Precise determination of the sigma pole from a dispersive analysis

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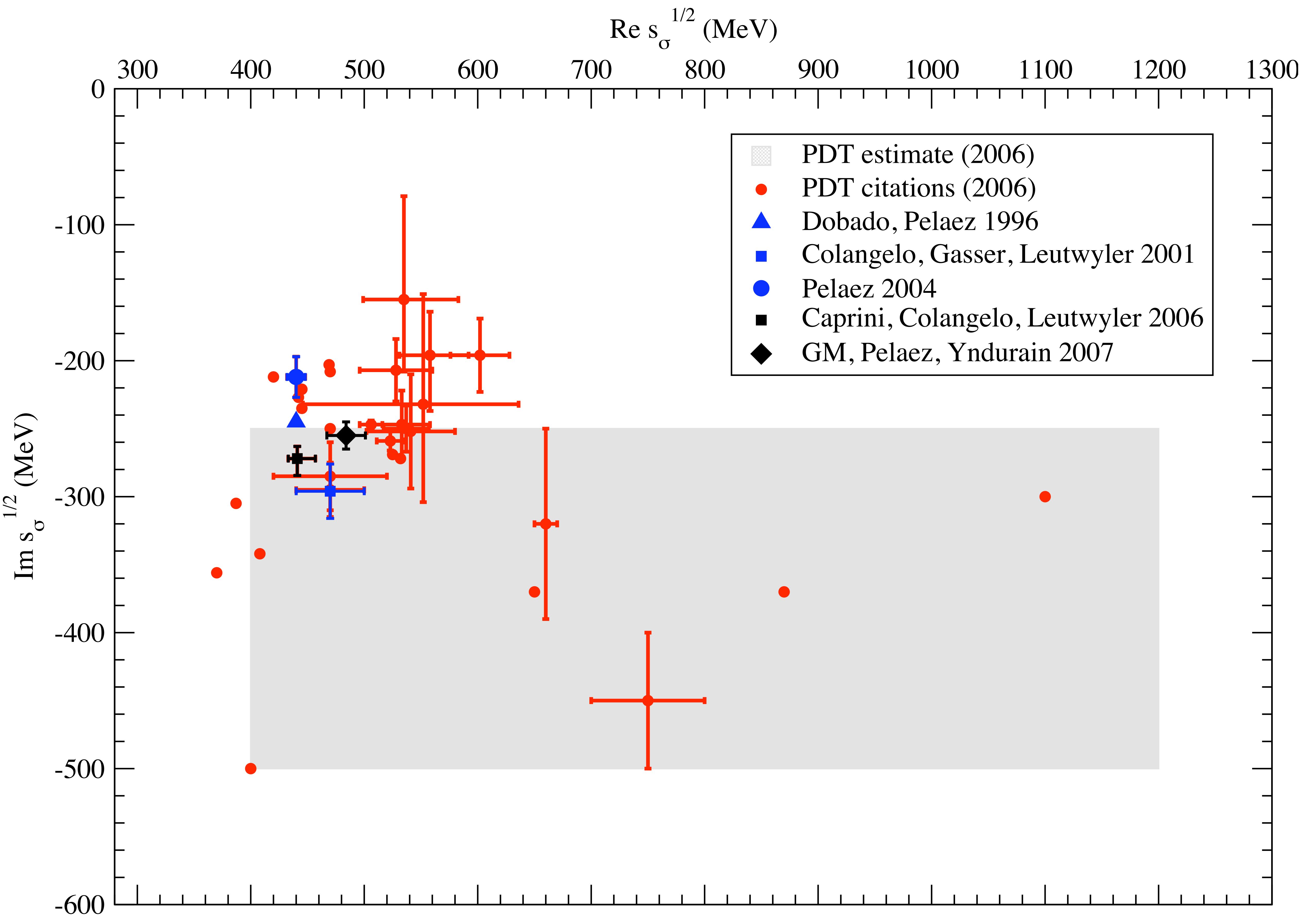
Dedicated to the memory of
Prof. F. J. Ynduráin

Motivation

- Values for the sigma in the Particle Data Table are very widely spread.


Re $s_\sigma^{1/2}$ (MeV)
300
400
500
600
700
800
900
1000
1100
1200
1300
0
-100
-200
-300
-400
-500
-600
Im $s_\sigma^{1/2}$ (MeV)
PDT estimate (2006)
PDT citations (2006)
Dobado, Pelaez 1996
Colangelo, Gasser, Leutwyler 2001
Pelaez 2004
Caprini, Colangelo, Leutwyler 2006
GM, Pelaez, Yndurain 2007

Motivation

- Values for the sigma in the Particle Data Table are very widely spread.
- Main reasons:

    Old data was poor.

    Different authors use different (incompatible) data sets for finding the sigma pole.

    Many model dependences in the extrapolation to the complex plane.

Motivation

- **Recently available:** New precise data on ππ scattering at low energy.
- Dispersion relations improve precision and are model independent.
- Already used for predicting the sigma pole:

    Dobado, Peláez (1997): Inverse Amplitude Method.

    Zhou et al. (2005): ChPT and unitarization.

    Caprini, Colangelo, Leutwyler (2006): Scattering lengths (prediction from ChPT) and Roy equations, but no low energy data (below 800 MeV).

- **OUR AIM:** To include the new data in a dispersive analysis, to obtain a precise and model independent determination of the sigma pole location, **EXCLUSIVELY FROM DATA, ANALITICITY AND CROSSING SYMMETRY** (without using ChPT, so that we can test its predictions).
- **For this, we will use Forward Dispersion Relations *and* Roy's equations.**

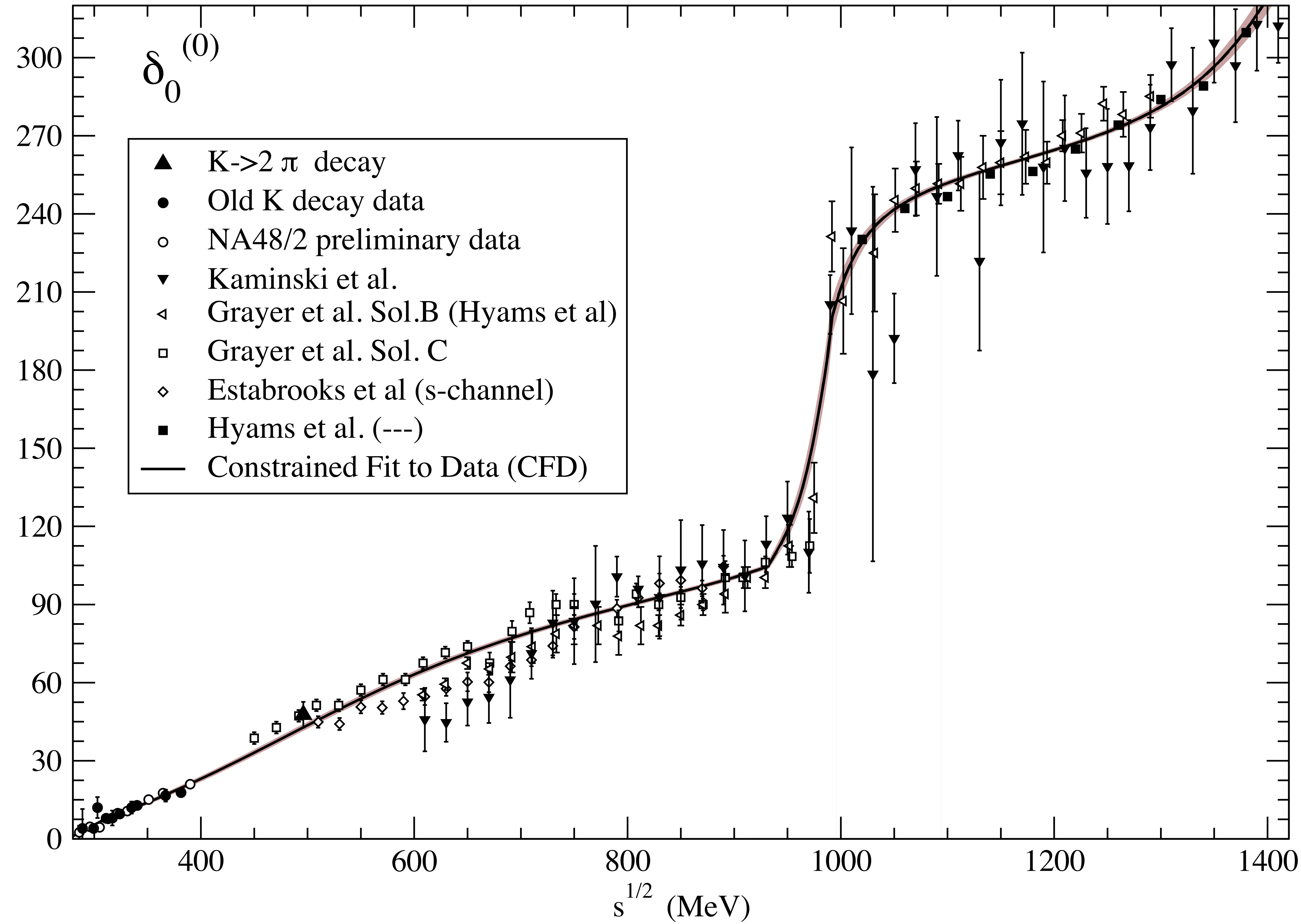
Approach

We have performed:

- Independent fits to data: **UFD** (*Unconstrained Fit to Data*)
  These fits satisfy Forward Dispersion Relations (FDR) and Roy equations within errors for each wave, except the S2, which lies at 1.3 standard deviations.

- Fits to data constrained to satisfy Roy equations and Forward Dispersion Relations: **CFD** (*Constrained Fit to Data*), which provide a remarkably precise and reliable description of the experimental data.

We're working in the isospin limit.



$\delta_0^{(0)}$
K->2 π decay
Old K decay data
NA48/2 preliminary data
Kaminski et al.
Grayer et al. Sol.B (Hyams et al)
Grayer et al. Sol. C
Estabrooks et al (s-channel)
Hyams et al. (---)
Constrained Fit to Data (CFD)
0
30
60
90
120
150
180
210
240
270
300
400
600
800
1000
1200
1400
$s^{1/2}$ (MeV)

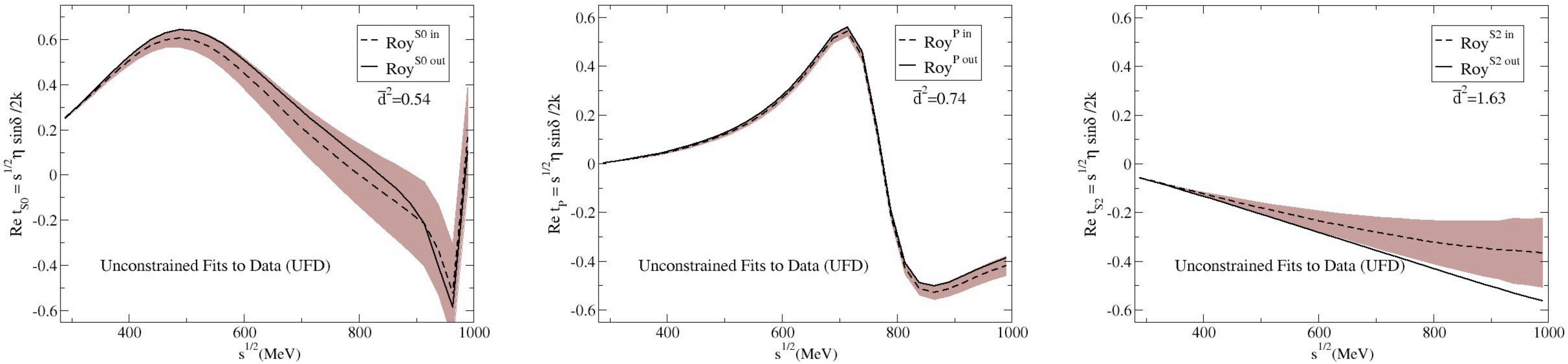
Approach

- A resonance corresponds to a pole on the second Riemann sheet of the complex plane S matrix.
- As it is well known, a pole on the second Riemann sheet corresponds to a zero on the first sheet.
- Thus, we just look for the S-matrix zeroes on the first sheet.
- The extension of the amplitudes from the real axis to the complex plane is provided by the Roy equations, which are a set of coupled integral equations for the partial wave amplitudes.
- Their domain of validity is proven to cover the sigma region
[Caprini, Colangelo, Leutwyler (2006)]

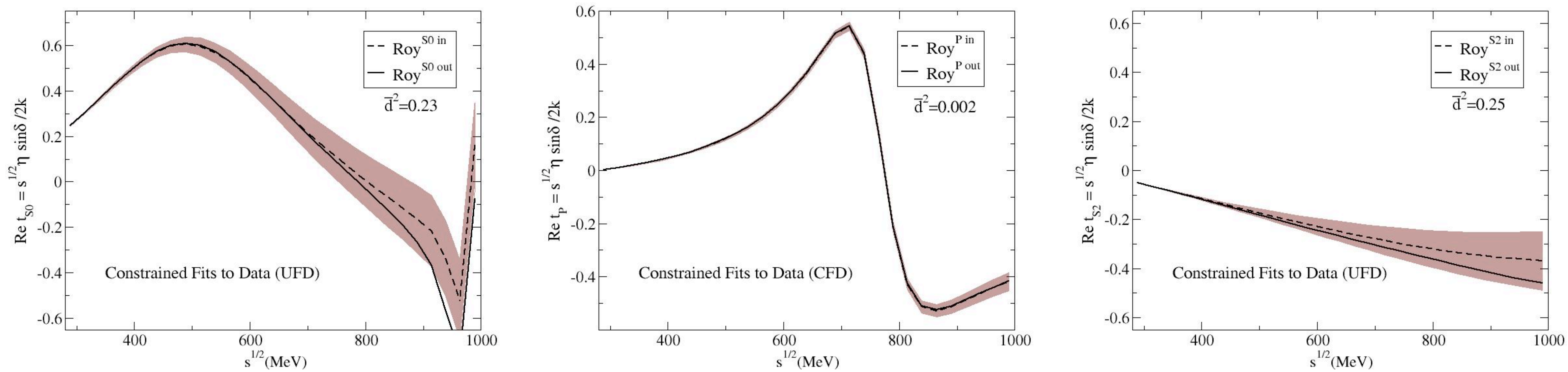
Approach

- Also, FDR are relevant due to good positivity properties (which lead to small uncertainties) and because they can be used to constraint data up to higher energies ($E \leq 1420$ MeV).
- Note that the input for Roy equations is <u>data on the real axis</u> ONLY.
- Both the Unconstrained and Constrained Fits to Data describe data well.
- CFD more reliable.

UFD: Independent data fits to data for each wave



CFD: Fits to all waves constrained to satisfy FDR + Roy Equations on the real axis



CDF very consistent!

Results (preliminary)

- Use Roy equations to go to the complex plane and find the poles:

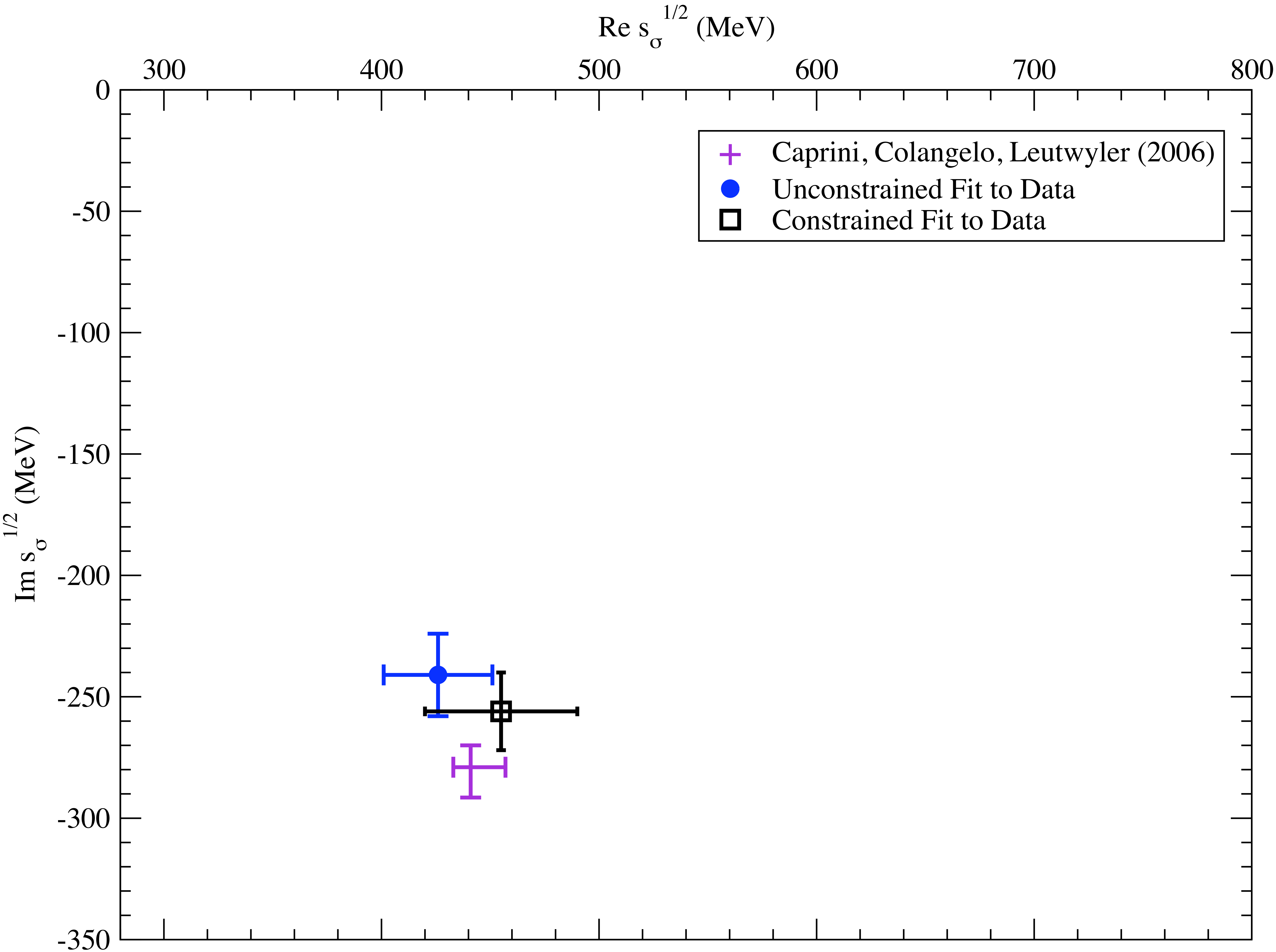
$$\text{UFD: } (426 \pm 25) - i\,(241 \pm 17)\ \text{MeV}$$

$$\boxed{\text{CFD: } (456 \pm 36) - i\,(256 \pm 17)\ \text{MeV}}$$

- This is a pure data dispersive analysis.
- Errors are still large and subject to further improvement (in progress).
- However, results are very compatible with each other and with theoretical predictions such as the one from ChPT by Caprini, Colangelo and Leutwyler:

$$\text{CCL: } 441^{+16}_{-8} - i\,272^{+9}_{-12.5}\ \text{MeV}$$

which has smaller errors due to the smaller uncertainties in the ChPT prediction of the scattering lengths.



Re $s_\sigma^{1/2}$ (MeV)
300
400
500
600
700
800
0
-50
-100
-150
-200
-250
-300
-350
Im $s_\sigma^{1/2}$ (MeV)
Caprini, Colangelo, Leutwyler (2006)
Unconstrained Fit to Data
Constrained Fit to Data

Results (in progress)

NEW RESULT (see talk by R. Kamiński):

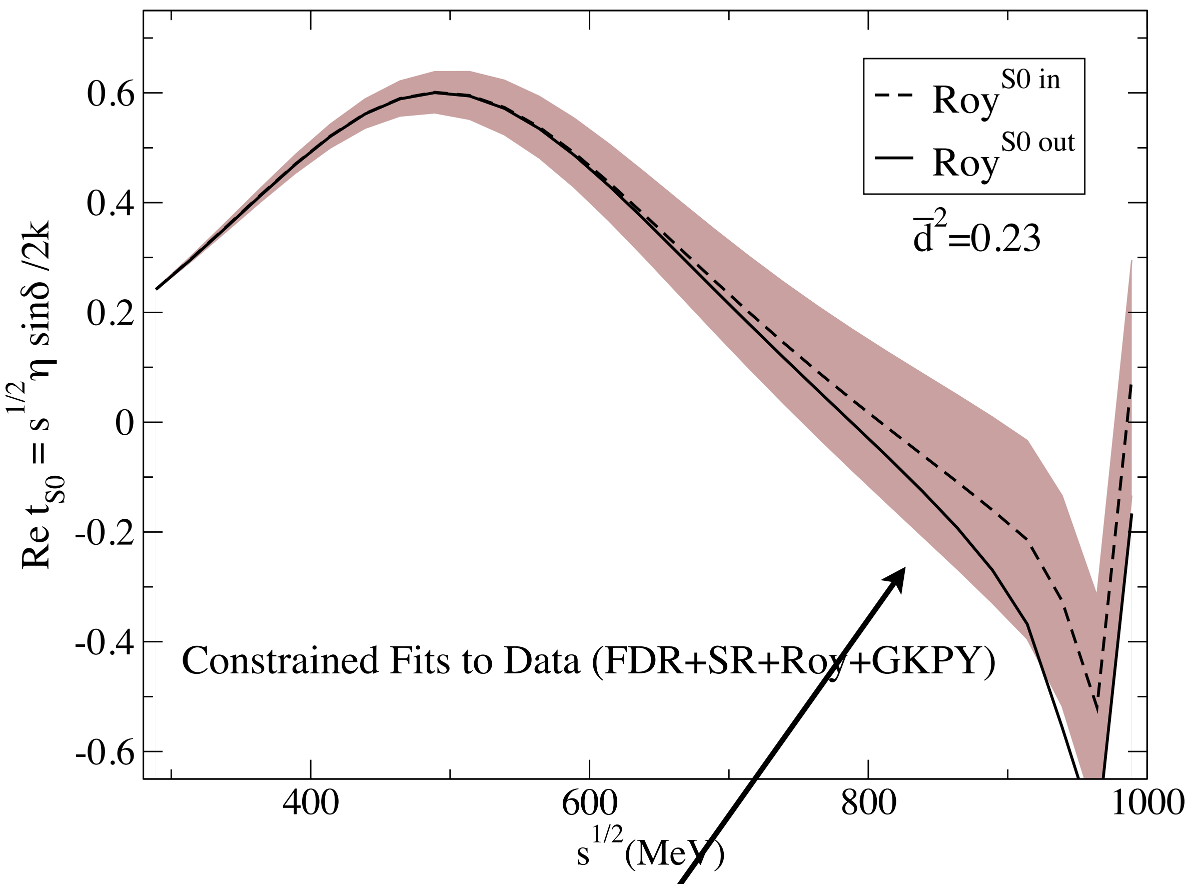
- Together with Kaminski, Peláez and Ynduráin, we have derived a new set of Roy-like equations, but with only 1 subtraction.
- The propagation of uncertainties coming from data fits has a different behaviour than in the standard (twice subtracted) Roy equations: for the same input, the uncertainties in these new equations are:

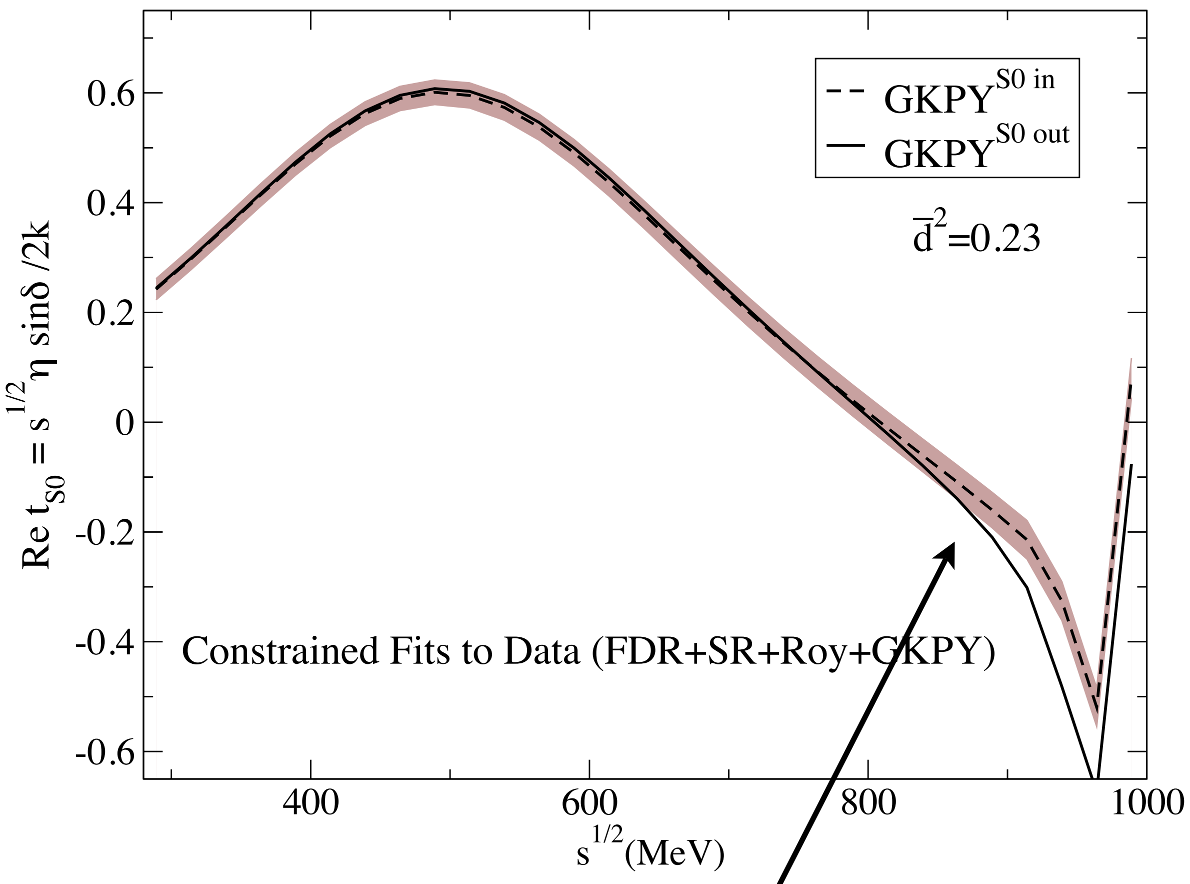
  larger than for Roy equations for $E \leq 350$ MeV

  much smaller than Roy equations for $E \geq 400$–500 MeV

- IN PROGRESS: New fit to data, constrained to fulfill not only FDR and Roy equations, but also the new GKPY equations.

In progress (very preliminary results): New fits including GKPY equations





Bigger errors, propagated from scattering lengths due to the term in twice subtracted Roy equations:

$$\sim (2a_0^0 - 5a_0^2)(s - 4M_\pi^2)$$

Smaller errors in the intermediate and high energy region, due to the way errors are propagated (not so sensitive to uncertainties in the scattering lengths)

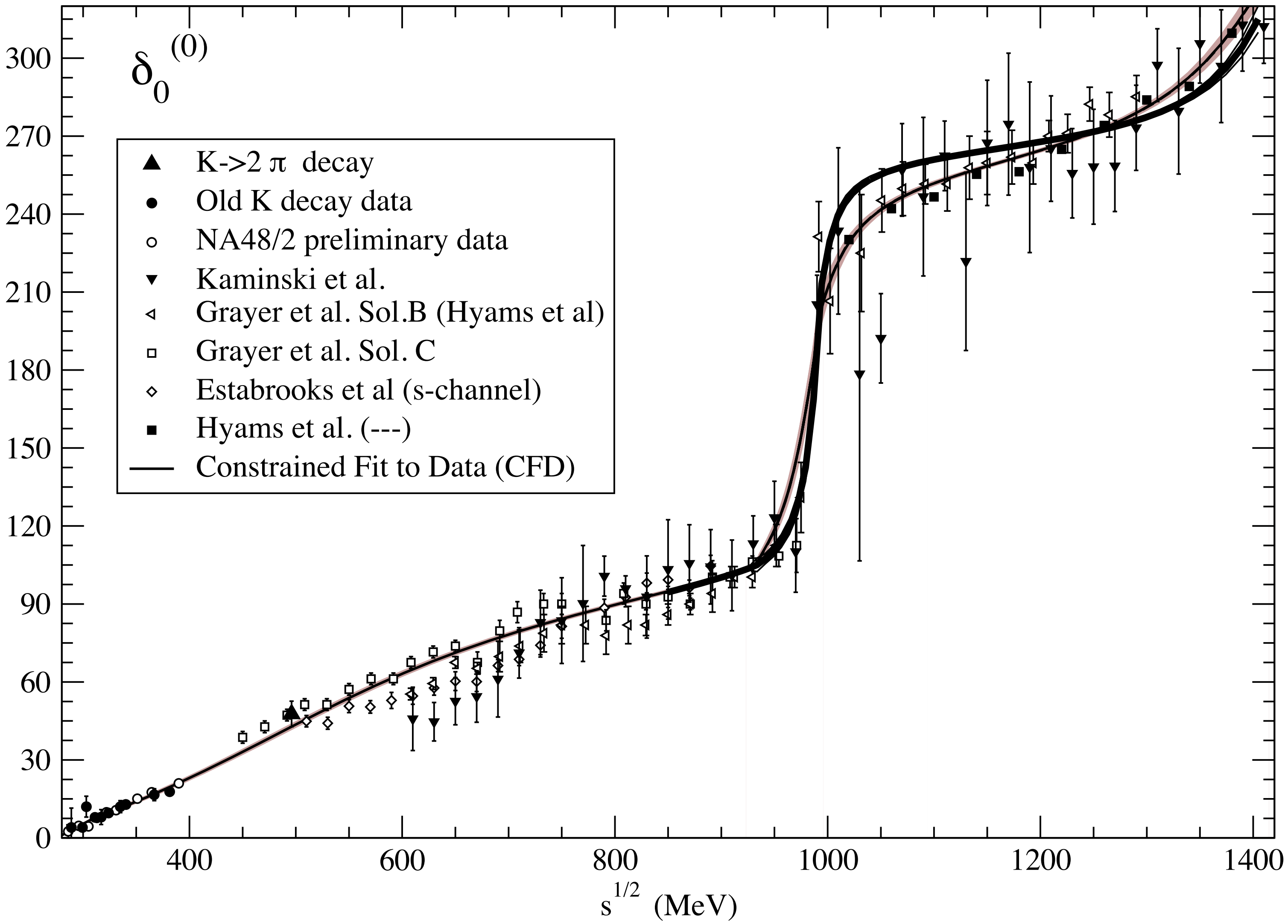
Forward Dispersion Relations, Roy equations and GKPY equations satisfied simultaneously below ~ 850 MeV

IN PROGRESS:

Sigma pole with new CFD and GKPY equations will improve errors:
(458 ± 15) – i (262 ± 15) MeV (very preliminary)

$$\text{CFD: } (456 \pm 36) - i\,(256 \pm 17)\ \text{MeV}$$

Some more work is needed on the f0(980) region—matching of the K-matrix with the low energy conformal expansion. But preliminary results tell us the pole position is quite stable.



Preliminary fit for the K-matrix with better matching.

Fullfills FDR, Roy and GKPY better, but at the price of a slight increase of the data fit's chi-squared.

Still, the sigma pole doesn't move much:
(461 ± 14) - i (255 ± 15) MeV

Conclusions

- We have obtained the sigma pole position **FROM DATA**, using a model independent dispersive approach based on fits to data constrained to satisfy Roy equations and Forward Dispersion Relations.

- We obtain: **M = 456 ± 36 MeV** and **Γ/2 = 256 ± 17 MeV**.

- We are **improving on the uncertainties** by using a **new set of Roy-like equations** with only one subtraction, which we expect will reduce uncertainties by a factor of 2.

- The K-matrix region of the fit can be improved. We don't expect this to have a big influence on the pole position, but could improve uncertainties.

Thanks!

We open a parenthesis to present a simple parametrization for approximating the results from Roy equations

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The conformal expansion

- The most correct way of going to the complex plain is by using Roy equations (or GKPY equations). However, dealing with the whole set of equations is tedious and complicated.
- If one needs to use our parametrizations, there exists a simple approximate solution, very easy to handle: the conformal expansion.
    - Model independent parametrization of experimental data at low energies.
    - Only based on elasticity and unitarity.
    - Describes experimental data accurately with few parameters.

Elastic partial wave amplitude:

$$t^{el}(s) = \frac{1}{\sigma(s)} \frac{1}{\cot\delta(s) - i} = \frac{1}{\psi(s) - i\sigma(s)}$$

Phase space factor: contains elastic cut

Phase shift

Effective range function

$\psi(s)$ does **not** have an elastic cut

Usual way: Series expansion in powers of the momentum

$$\psi(s) = f(s) \times \left(a + bk^2 + ck^4 + \ldots\right)$$

Problem: Series convergence is limited to E < 396 MeV

A solution: The following maps the entire uncut complex plane inside the unit circle:

$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}, \qquad s_0 : \text{ first inelastic threshold}$$

so that the expansion $\psi(s) = f(s) \times \left(b_0 + b_1\omega(s) + b_2\omega(s)^2 + \ldots\right)$

absolutely and uniformely converges on the whole uncut complex plane.

The conformal expansion

- Difference between constrained dispersive approach and conformal expansion calculated with THE SAME FIXED INPUT (this substracts statistical errors in input):

  Constrained to FDR+Roy+GKPY eqs.: $(458 \pm 15) - i\,(262 \pm 15)$ MeV
  Conformal expansion: $(478 \pm 17) - i\,(262 \pm 7)$ MeV

- The systematic error of the conformal expansion is:

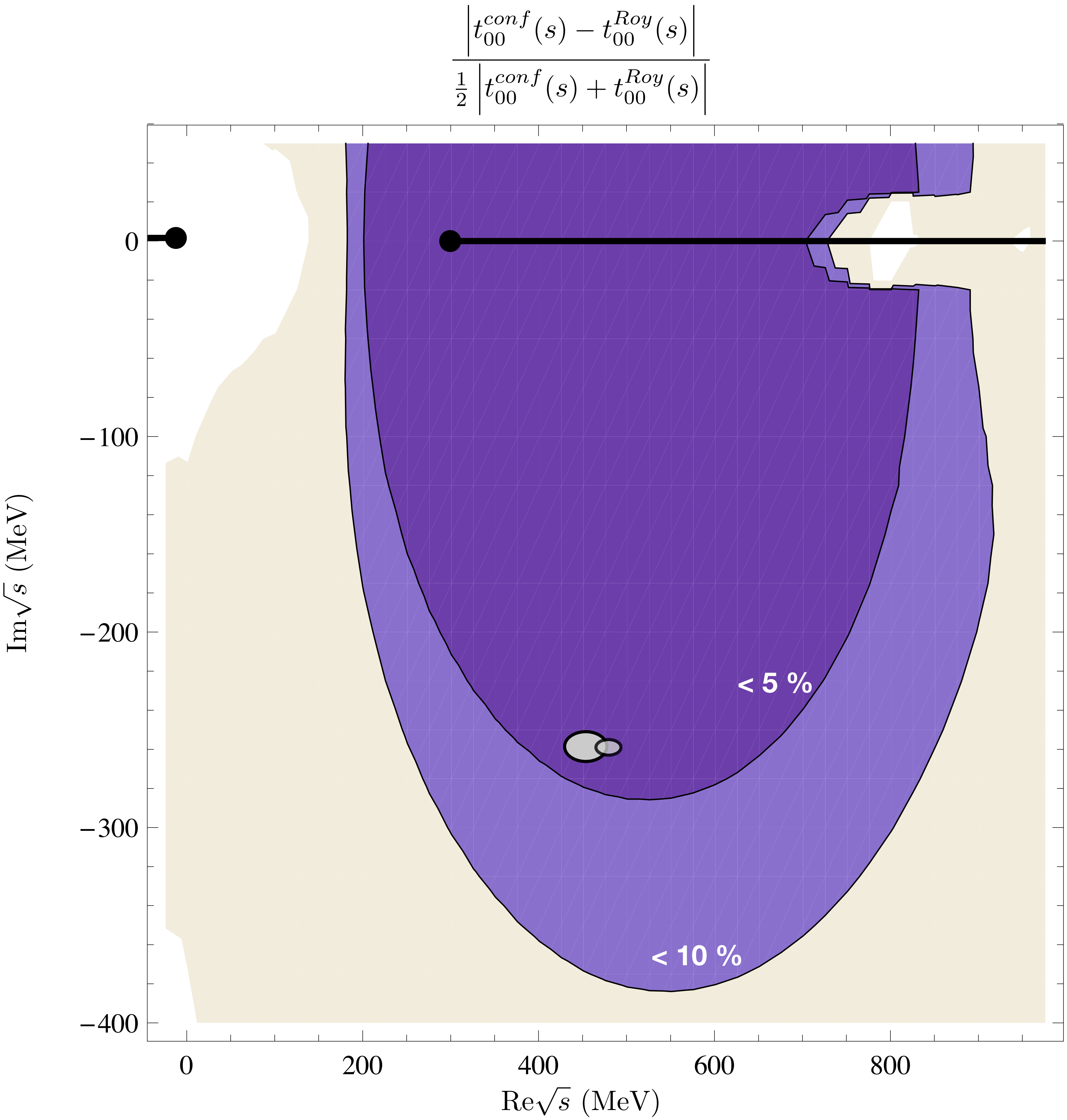
$$\Delta M \text{ (syst.)} = \pm 20 \text{ MeV}, \; \Delta\Gamma/2 \text{ (syst.)} = \pm 8 \text{ (MeV)}.$$

- In our previous work we estimated:

$$\Delta M \text{ (syst.)} = \pm 11 \text{ MeV}, \; \Delta\Gamma/2 \text{ (syst.)} = \pm 2 \text{ (MeV)}.$$

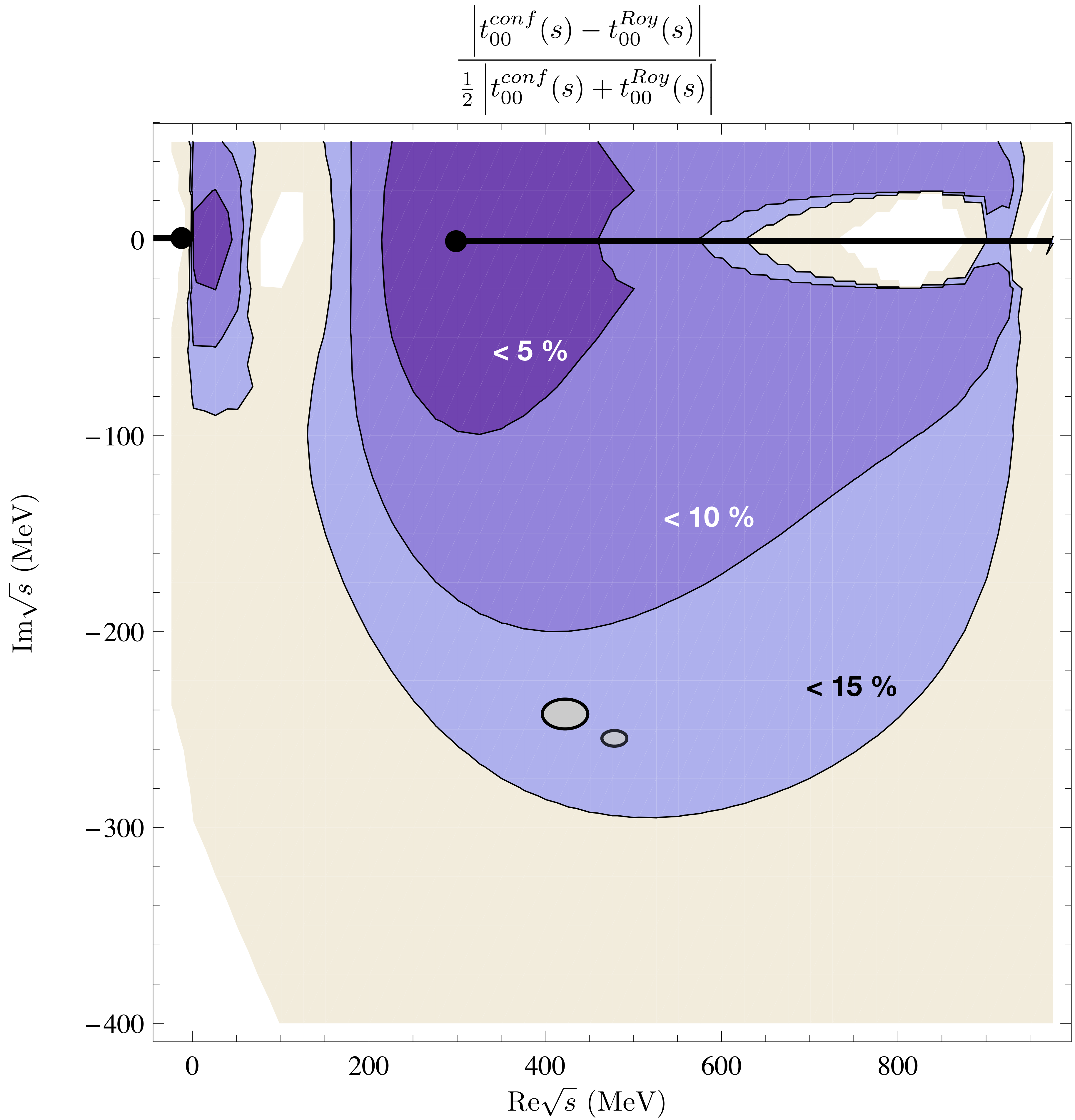
- By actual calculation we find systematic errors bigger than our previous estimation. Still, they are not as big as the values suggested yesterday by I. Caprini.
- In fact, deviation from Roy equations is less than 5 % in the region of interest.

CFD

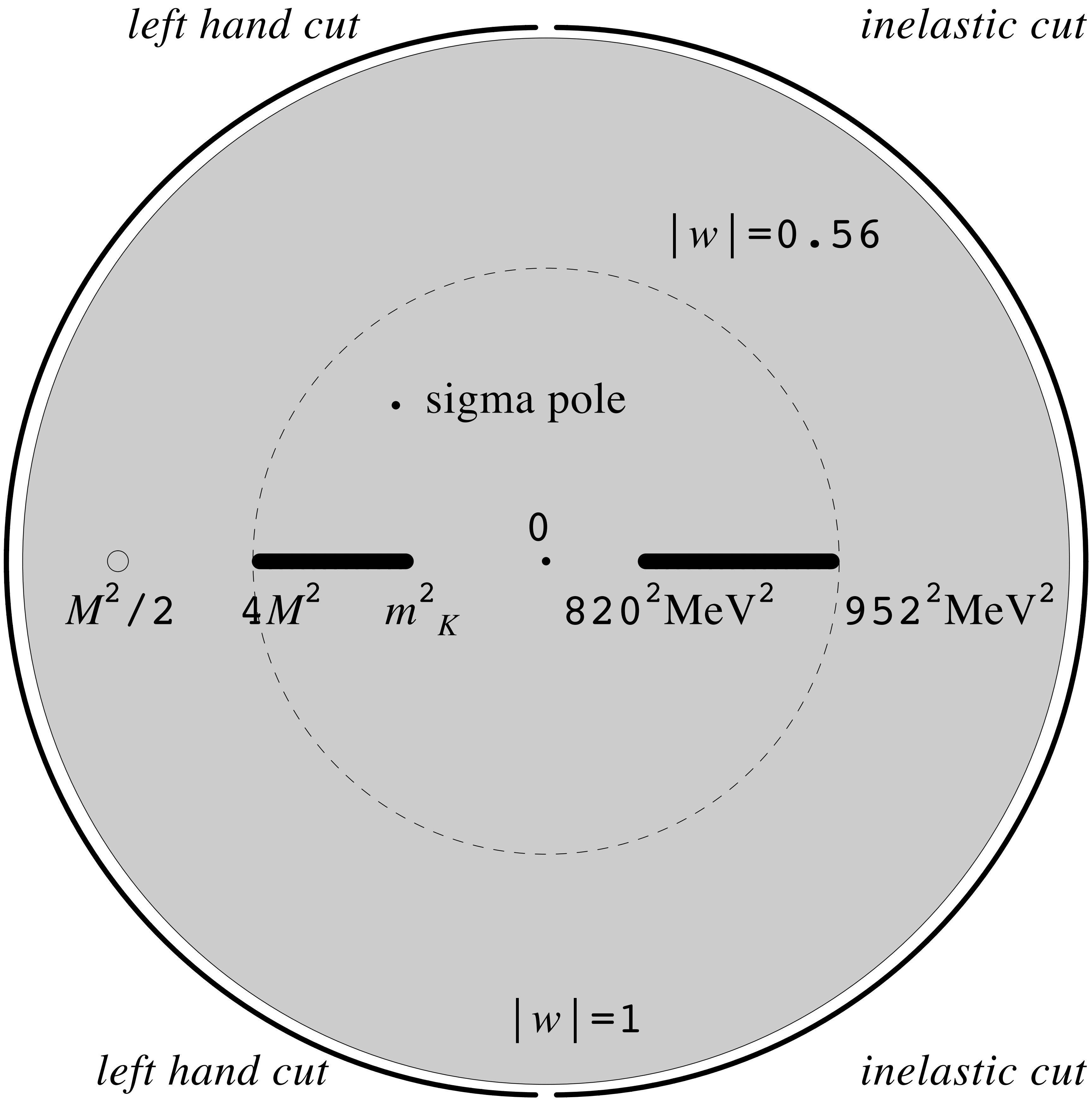

$$\frac{\left|t_{00}^{conf}(s) - t_{00}^{Roy}(s)\right|}{\frac{1}{2}\left|t_{00}^{conf}(s) + t_{00}^{Roy}(s)\right|}$$
< 5 %
< 10 %
0
−100
−200
−300
−400
0
200
400
600
800
Im$\sqrt{s}$ (MeV)
Re$\sqrt{s}$ (MeV)

)

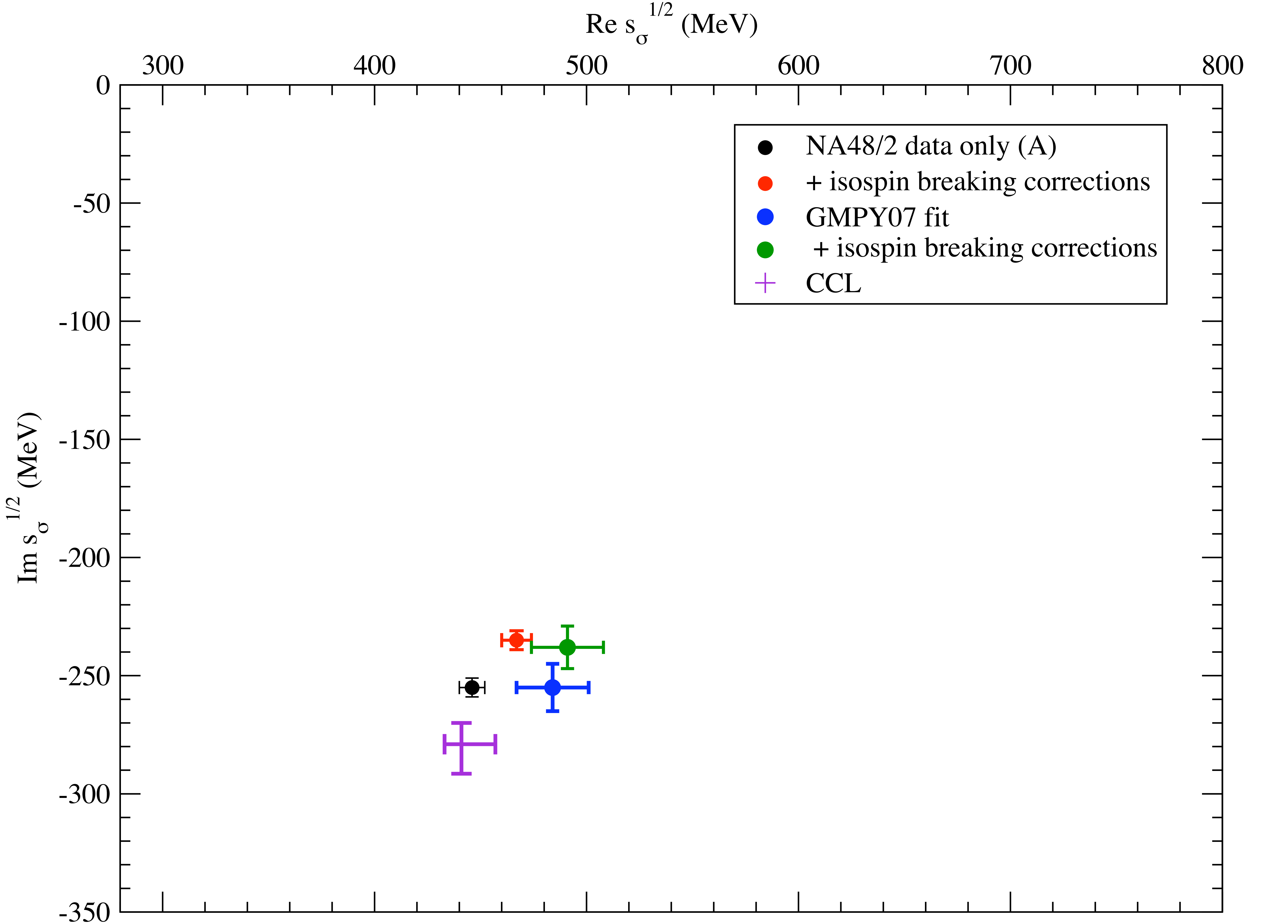
UFD



$$\frac{\left|t_{00}^{conf}(s) - t_{00}^{Roy}(s)\right|}{\frac{1}{2}\left|t_{00}^{conf}(s) + t_{00}^{Roy}(s)\right|}$$
< 5 %
< 10 %
< 15 %
0
−100
−200
−300
−400
0
200
400
600
800
Re$\sqrt{s}$ (MeV)
Im$\sqrt{s}$ (MeV)



left hand cut
inelastic cut
|w|=0.56
sigma pole
0
M^2/2
4M^2
m^2_K
820^2MeV^2
952^2MeV^2
|w|=1
left hand cut
inelastic cut


Re s_σ^{1/2} (MeV)
300
400
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600
700
800
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-300
-350
Im s_σ^{1/2} (MeV)
NA48/2 data only (A)
+ isospin breaking corrections
GMPY07 fit
+ isospin breaking corrections
CCL

Conclusions

- We have obtained the sigma pole position **FROM DATA**, using a model independent dispersive approach based on fits to data constrained to satisfy Roy equations and Forward Dispersion Relations.
- We obtain: **M = 456 ± 36 MeV** and **Γ/2 = 256 ± 17 MeV**.
- We are **improving on the uncertainties** by using a **new set of Roy-like equations** with only one subtraction, which we expect will reduce uncertainties by a factor of 2.
- For simple phenomenological applications, given the same input on the elastic region, the **conformal expansion** provides a good approximation within a 5% uncertainty both in the real axis and in the sigma pole region.

SUMMARY SO FAR

Sigma pole with previous CFD and Roy equations:

(456 ± 36) – i (256 ± 17) MeV   (preliminary)

IN PROGRESS:

Sigma pole with new CFD and GKPY equations will improve errors:

(458 ± 15) – i (262 ± 15) MeV   (very preliminary)

Some more work is needed on the f0(980) region and K-matrix matching with the conformal expansion

Asymmetric errors