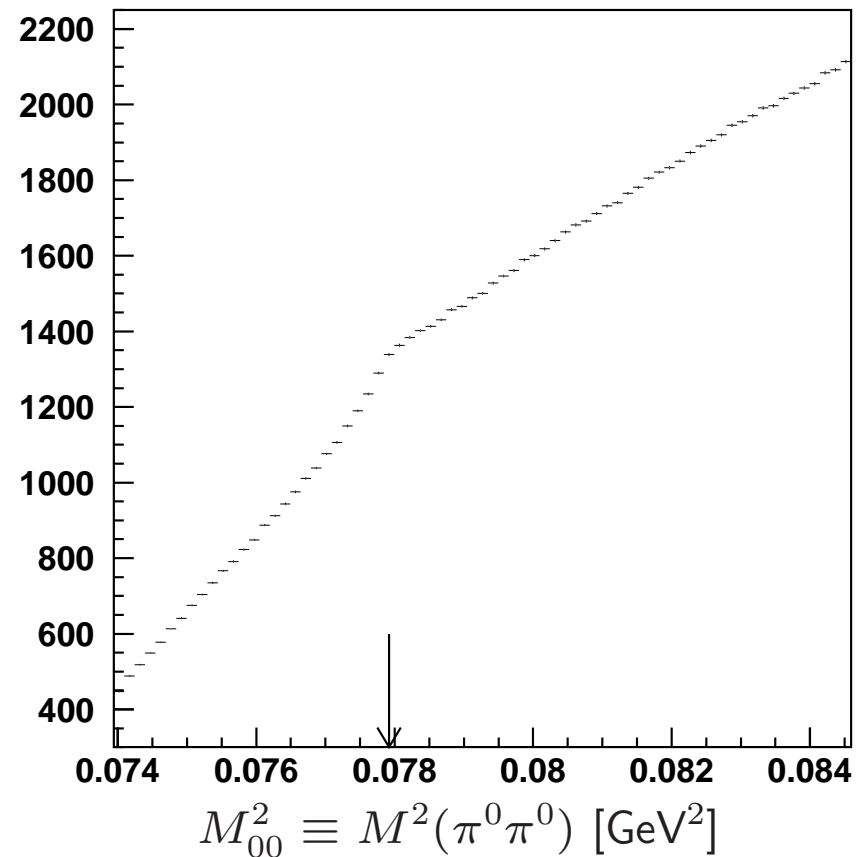


$M_{\pi^0\pi^0}^2$ distribution in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays

$\times 10^2$ NA48/2 (2003+2004 data)



$\times 10^2$ Zoom on the cusp region



Evidence of a **cusp** at $M_{00}^2 = (2m_{\pi^+})^2 \simeq 0.078 \text{ GeV}^2$

Cusp interpretation

$\pi\pi$ rescattering: one -loop calculation

[N. Cabibbo, PRL 93(2004) 12181]

$$\mathcal{M}(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = \mathcal{M}_0 + \mathcal{M}_1$$

\mathcal{M}_0 : $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ (direct)

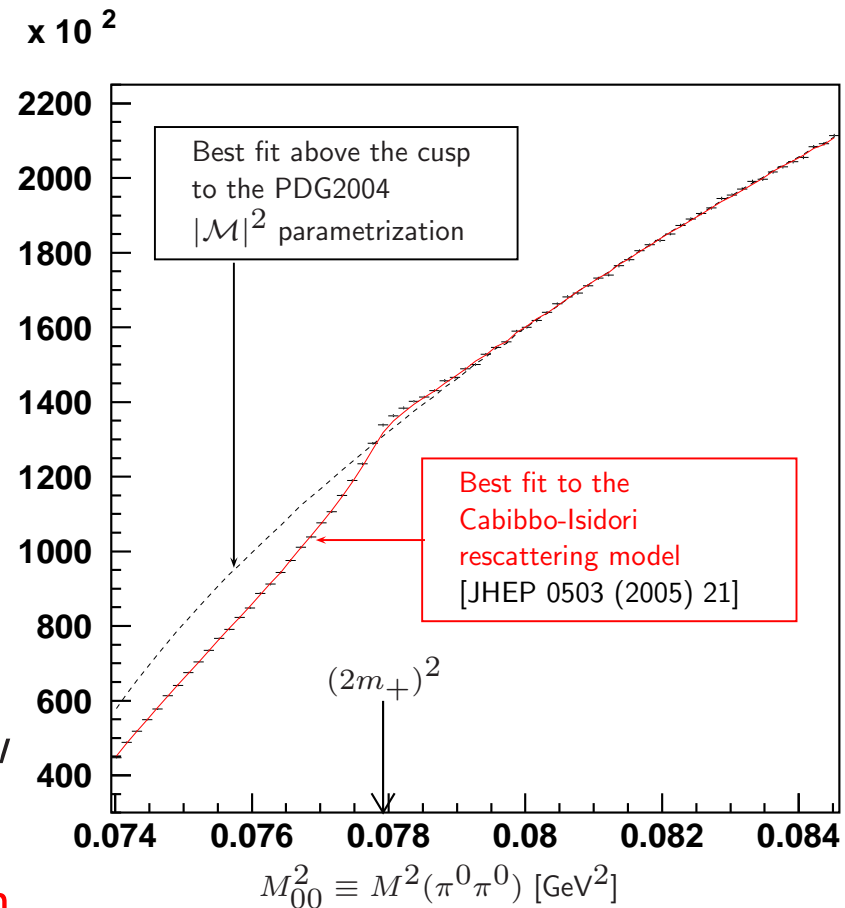
\mathcal{M}_1 : $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$, $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$

$$\mathcal{M}_1 = -\frac{2}{3}(a_0 - a_2)m_+ \mathcal{M}_{+,thr} \sqrt{1 - \frac{4m_+^2}{M_{00}^2}}$$

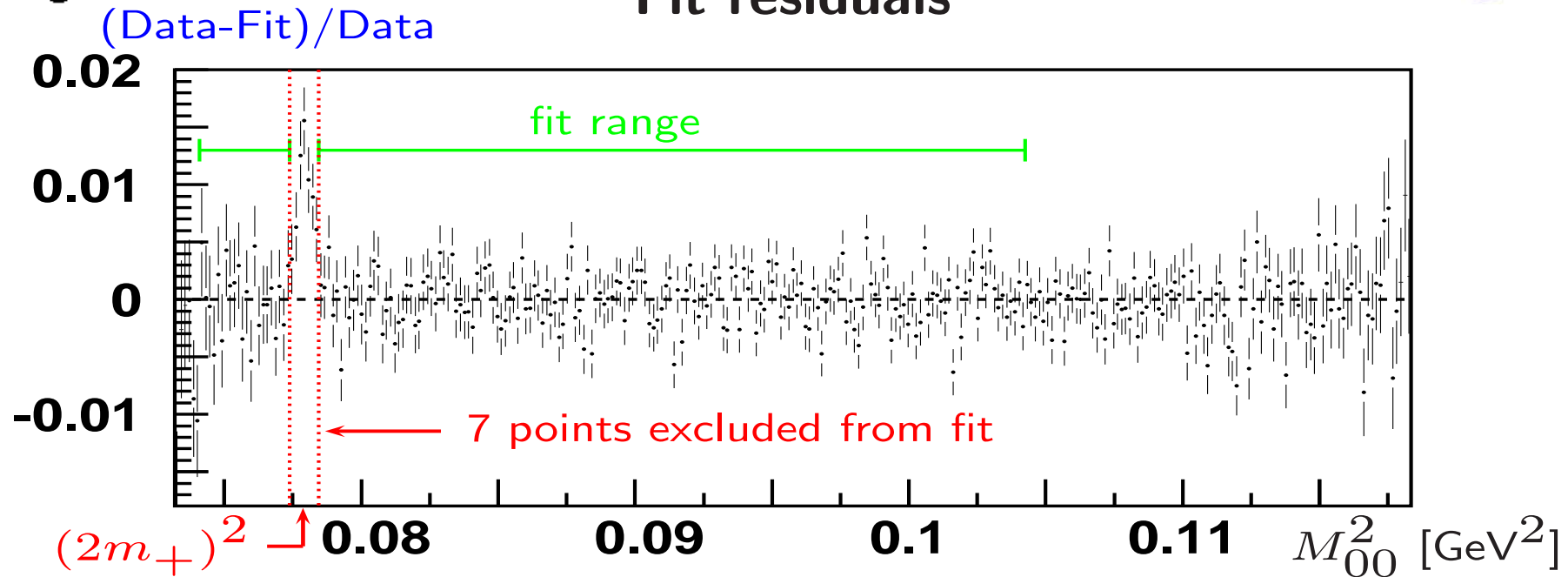
[$m_+ \equiv m_{\pi^+}$; $\mathcal{M}_+ \equiv \mathcal{M}(K^+ \rightarrow \pi^+ \pi^+ \pi^-)$]

interferes destructively with \mathcal{M}_0 below $\pi^+ \pi^-$ threshold: integrated deficit $\approx 13\%$

Possibility to determine $(a_0 - a_2)$ and its sign



Fit residuals



Observe an excess of events in the M_{00}^2 interval excluded from fit.

Rate = $(5.6 \pm 1.0) \cdot 10^{-5}$ per $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decay

(adding a delta-function at $M_{00} = (2m_+)$ before taking into account detector resolution and using its integral as a free parameter)

Two contributions to the peak in the $M_{00} \simeq 2m_+$ region:

- ponium formation: $(\pi^+ \pi^-)_{atom} \rightarrow \pi^0 \pi^0$ (Silagadze, JETP Lett.60 (1994) 689)
- additional $\pi^+ \pi^-$ unbound states with resonant structure $\rightarrow \pi^0 \pi^0$ (Gevorkian, Tarasov, Voskresenskaya, hep-ph/0612129)



Cusp: fit results (I)

Data fitted using two different theoretical models:

- Cabibbo-Isidori (CI) rescattering model [JHEP 0503 (2005) 21]
- Colangelo-Gasser-Kubis-Rusetsky (CGKR) effective field theory [PLB 638 (2006) 187]

Fit with **analyticity and chiral symmetry constraint** between a_2 and $a_0 - a_2$

$$a_2 m_+ = (-0.0444 \pm 0.0008) + 0.236(a_0 m_+ - 0.22) - 0.61(a_0 m_+ - 0.22)^2 - 9.9(a_0 m_+ - 0.22)^3$$

[Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008]

$$\text{(CI): } (a_0 - a_2)m_+ = 0.268 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

$$\text{(CGKR): } (a_0 - a_2)m_+ = 0.266 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

External uncertainty: from the uncertainty on the ratio (PDG 2006)

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0 \pi^0)} = 3.182 \pm 0.047 \Rightarrow \frac{A(K^+ \rightarrow \pi^+ \pi^+ \pi^-)}{A(K^+ \rightarrow \pi^+ \pi^0 \pi^0)} = 1.975 \pm 0.015$$

at the Dalitz plot centres (**exact isospin symmetry predicts 2**)

Theoretical uncertainty on $(a_0 - a_2)m_+$: ± 0.013 (CI), not given (CGKR)
(estimated from neglecting higher order diagrams and radiative corrections)



Cusp: fit results (II)

Fits imposing chiral symmetry constraint:

$$(CI): (a_0 - a_2)m_+ = 0.268 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

$$(CGKR): (a_0 - a_2)m_+ = 0.266 \pm 0.003_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$

Fits **not** imposing the chiral symmetry constraint: possibility to measure a_2

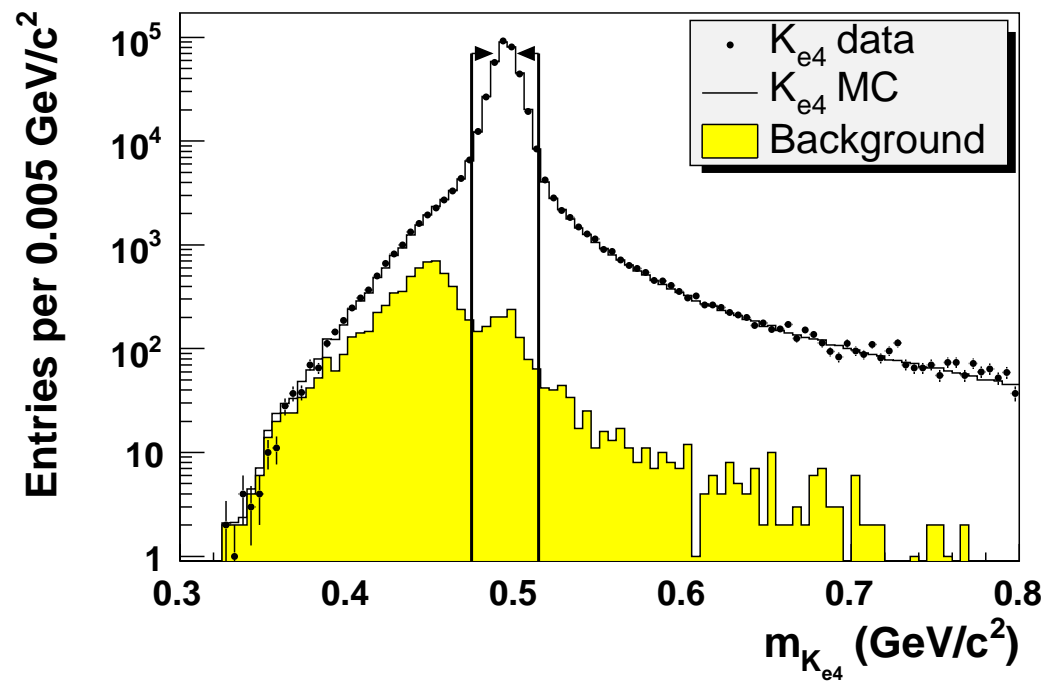
$$(CI): (a_0 - a_2)m_+ = 0.266 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$
$$a_2 m_+ = -0.039 \pm 0.009_{stat} \pm 0.006_{syst} \pm 0.002_{ext}$$

$$(CGKR): (a_0 - a_2)m_+ = 0.273 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext}$$
$$a_2 m_+ = -0.065 \pm 0.015_{stat} \pm 0.010_{syst} \pm 0.002_{ext}$$

Theoretical uncertainty: $\pm 5\%$ (CI); not given (CGKR).

Ke4 ($K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$)

Analysis of 2003 data (Eur. Phys. J. C 54 (2008) 411)



Rare decay:

$$\text{B.R.} = (4.09 \pm 0.09) \cdot 10^{-5}$$

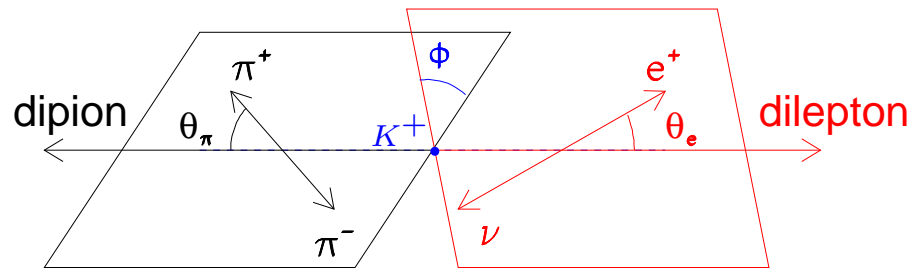
Event selection:

- 3 tracks
- missing energy and p_t
- E/p for e and π ID

$6.8 \cdot 10^5$ selected events

Background estimated from “wrong sign” events ($\pi^\pm\pi^\pm e^\mp$): $\approx 0.5\%$

Ke4 kinematics



Kinematics described by 5 independent Cabibbo-Maksymowicz variables:

$$M_{\pi\pi}^2, M_{e\nu}^2, \theta_\pi, \theta_e, \phi$$

Assuming CP symmetry:

$$K^+ \rightarrow K^- \Rightarrow \begin{aligned} \phi &\rightarrow \phi + \pi, \\ \theta_e &\rightarrow \pi - \theta_e \end{aligned}$$

Decay amplitude is described by two axial (F , G) and one vector (H) form factors, which can be **partial wave** expanded (s and p waves; d neglected):

$$F = F_s(q^2)e^{i\delta_s} + F_p(q^2)e^{i\delta_p} \cos \theta_\pi$$

$$G = G_p(q^2) e^{i\delta_p}$$

$$H = H_p(q^2) e^{i\delta_p}$$

$$\delta(q^2) \equiv \delta_s - \delta_p$$

$$q^2 \equiv (M_{\pi\pi}^2/4m_\pi^2) - 1$$

and further expanded in **powers of q^2** :

$$F_s(q^2) = f_s + f'_s q^2 + f''_s q^4 + f_e (M_{e\nu}^2/4m_\pi^2) + \dots$$

$$F_p(q^2) = f_p + f'_p q^2 + \dots$$

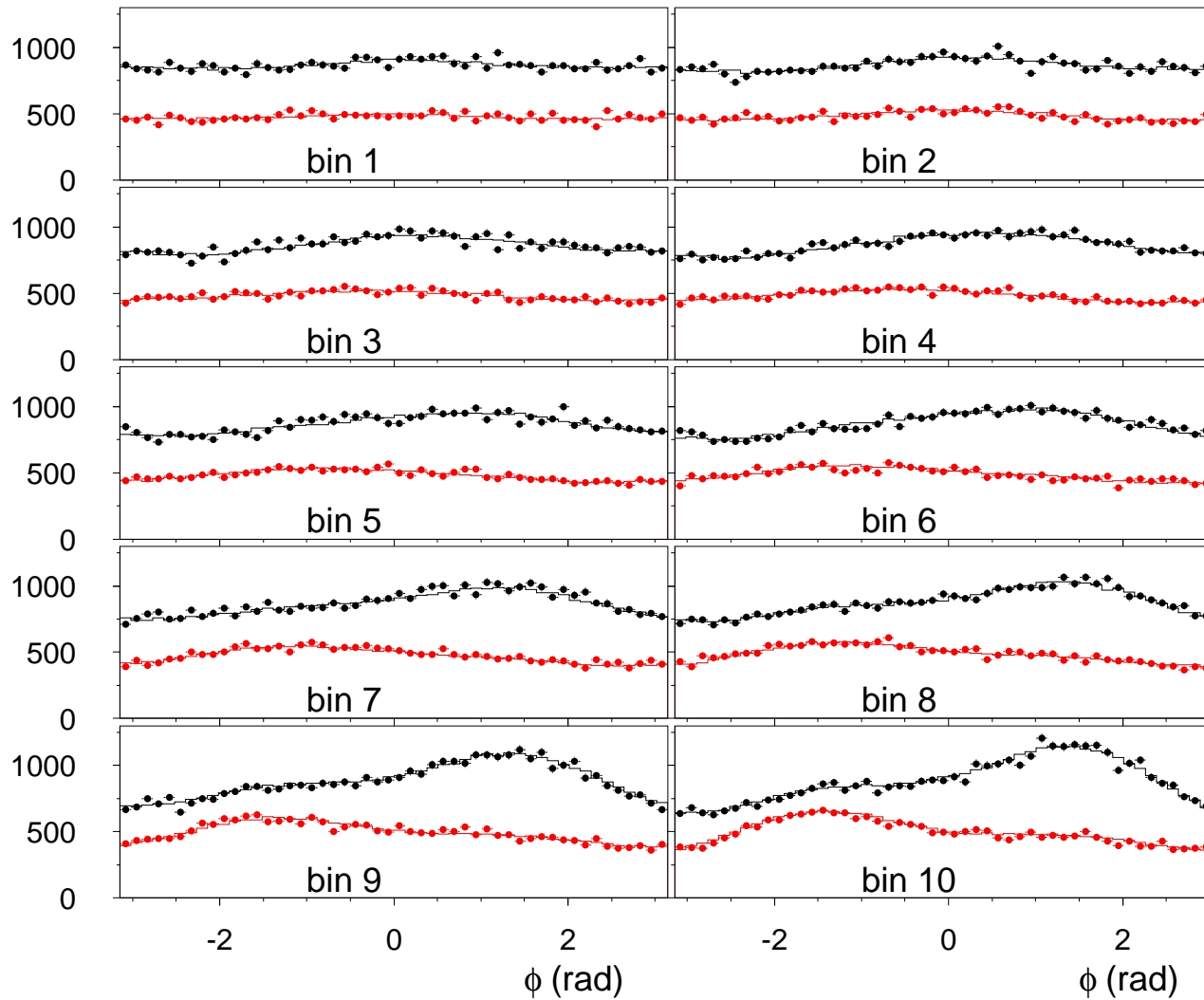
$$G_p(q^2) = f_p + g'_p q^2 + \dots$$

$$H_p(q^2) = h_p + h'_p q^2 + \dots$$

$\delta \neq 0 \Rightarrow$ **asymmetry in ϕ distribution, increasing with $M_{\pi\pi}$**



Ke4 ϕ distributions



one plot
per
 $M_{\pi\pi}$ bin

K^+ (black)
and K^- (red):

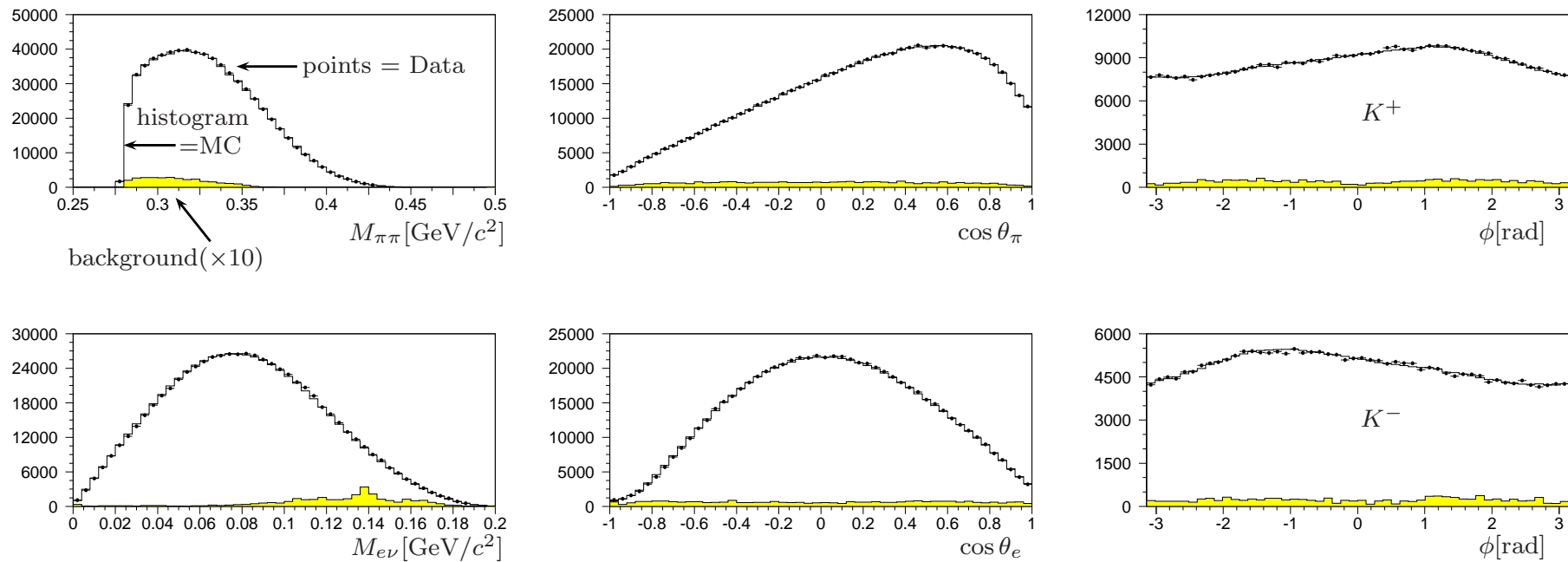
points = data

histogram = MC

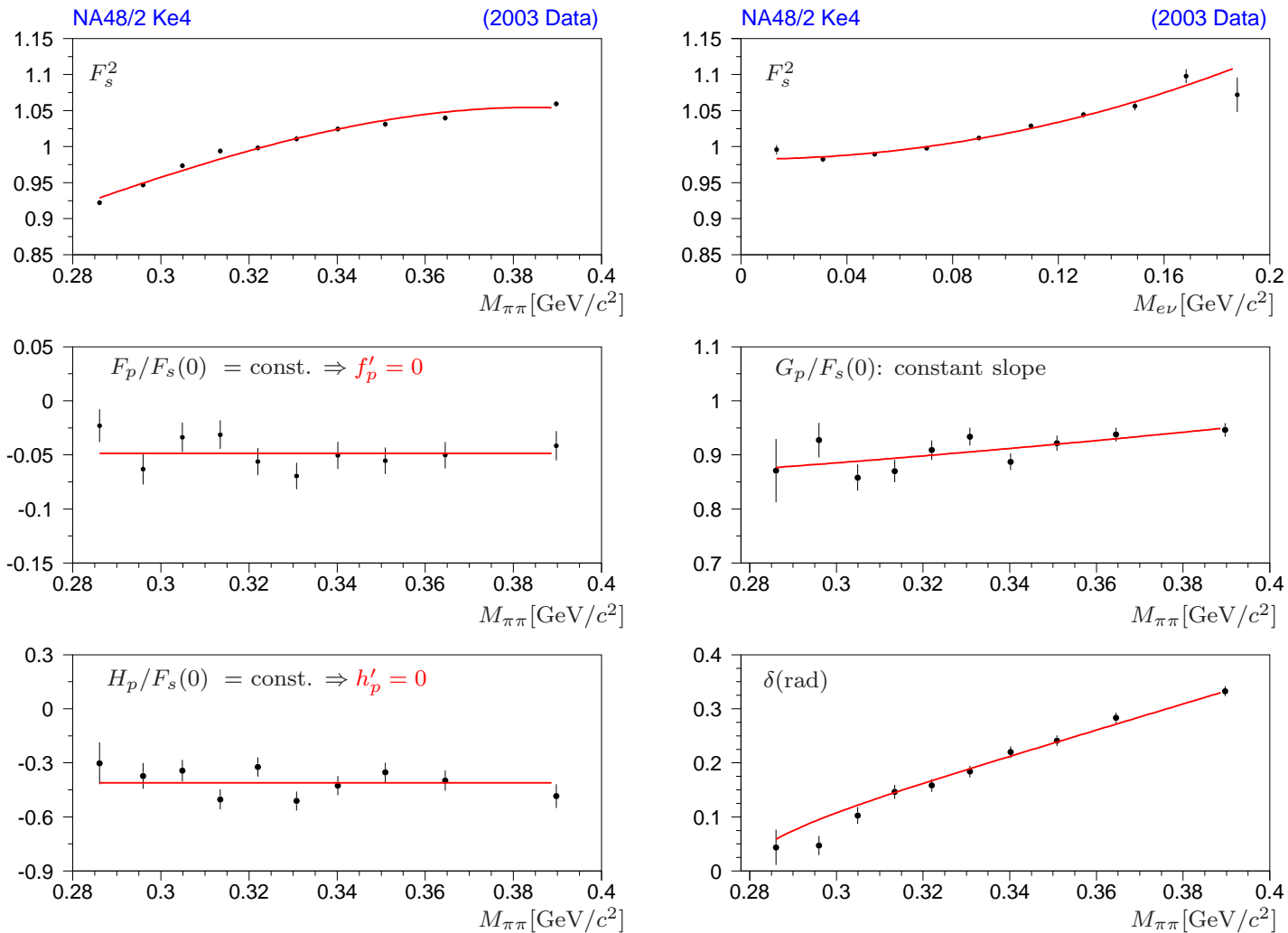
Ke4 form factors fit (I)

F_s, F_p, G_p, H_p and δ are extracted from a fit to
 $10(M_{\pi\pi}) \times 5(M_{e\nu}) \times 5(\cos\theta_e) \times 5(\cos\theta_\pi) \times 12(\phi) = 15000$ equi-populated bins.

Event distributions in Cabibbo-Maksymowicz variables (2003 data, $6.8 \cdot 10^5$ events)



Ke4 form factors fit (II)





Ke4 form factors results (2003 data)



$$f'_s/f_s = 0.172 \pm 0.009_{stat} \pm 0.006_{syst}$$

$$f''_s/f_s = -0.090 \pm 0.009_{stat} \pm 0.007_{syst}$$

$$f'_e/f_s = 0.081 \pm 0.008_{stat} \pm 0.009_{syst}$$

$$f_p/f_s = -0.048 \pm 0.004_{stat} \pm 0.004_{syst}$$

$$g_p/f_s = 0.873 \pm 0.013_{stat} \pm 0.012_{syst}$$

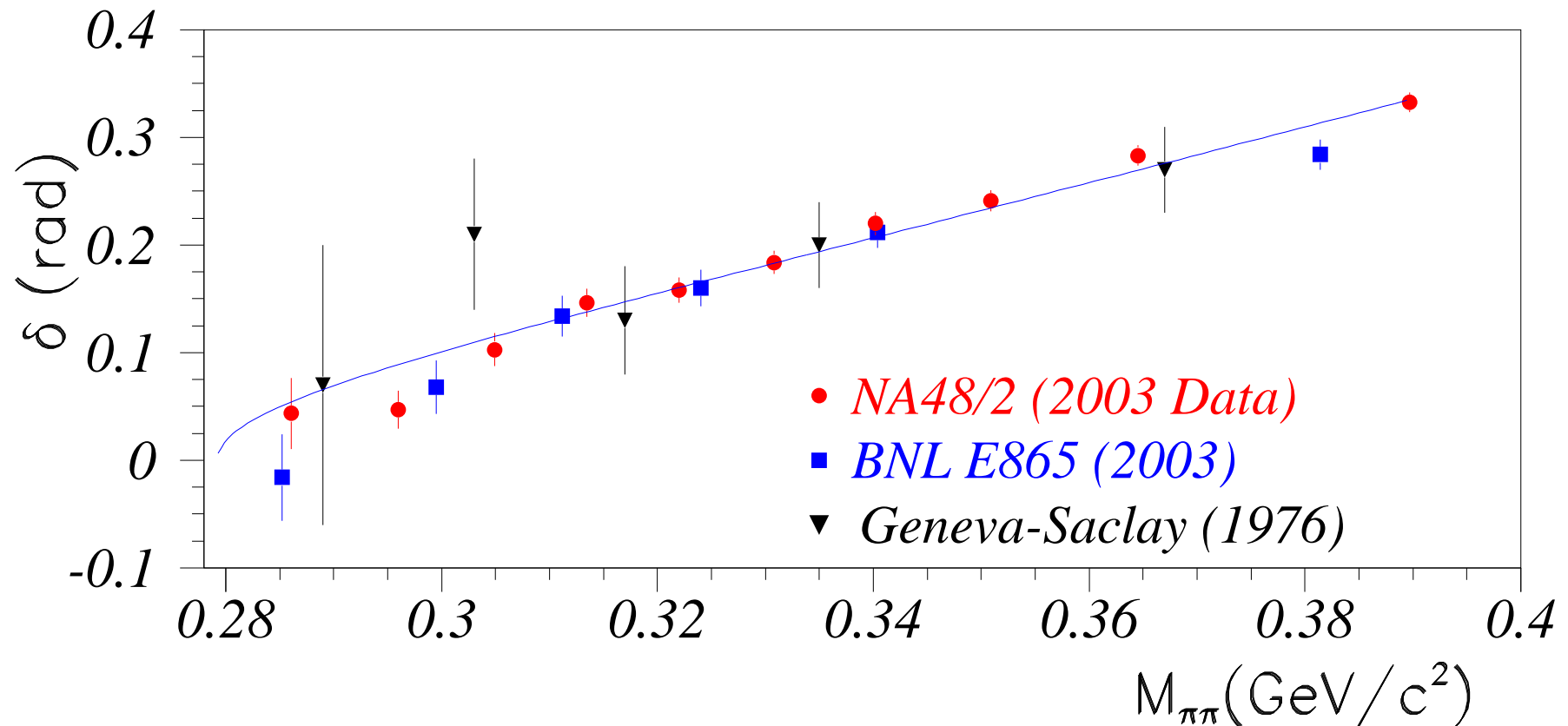
$$g'_p/f_s = 0.081 \pm 0.022_{stat} \pm 0.015_{syst}$$

$$h_p/f_s = -0.411 \pm 0.019_{stat} \pm 0.008_{syst}$$

Relative form factors (normalized to f_s)

Separate measurements for K^+ and K^- , then combined

Ke4: $\delta(M_{\pi\pi})$ and scattering lengths



Fit to Universal Band (UB) central line in (a_0, a_2) space

(UB) = limits from experimental data at higher energies, extrapolated using Roy equations

[Ananthanarayan, Colangelo, Gasser, Leutwyler, Phys. Rep. 353 (2001) 207]



Ke4: $\pi\pi$ scattering lengths

Fit to Universal Band (UB) central line in (a_0, a_2) space:

$$a_0 m_+ = 0.256 \pm 0.006_{stat} \pm 0.002_{syst}^{+0.018}_{-0.017_{ext}}$$

$$a_2 m_+ = -0.0312 \pm 0.0011_{stat} \pm 0.0004_{syst}^{+0.0129}_{-0.0122_{ext}}$$

(“External” error: from central line to limits of UB)

Fit with 2 free parameters (a_0 and a_2):

$$a_0 m_+ = 0.233 \pm 0.016_{stat} \pm 0.007_{syst}$$

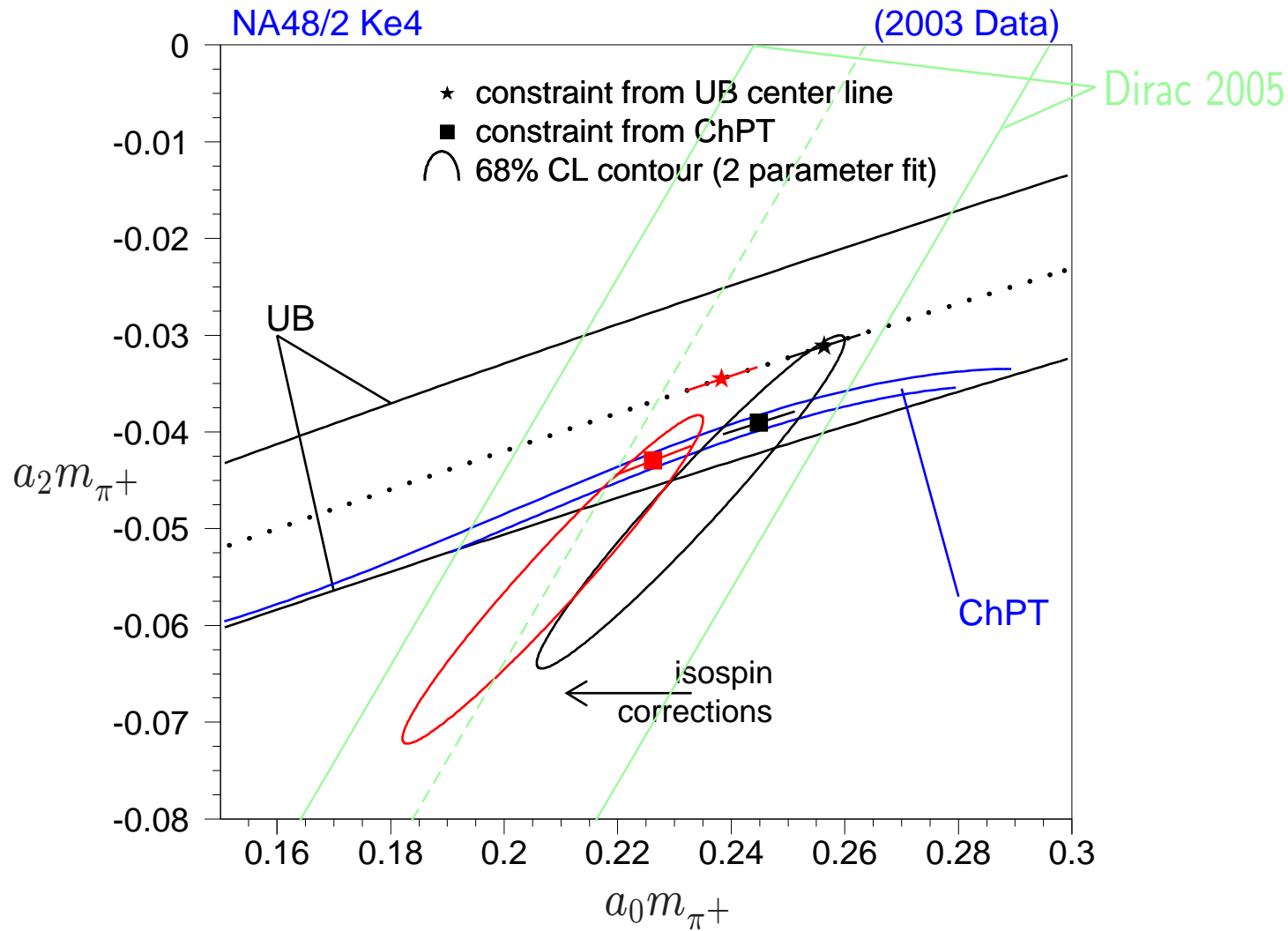
$$a_2 m_+ = -0.047 \pm 0.011_{stat} \pm 0.004_{syst}$$

Fits performed assuming isospin symmetry

Expected corrections for isospin symmetry breaking: $10 \sim 15$ mrad on δ ;
 ≈ -0.02 on $a_0 m_+$.

The analysis of full (2003+2004) data sample ($10^6 K_{e4}$ decays), including isospin symmetry breaking effects, is nearly finished.

Ke4: $\pi\pi$ scattering lengths





Conclusions and outlook

- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$: full 2003+2004 sample (60 million events) analysed.
- $a_0 - a_2$ extracted from cusp analysis using both [Cabibbo-Isidori](#) and [Colangelo-Gasser-Kubis-Rusetsky](#) formulae, with reasonable agreement, especially in the ChPT-constrained fit.
- Results consistent with a_0 and a_2 determined from K_{e4} analysis of 2003 data [Eur.Phys.C 54 (2008) 411]
- K_{e4} analysis of 2003+2004 data (1 million events) nearly finished, will reach a statistical precision at the level of theoretical precision.