Experimental studies of K_{e4} and $K_{3\pi}$ decays from the NA48/2 experiment at CERN
Interpretation in terms of $\pi\pi$ scattering lengths

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On behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

NA48/2 data taking: completed

A view of the NA48/2 beam line

2003 run: ~ 50 days
2004 run: ~ 60 days

Total statistics in 2 years:

$K^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$: $\sim 4 \cdot 10^{9}$

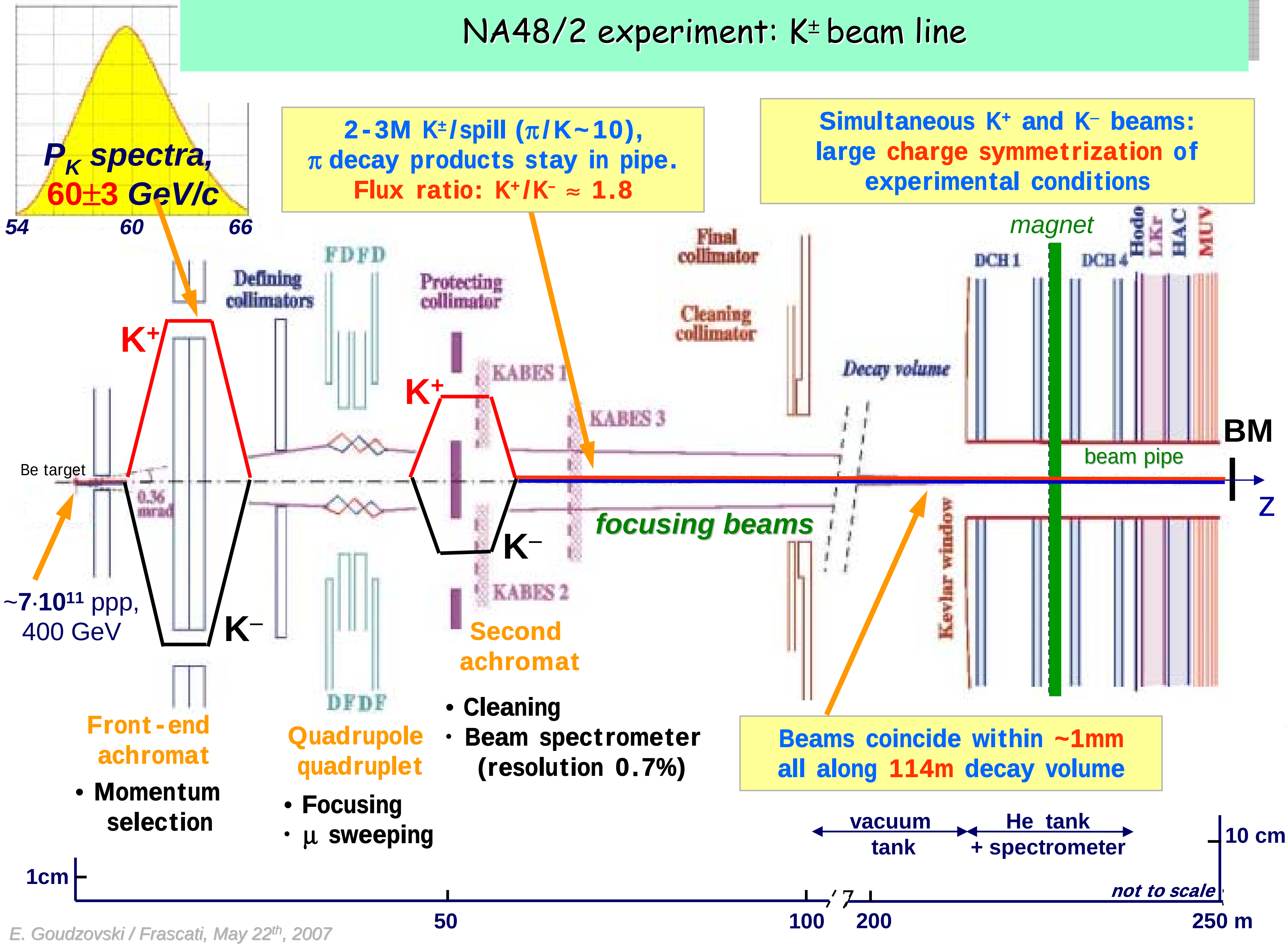
$K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$: $\sim 1 \cdot 10^{8}$

$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$: $\sim 1 \cdot 10^{6}$

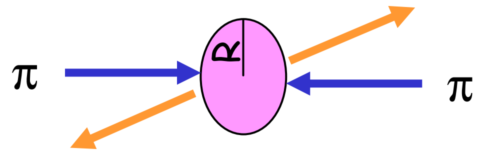
Rare $K^{\pm}$ decays:
BR's down to 10^{-9}
can be measured

>200 TB of data recorded

NA48/2 experiment: K± beam line



$\pi\pi$ scattering lengths: why interesting?



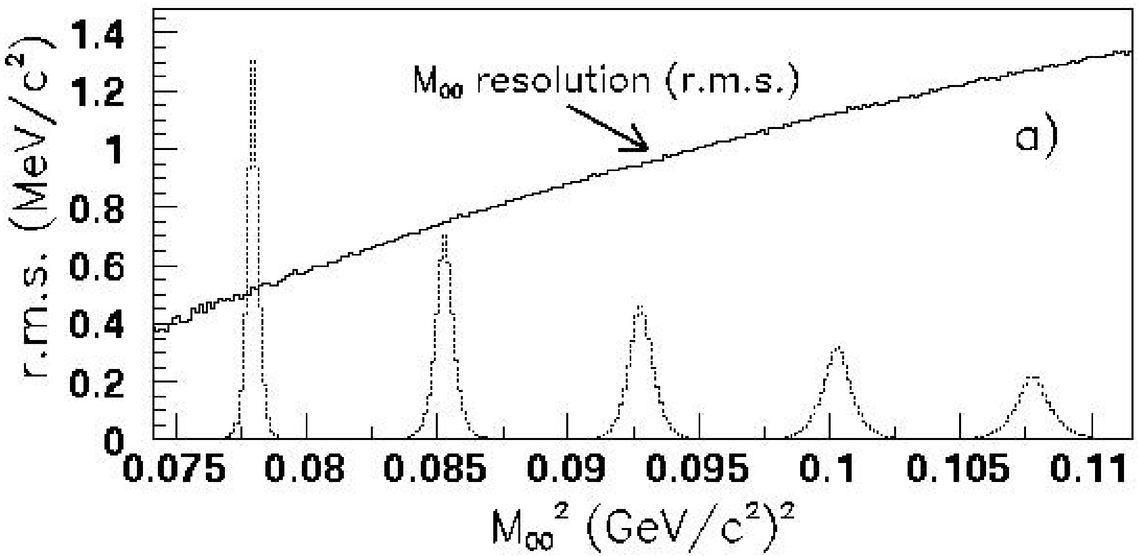
$$k = \frac{\sqrt{2mE}}{\hbar}$$

- at low energy kR <<1 ⇨ S-wave dominates the total cross section
- furthermore Bose statistics ⇨ isospin I = 0, 2 allowed
- scattering matrix ⇨ $S|\pi\pi\rangle = \exp(2i\delta)|\pi\pi\rangle$

  i.e. parameterization with 2 phases: $\delta_{0,2} = a_{0,2}\, k$ related to scattering lengths a_0, a_2
- 2 clean measurements of scattering lengths a_0, a_2
  - cusp-effect in $K^\pm \to \pi^\pm\pi^0\pi^0$ decays
  - $\delta_{\pi\pi}$ and form factors in $K^\pm \to \pi^\pm\pi^- e^+\nu$ decays
- at low energy the *S*-wave scattering lengths a_0, a_2 are the essential parameters of χPT

$\pi\pi$ re-scattering in $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ decays

Initial motivation was to observe $\pi^+\pi^-$ atoms ("pionium") formation in $K^\pm \to \pi^\pm\pi^+\pi^-$ decays, followed by pionium annihilation to $\pi^\circ\pi^\circ$
Note M_{00} resolution σ = 0.56 MeV/c² at $M_{00} = 2m_+$ (2x139.57)



$M_{\pi^\circ\pi^\circ} \equiv M_{00}$ resolution optimized for low M_{00} values

What did happen? → The unexpected discovery of $\pi\pi$ re-scattering in high-statistics sample of $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ decays

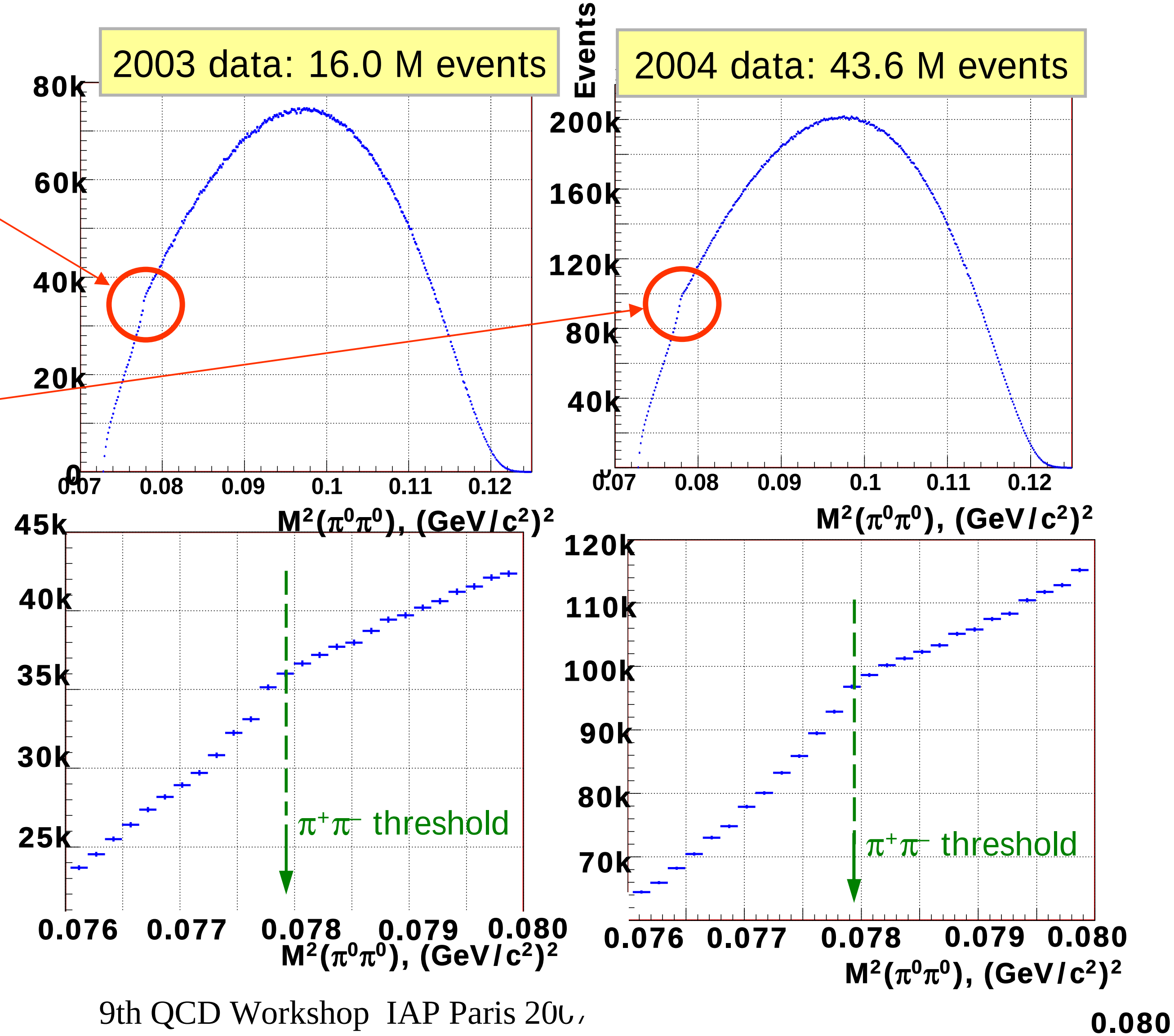
Observation of cusp in the $\pi^\circ\pi^\circ$ invariant mass distribution at $M_{\pi^\circ\pi^\circ} = 2m_{\pi+}$

First observation of cusp made with the 2003 data

Inclusion of 2004 data: increased statistics by factor 3.7 !

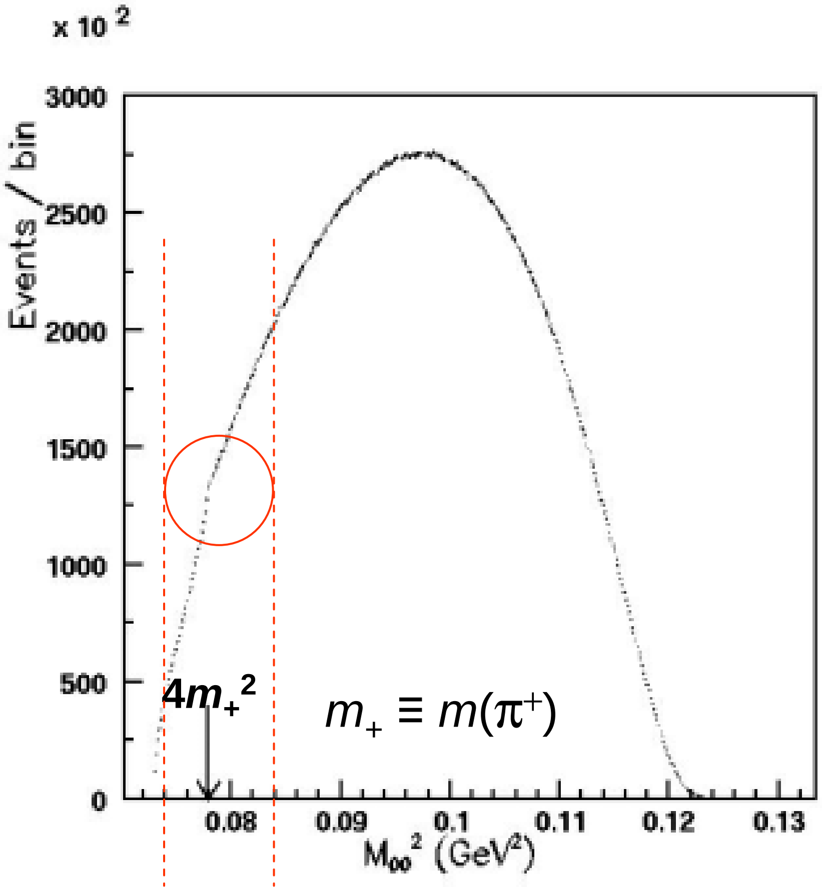
2003 and 2004 data: slightly different selection conditions and different data/MC sample ratios.

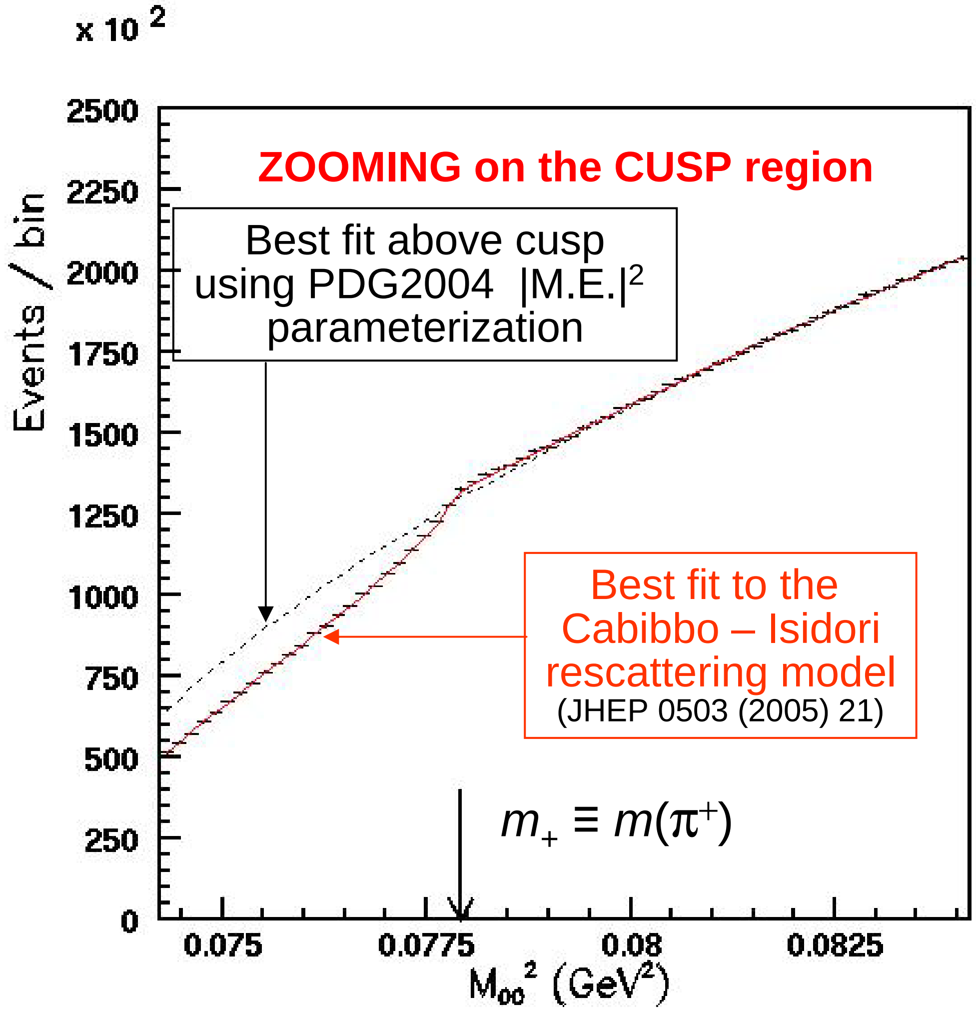
Analysis made independently for the 2 data sets, and then the results are averaged.



Cusp NA48/2 (preliminary)

59,624,170 fully reconstructed $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ events





Below threshold: destructive interference between virtual $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ states followed by $\pi^{+}\pi^{-}\ \pi \rightarrow \pi^{\circ}\pi^{\circ}$ and the $K^{\pm} \rightarrow \pi^{\pm}\ \pi^{\circ}\pi^{\circ}$ amplitude. Integrated event deficit ~13%

Theory: final state rescattering. N. Cabibbo, PRL 93 (2004) 121801

$$M(K^\pm \to \pi^\pm\pi^0\pi^0) = M_0 + M_1$$

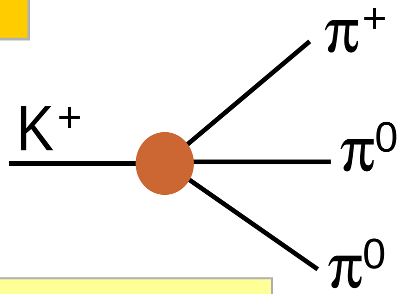
Direct emission:

$$M_0 = A_0(1+g_0 u/2+h' u^2/2+k' v^2/2)$$

Kaon rest frame:

$u = 2m_K \cdot (m_K/3 - E_{odd})/m_\pi^2$

$v = 2m_K \cdot (E_1 - E_2)/m_\pi^2$



Rescattering amplitude:

$$M_1 = -2/3(a_0 - a_2)m_+ M_+ \sqrt{1-\left(\frac{M_{00}}{2m_+}\right)^2}$$

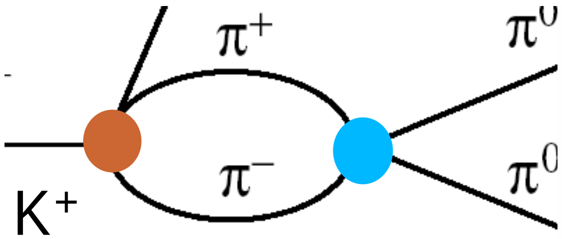
Negative interference under threshold

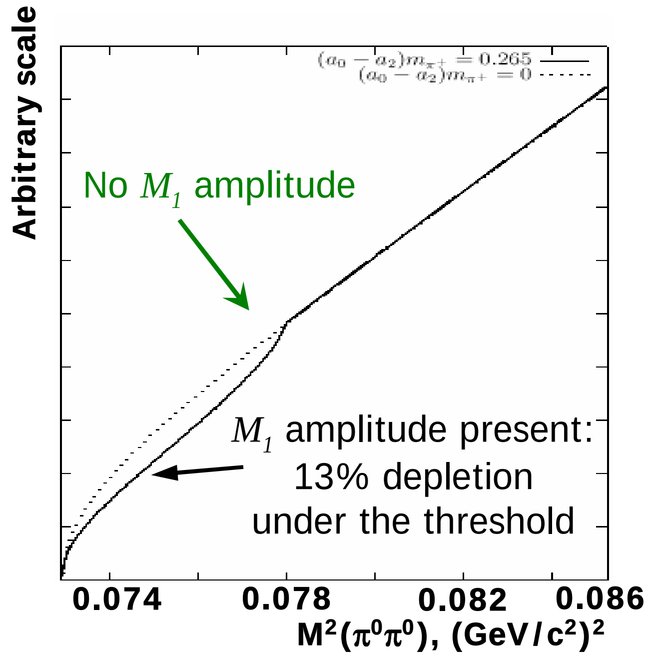
Combination of S-wave ππ scattering lenghts

(isospin symmetry assumed here)

$K^\pm \to 3\pi^\pm$ amplitude at threshold

1 loop theory not sufficient for accurate description of the data!

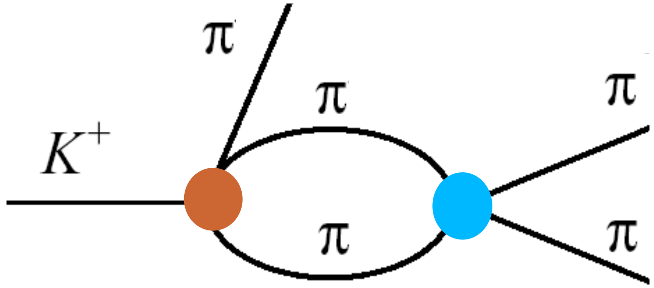




B. Peyaud

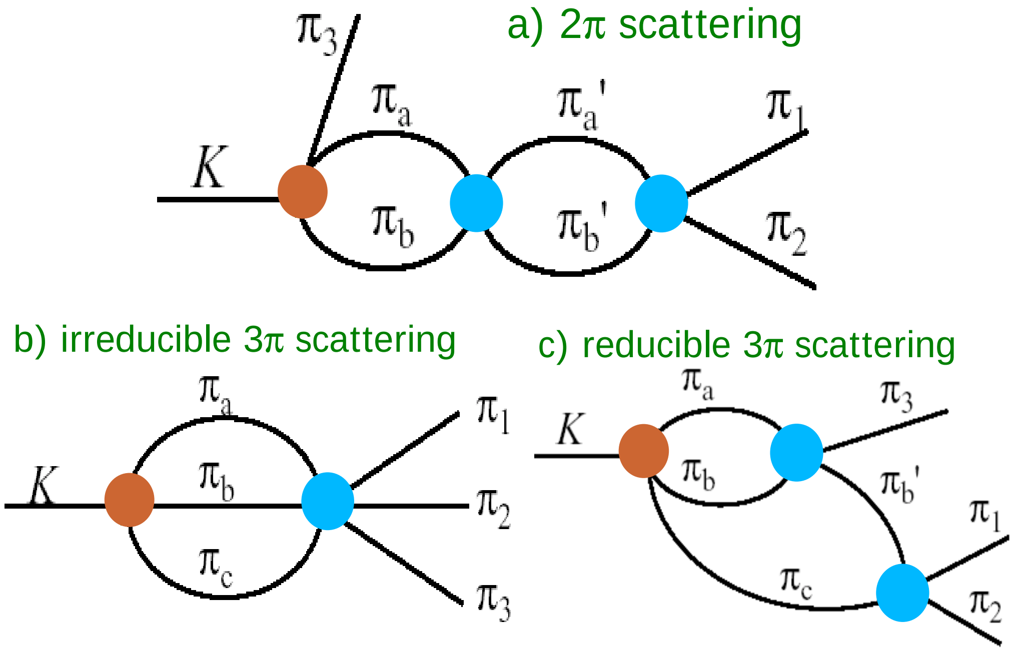
Theory: two-loop diagrams N. Cabibbo and G. Isidori, JHEP 503 (2005) 21

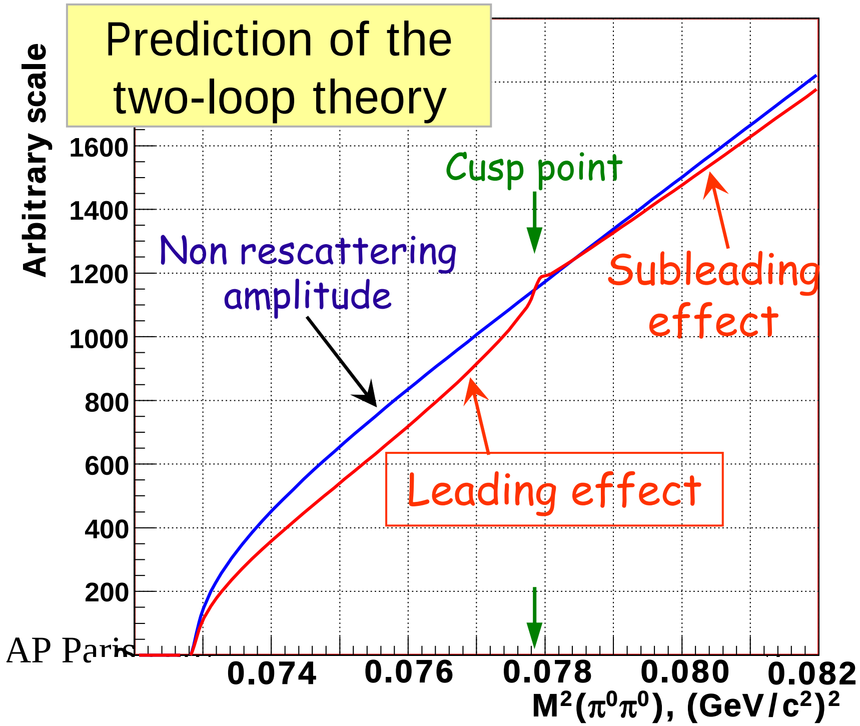
One-loop diagrams:



- rescattering processes at two-loop level included
- S-wave scattering lengths (a_x, a_{++}, a_{+-}, a_{+0}, a_{00}) are functions of a_0 and a_2
- Isospin symmetry breaking following J. Gasser e.g. $a_x = (1+\varepsilon/3)(a_0-a_2)/3$, where $\varepsilon=(m_+^2-m_0^2)/m_+^2=0.065$ is isospin breaking parameter
- Radiative corrections missing: (a_0-a_2) precision ~5%

Two-loop diagrams:

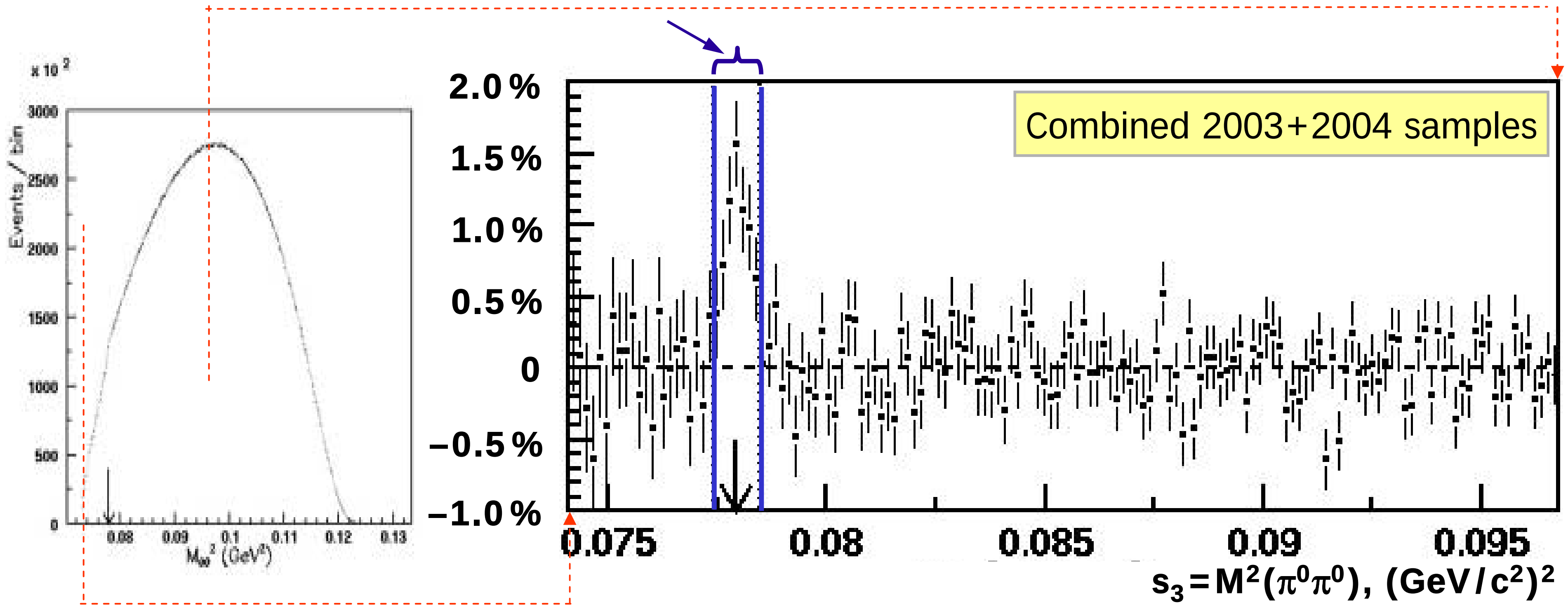




Fit quality and pionium signature

7 data bins currently excluded from the fit
EM corrections not yet included in the theoretical model

Theory describes the data!



A number of alternative theoretical approaches developed to describe final-state scattering and formation of bound $\pi^+\pi^-$ states near threshold, work going on…

Results and uncertainties

preliminary

NA48/2 result with 2003+2004(80%) statistics

$$(a_0-a_2)m_+ = \quad 0.261 \pm 0.006_{stat.} \pm 0.003_{syst.} \pm 0.0013_{ext.}$$
$$a_2 m_+ = -0.037 \pm 0.013_{stat.} \pm 0.009_{syst.} \pm 0.0018_{ext}$$

- External uncertainty: due to $R = (A_{++-}/A_{+00})|_{threshold} = 1.975 \pm 0.015$;
- Theory precision (rad.corr. & higher order terms neglected): $\delta(a_0-a_2)m_+ = 0.013$

measurement has still large theoretical errors;
complementary to DIRAC measurement with of pionium lifetime

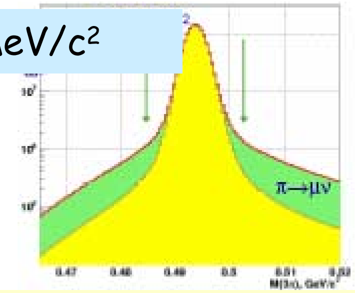
Systematic effect	$(a_0-a_2)\times 10^2$	$a_2\times 10^2$
Analysis technique	±0.10	±0.20
Trigger inefficiency	negl.	±0.50
Description of resolution	±0.06	±0.11
LKr non-linearity	±0.06	±0.26
Geometric acceptance	±0.02	±0.01
MC sample	±0.03	±0.21
Simulation of LKr showers	±0.17	±0.50
V-dependence of amplitude	±0.17	±0.38
Total	±0.28	±0.90

$\pi\pi$ scattering in $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ decays (Ke4)

- K_{e4} kinematic variables and form factors
- event selection, reconstruction and physics parameters measurement
- K_{e4} results : form factors and phase shift $\delta_{\pi\pi}$
- interpretation in terms of $\pi\pi$ scattering lengths
- in $\pi\pi$ scattering process, dispersion relations (Roy equations) relate amplitudes with different Isospin ⇨ dependence essentially on two parameters, the scattering lengths a_0^0 and a_0^2
- predictions from χPT for $\pi\pi$ interaction at low energy introduce further constrains between a_0^0 and a_0^2 which are related to the size of the quark condensate.

Important detector components for Ke4 analysis

$\sigma = 1.7$ MeV/c^2

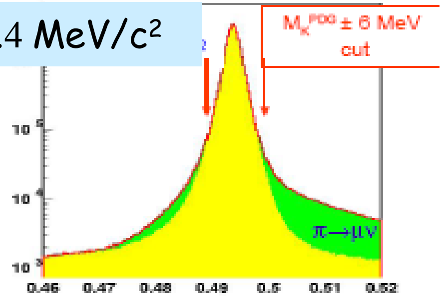


$(\pi^{\pm}\,\pi^{+}\pi^{-})$ mass GeV/c^2

Magnetic spectrometer : 4 high-resolution DCH's
$\Delta p/p = (1.0 \oplus 0.044\, p)\%$ (p in GeV/c)
→ Very good resolution for charged invariant masses

LKr electromagnetic calorimeter : quasi-homogenous and high granularity
$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)
$\sigma_x = \sigma_y \sim 1.5$ mm for E=10 GeV
→ Very good resolution for neutrals (π0)
→ E/p ratio for e/π discrimination

$\sigma = 1.4$ MeV/c^2



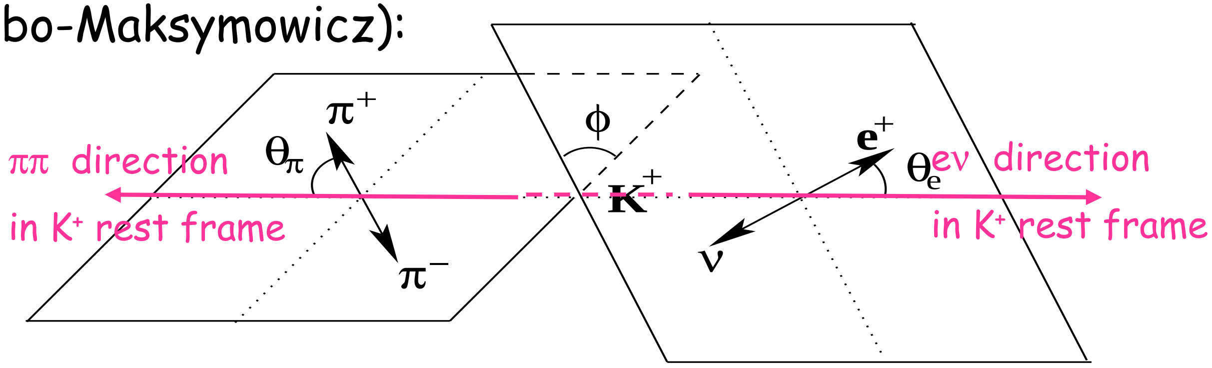
$(\pi^{\pm}\,\pi^{0}\pi^{0})$ mass GeV/c^2

Ke4 charged decays : formalism

5 kinematic variables (Cabibbo-Maksymowicz):

$S_\pi = M^2_{\pi\pi}$, $S_e = M^2_{e\nu}$,

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



partial wave expansion of the amplitude:

F, G = Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi + \text{d-wave term...}$

$G = G_p e^{i\delta g} + \text{d-wave term...}$

H = Vector Form Factor

$H = H_p e^{i\delta h} + \text{d-wave term...}$

expansion in powers of q^2 , $Se/4m\pi^2$

$(q^2 = (S_\pi / 4m_\pi^2 - 1))$

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e \left(S_e / 4m_\pi^2\right) + ..$$

$$F_p = f_p + f_p' q^2 + ..$$

$$G_p = g_p + g_p' q^2 + ..$$

$$H_p = h_p + h_p' q^2 + ..$$

The fit parameters are : F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

Ke4 decays: event selection and background rejection

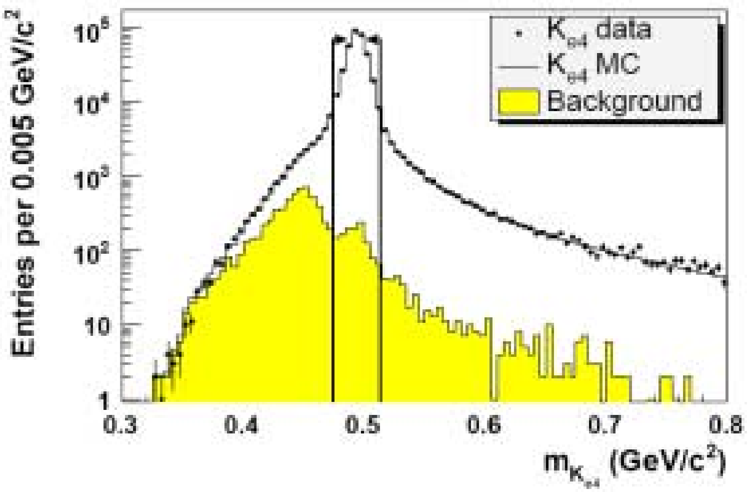
- topology = 3 charged tracks.
- Signal $\pi^+\pi^- e^{\pm}\nu$: 2 πs with opposite sign + 1 e identified with E/p, missing ν energy and pt
- Background : main sources

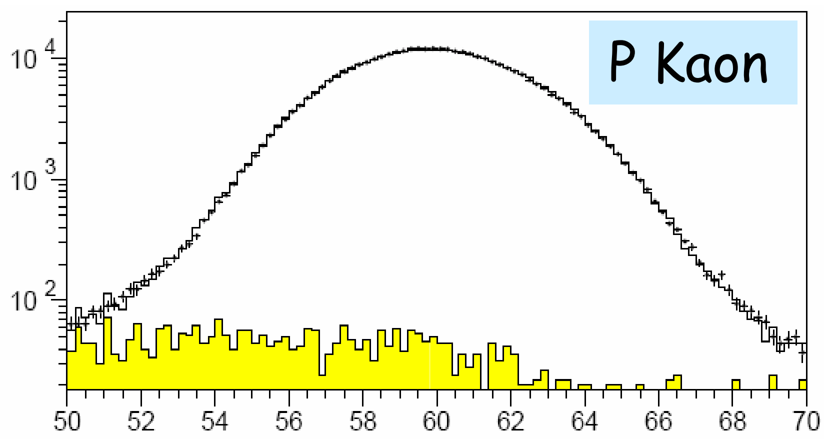
  $\pi^{\pm}\pi^+\pi^-$ decay + $\pi \rightarrow e\,\nu$ decay (dominant) or + π misidentified as e

  $\pi^{\pm}\pi^0(\pi^0)$ decay + π^0 Dalitz decay ($e^+e^-\gamma$) + e misidentified as π and γ(s) undetected
- control background with data: Wrong Sign events have same total charge as signal events but $\pi^+\pi^+e^-$ signature for K^+ decays ($\pi^-\pi^-e^+$ for K^- decays).

  Background events appear in signal (Right Sign) events with x1 or x2 WS rate

Total background level is kept at ~ 0.5 % relative level





Ke4 charged decays : 2003 Data sample

the 5D space of the cinematic variables is binned into 15000 boxes that are equally populated

10 $M_{\pi\pi}$ x5 $M_{e\nu}$ x5 $\cos\theta_\pi$ x5 $\cos\theta_e$ x12 ϕ bins ⇨15000 boxes.

form factors and δ used as parameters to minimize a log-likelihood estimator suited for small numbers of events/bin and taking into account the MC statistics (simulated and expected events/bin)

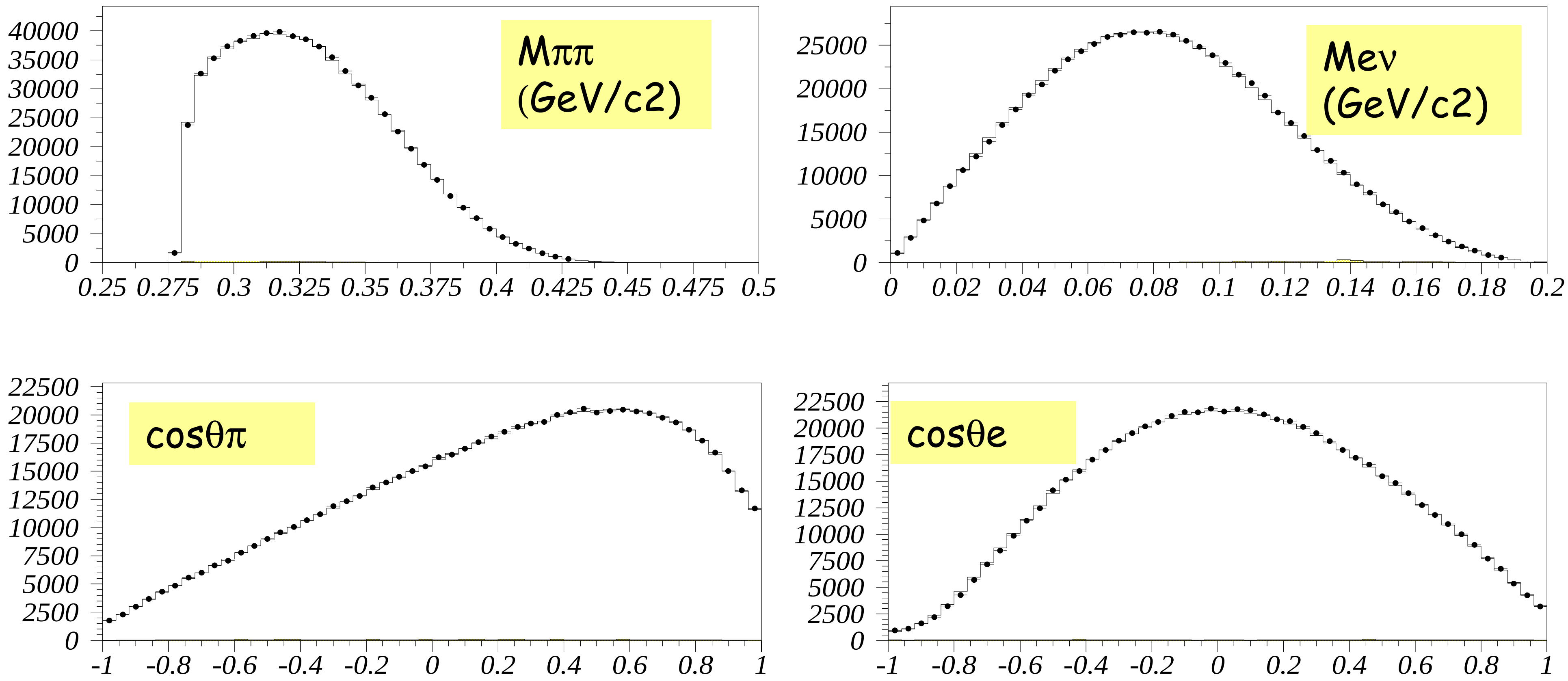
K^+ sample (435654 events)	29 events/box
K^- sample (241856 events)	16 events/box
MC K+ sample (10.0 Millions events)	~667 events/box
MC K- sample (5.6 Millions events)	~373 events/box

Ratio K+/K- ~ 1.8 both in Data and MC (run by run basis)
Ratio MC/Data ~ 23. both for K+ and K- (run by run basis)

Ke4 charged decays : mass and angular distributions

K^+ and K^- samples fitted separately, results combined

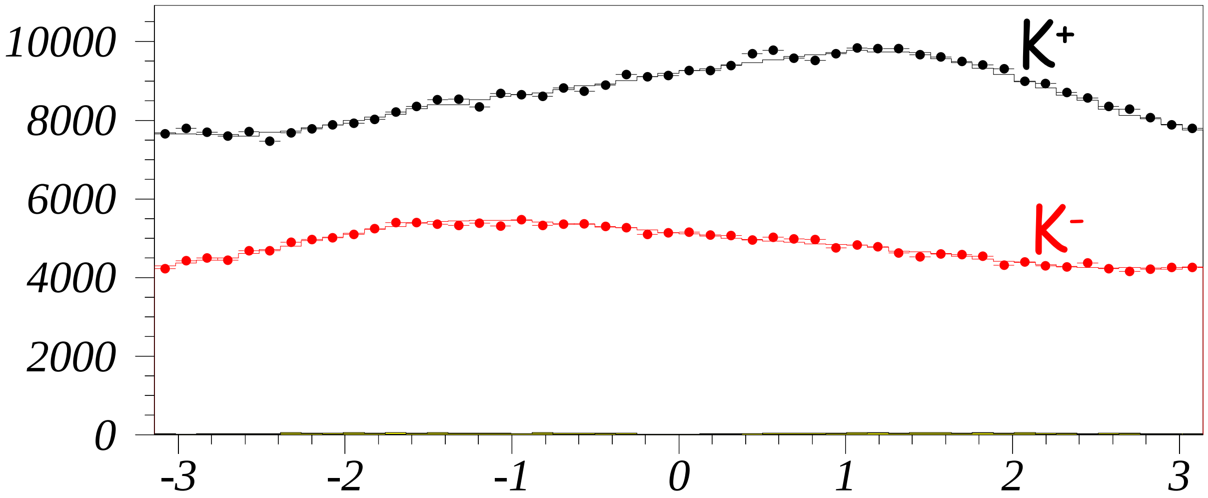
data (symbols), simulation after fit (hist.) and background (hardly visible)



Ke4 charged decays : the ϕ azimuth distributions

CP symmetry : (K+) ϕ distribution is opposite of (K-) ϕ distribution

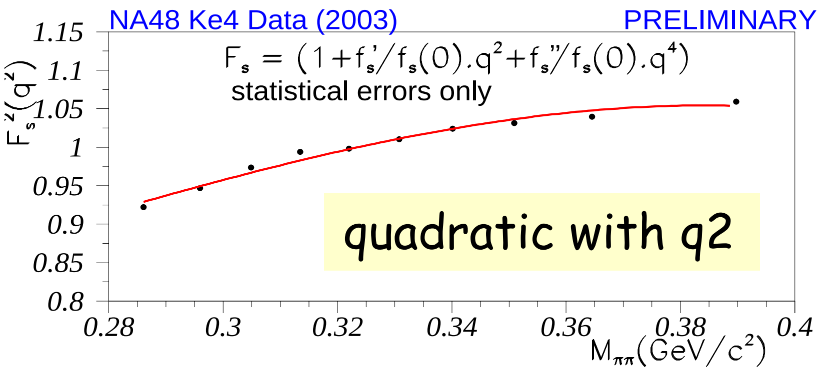
Ratio K+/K- = 1.8

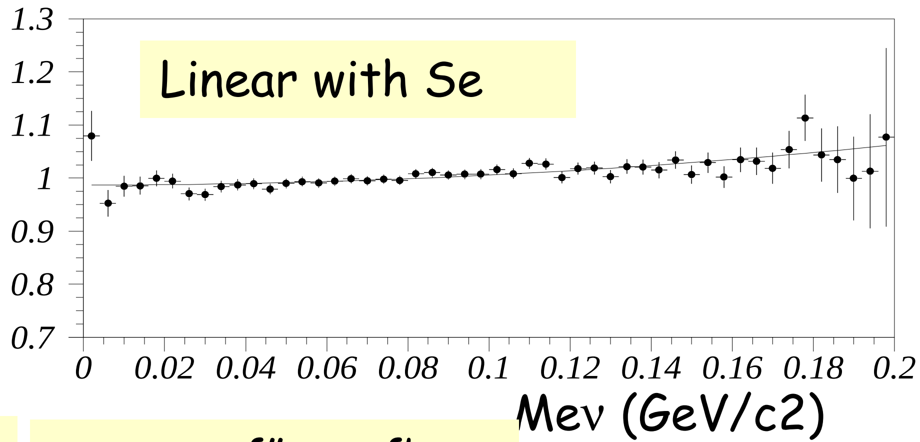


ϕ (rad)

Ke4 charged decays : getting form factors and phase shift

- independent fits in 10 $M_{\pi\pi}$ bins, assuming ~constant form factors
  ⇨ allow model independent analysis.
- relative form factors variations with q^2, q^4 ($q^2= (S_\pi/4m_\pi^2 -1)$ and $Se/4m_\pi^2$
- Fs^2 obtained from relative bin to bin normalization Data/MC after fit
- projection along Men ⇨ residual variation is observed.
  2D fit of the normalization is performed to get the slopes



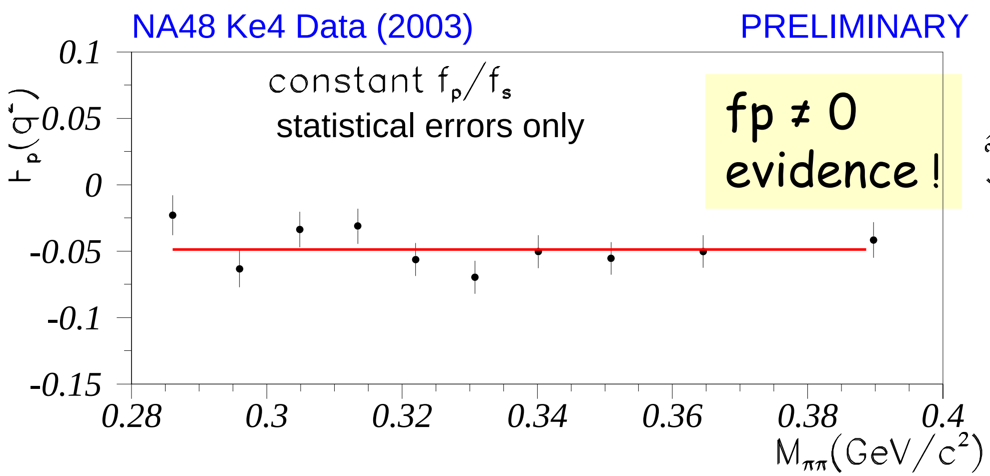


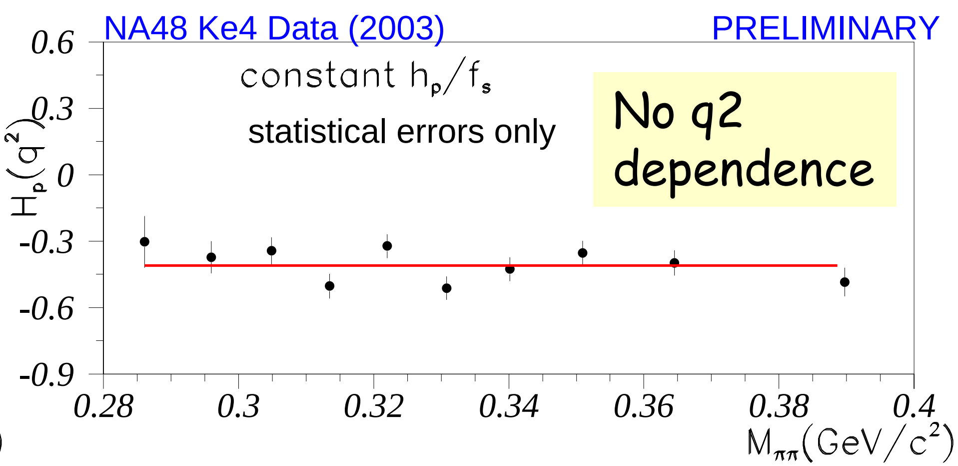
The 3 slopes are correlated

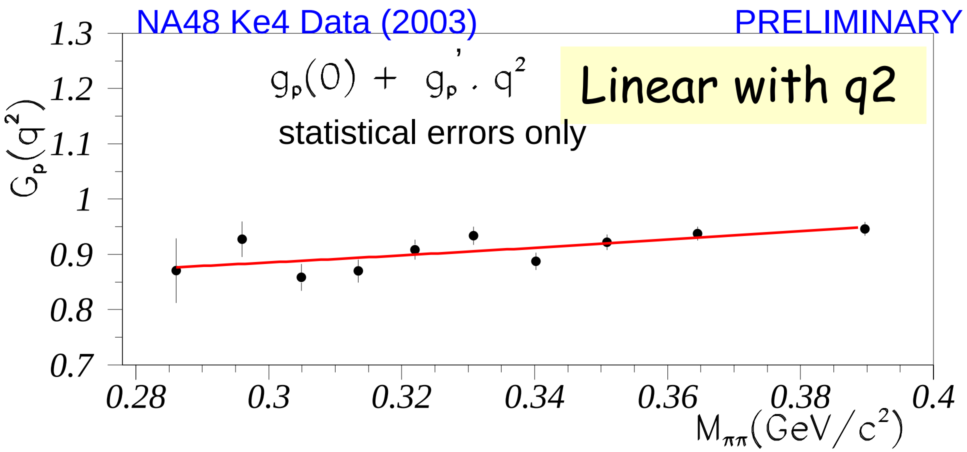
$Fs^2 \propto (1 + f's\ q^2 + f''s\ q^4 + f'e\ Se/4m\pi^2)^2$

	f"s	f'e
f's	-0.96	0.03
f"s		-0.06

Form Factors Fp, Gp, Hp







Correlation

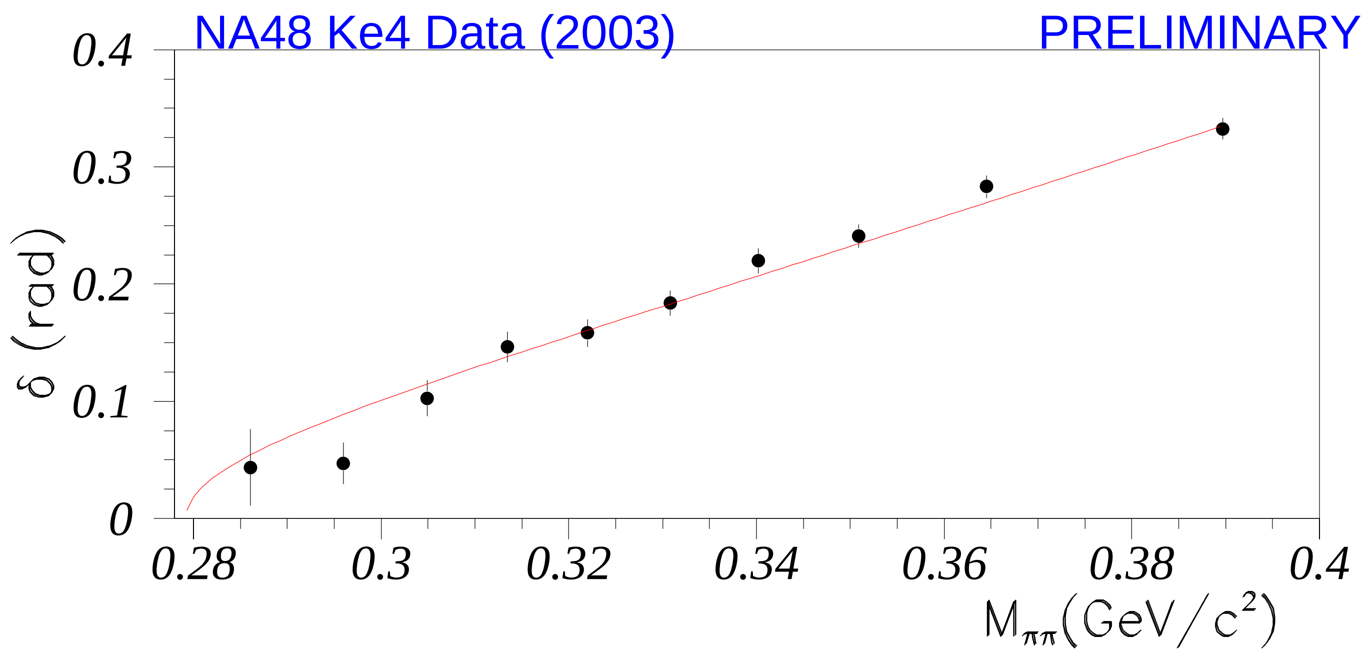
	g'_p
$g_p(0)$	-0.914

Ke4 charged decays : phase shift δ and a_0^0

- to extract information from the $\delta = \delta_0^0 - \delta_1^1$ variation, some external data (I=2 ππ data @higher energy) and theoretical work are needed :

e.g. numerical solution of Roy equations (ACGL Phys. Rep.353 (2001), DFGS EPJ C24 (2002)) which relates δ and (a_0^0 , a_0^2) .

- the Universal Band centre line parameterization corresponds to a 1-parameter fit with a fixed relation $a_0^2 = f(a_0^0)$.



Ke4 charged decays : Form Factors results (677 500 decays)

relative Form Factors = FF/Fs(0)

- measured separately for K^+ and K^-,
- combined according to statistical errors,
- Fs obtained from bin to bin normalization,
- Fp, Gp, Hp de-convoluted from observed Fs(q2,Se) variation .

All form factors and their $M_{\pi\pi}$ dependence now measured with 5 -15 % relative precision

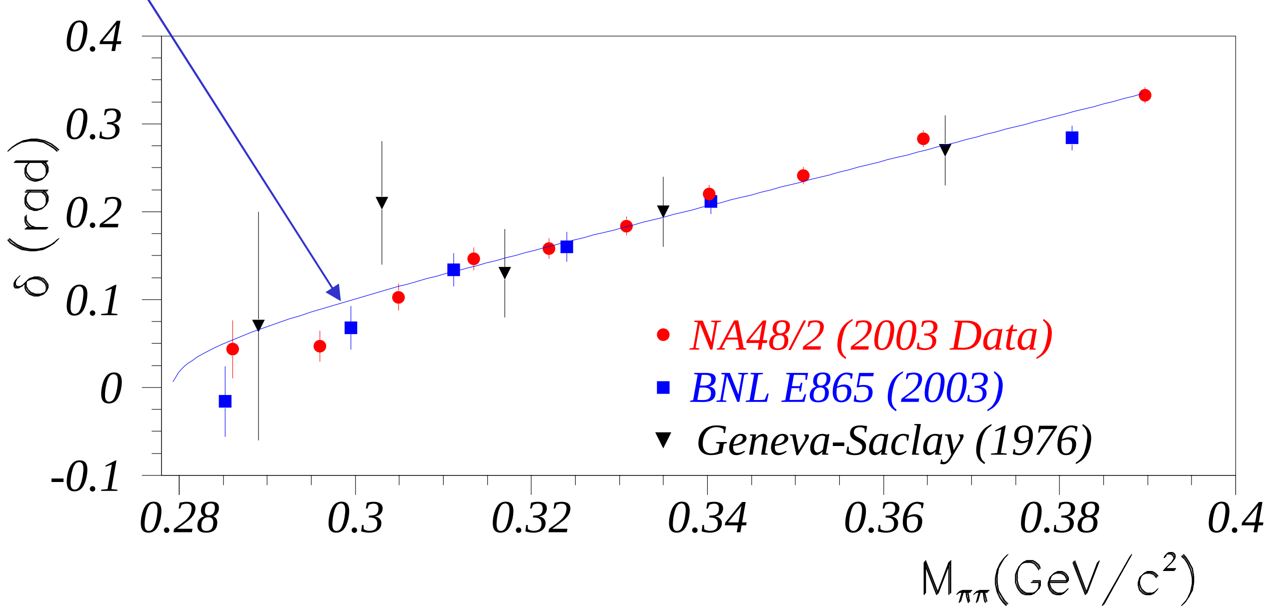
	value ± stat. ± syst.
$f_s^{'}/f_s$	= 0.165 ± 0.011± 0.006
$f_s^{''}/f_s$	= -0.092 ± 0.011 ± 0.007
$f_e^{'}/f_s$	= 0.081± 0.011 ± 0.008
f_p/f_s	= -0.048± 0.004 ± 0.004
g_p/f_s	= 0.873± 0.013 ± 0.012
$g_p^{'}/f_s$	= 0.081 ± 0.022 ± 0.014
h_p/f_s	= -0.411 ± 0.019± 0.007

Precision results from K_{e4} charged decays

Comparison with previous published Ke4 results

- CERN/PS Geneva-Saclay ~30000 decays (K^+) , Phys. Rev. D15 (1977)
- BNL E865 ~390 000 decays (K^+) (PRL 87 (2001), Phys. Rev. D67 (2003))
- CERN/SPS NA48/2 : preliminary result from ~677 500 decays (K^++K^-) highest acceptance at large $m_{\pi\pi}$, high resolution + low background level.

Universal Band centre line (stat + syst errors added)

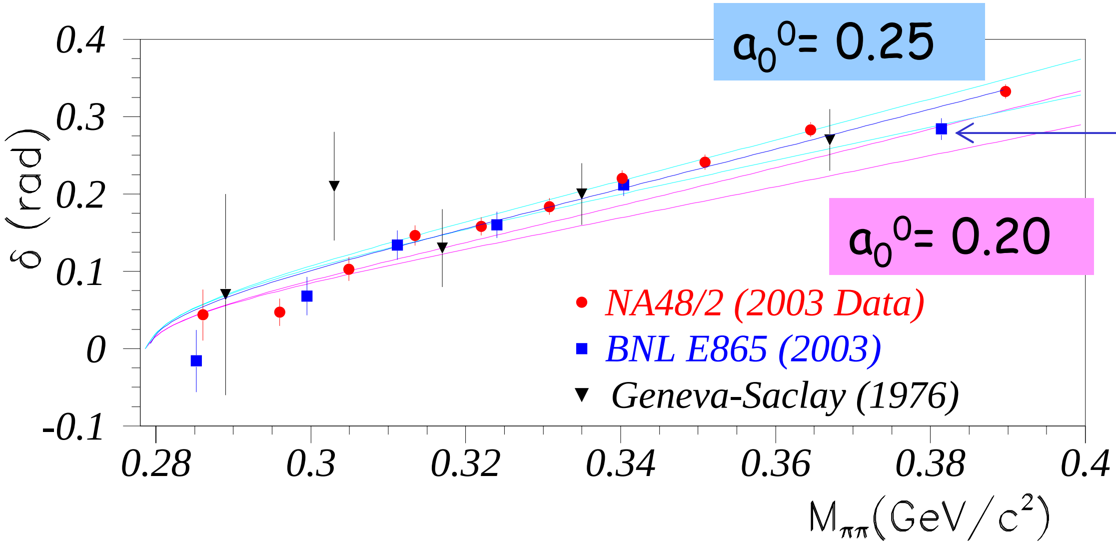


Ke4 phase shift measurements : are they compatible ?

• advantage of "independent bin" analysis: scattering lengths extraction with new models + combination of results from various experiments even after completion of collaborations !

• E865 quotes various values extracted from their Ke4 phase measurements, ranging from a_0^0 = 0.203 to a_0^0= 0.237

• NA48/2 tends to favor slightly higher values

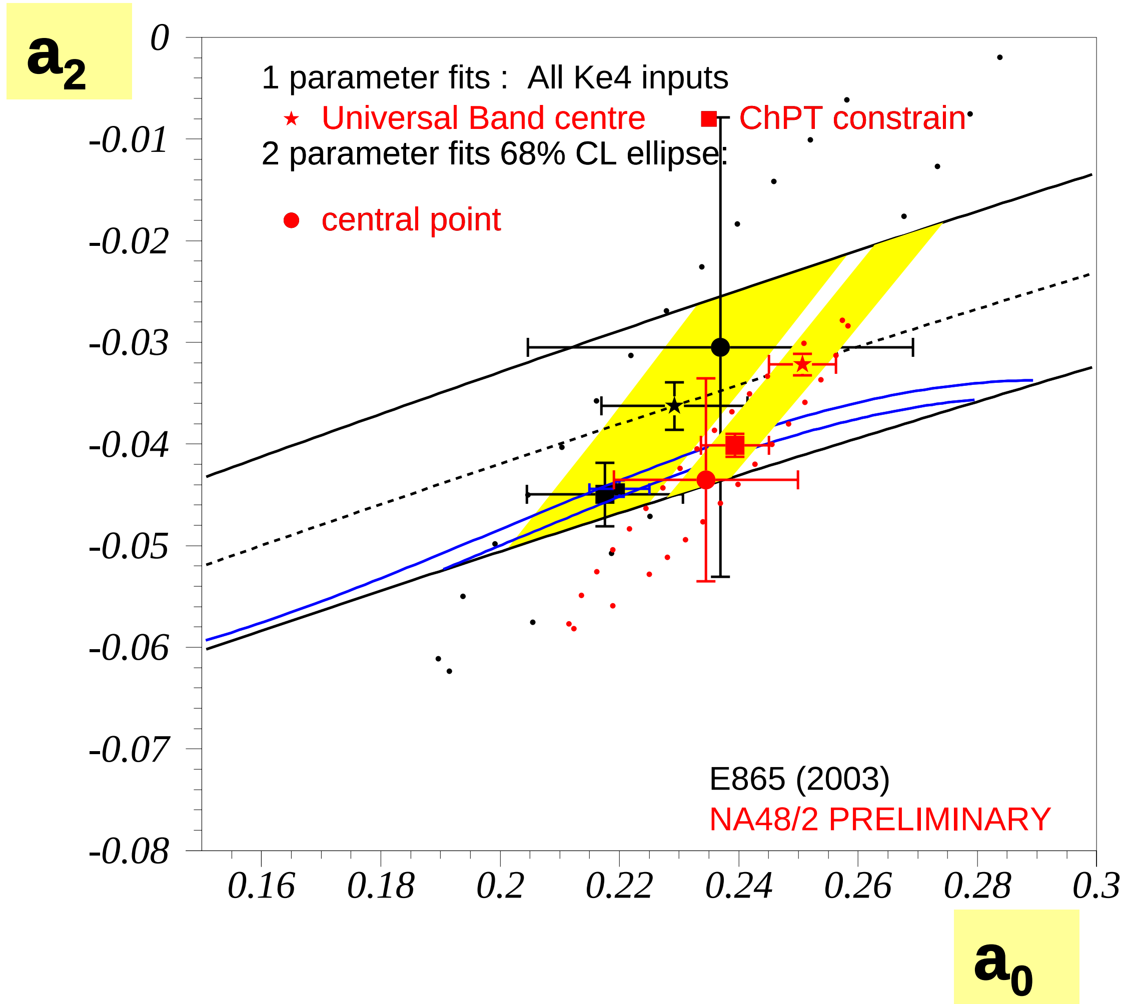


Last point of E865 is at variance with other points

More on scattering lengths extraction

- Several approaches relate the phase variation to [a_0^2,a_0^0]
- scan of the allowed regions or best 2D parameter fit
- the 2 Ke4 experiments point to slightly different regions of the UB with similar correlation between a_0^0 and a_0^2 (~96%).

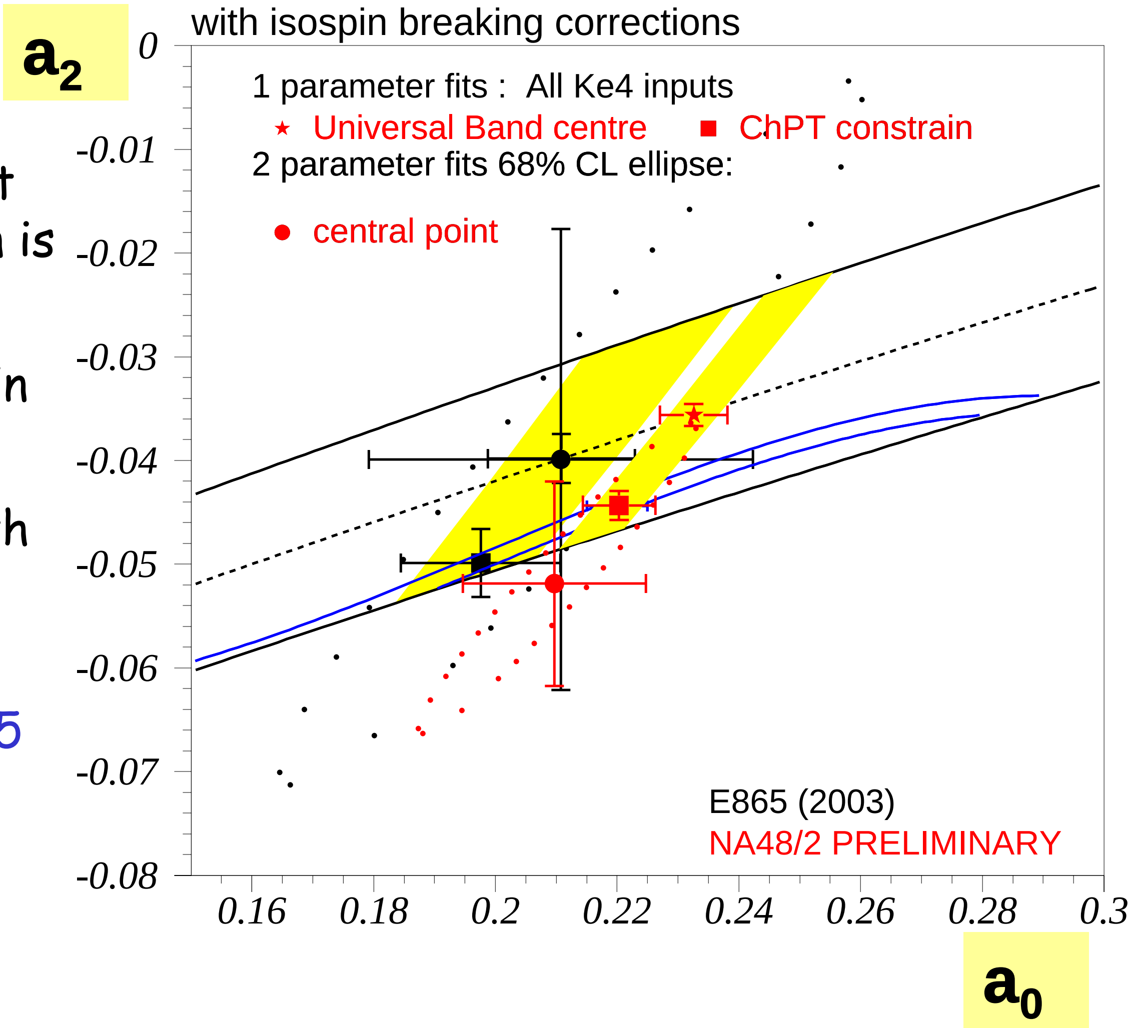
(removing the last E865 point brings the two bands closer and decreases the BNL $\chi 2$)



Correcting for isospin symmetry breaking…

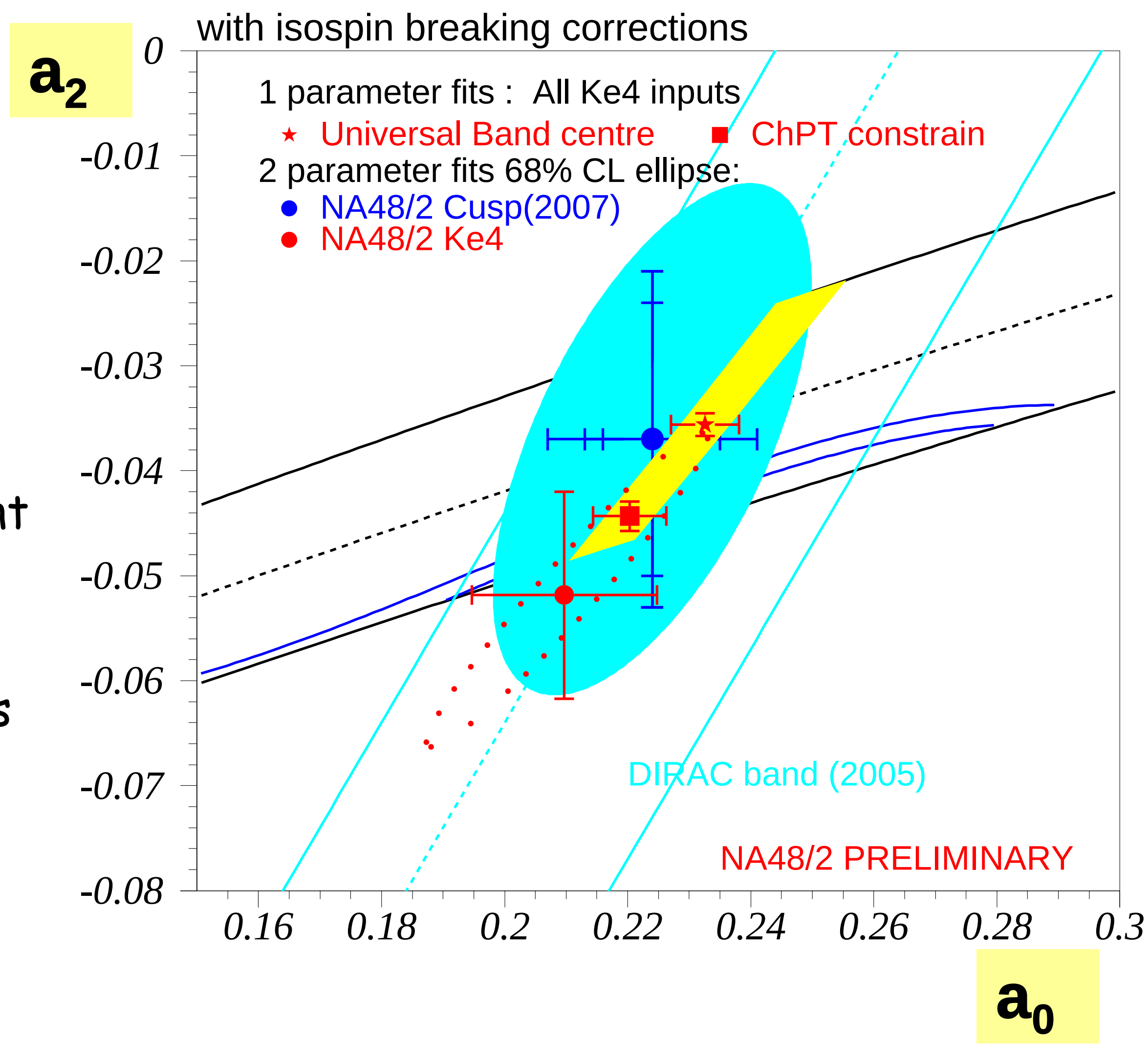
- Following recent developments (J.Gasser), the measured Ke4 phases need to be corrected for isospin symmetry breaking effect before extracting a_0^0. correction is negative ~10-12 mrad
- Both bands shift left and down in the $[a_0^2, a_0^0]$ plane
- One can replay this exercise with all data points (E865, Geneva-Saclay and NA48).

The contribution of the last E865 point to the $\chi 2$ is large, 5 units decrease if removed



Comparing with results from other channels

- NA48 also measures a_0^0 and a_0^2 from cusp in $K3\pi$ decays
- DIRAC experiment provides information from pionium life time
- K_{e4} phase measurements are complementary and independent
- global picture is quite consistent when isospin symmetry breaking corrections are taken into account in K_{e4}



Ke4 analysis in NA48/2 : Summary and prospects

Very fruitful collaboration with theory groups from Bern, Orsay, Madrid, Dubna has brought improvement of $\pi\pi$ phase shift measurement

analysis ~0.68 M Ke4 events (2003) from NA48 resulted in:

- improved measurements of Axial /Vector form factors, including their $m_{\pi\pi}$ and $m_{e\nu}$ dependence + first evidence of non zero fp term.
- up-to-date theoretical framework used to extract the $\pi\pi$ scattering lengths.
- result is in agreement with other Ke4 measurements and with $\pi\pi$ re-scattering (cusp) in $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decays
- values of a_0^0 and a_0^2 allow successful test of χPT predictions.

Additional data from 2004 will bring a total of 10^6 Ke4 events and improved scattering lengths.

Spare slides

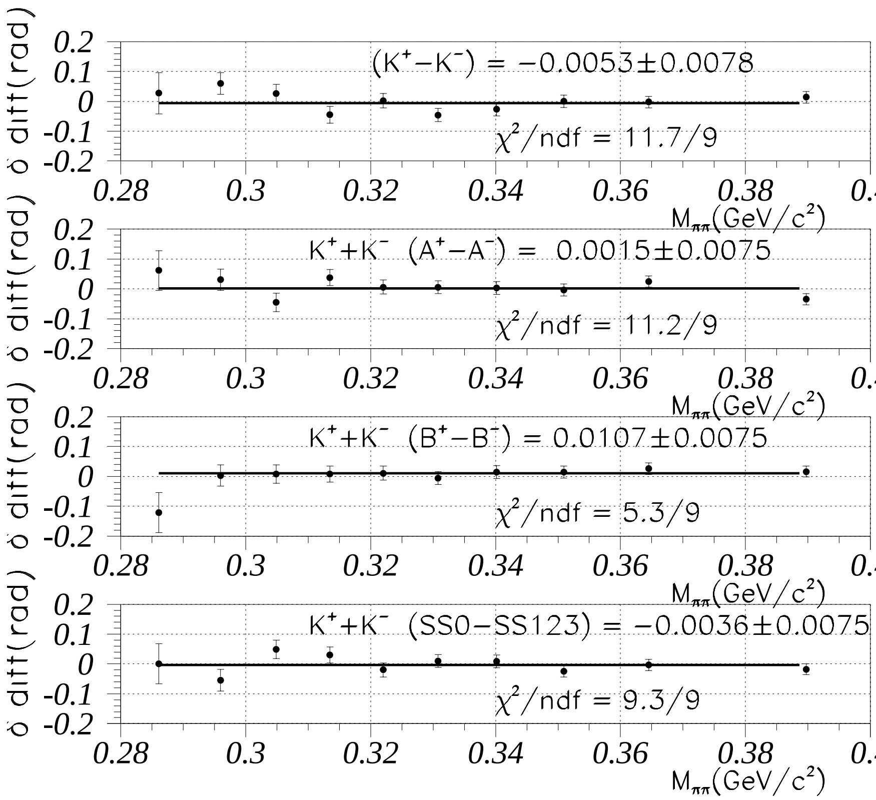
2003: SS0= first 20 days, SS123=last 30 days

Ke4 charged decays : systematic uncertainties

- 2 independent analyses with slightly different approaches (binning, trigger efficiency, fit method..)
- acceptance control
- background level and shape control
- e/π rejection control
- radiative corrections implementation
- neglect S_e dependence in MC

Possible bin to bin correlations were investigated and taken into account in the overall fit procedure (non diagonal covariance matrix)

Acceptance control



2003: SS0= first 20 days, SS123=last 30 days

pionium signature and more

Direct count of the excess in 7 bins excluded from the fit: (960 ± 320 events more than the pionium best fit) **Rate = 2.58 ± 0.31 times the predicted pionium formation rate**

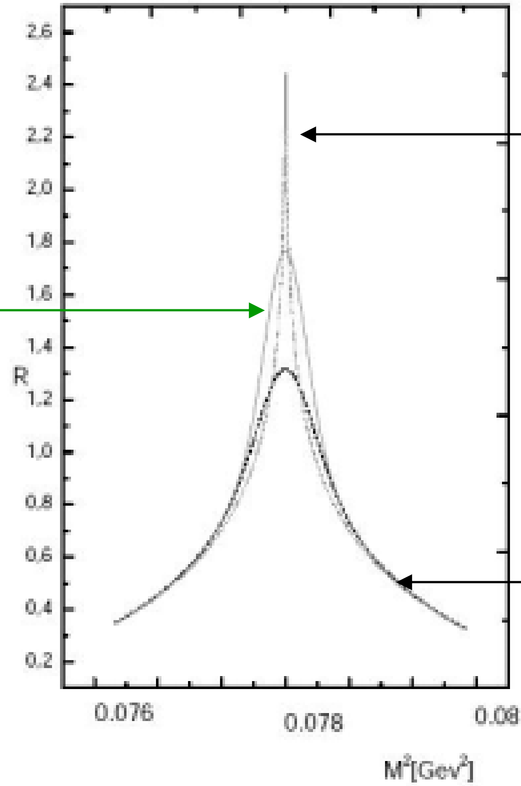
IS THE EVENT EXCESS ONLY DUE TO PIONIUM ATOMS?

EM corrections to final state interactions in $K \rightarrow 3\pi$ decays
(Gevorkian, Tarasov, Voskresenskaya, hep-ph / 0612129)

2 contributions from $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ decay to the $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ cusp region:

- **Pionium formation : $\pi^{+}\pi^{-}$ atom $\rightarrow \pi^{\circ}\pi^{\circ}$ (negligible width)**
- **Additional $\pi^{+}\pi^{-}$ unbound states with resonance structure $\rightarrow \pi^{\circ}\pi^{\circ}$**

$\pi^{+}\pi^{-}$ atoms and
$\pi^{+}\pi^{-}$ resonant structure
with experimental resolution



$\pi^{+}\pi^{-}$ resonant structure
(no experimental resolution)

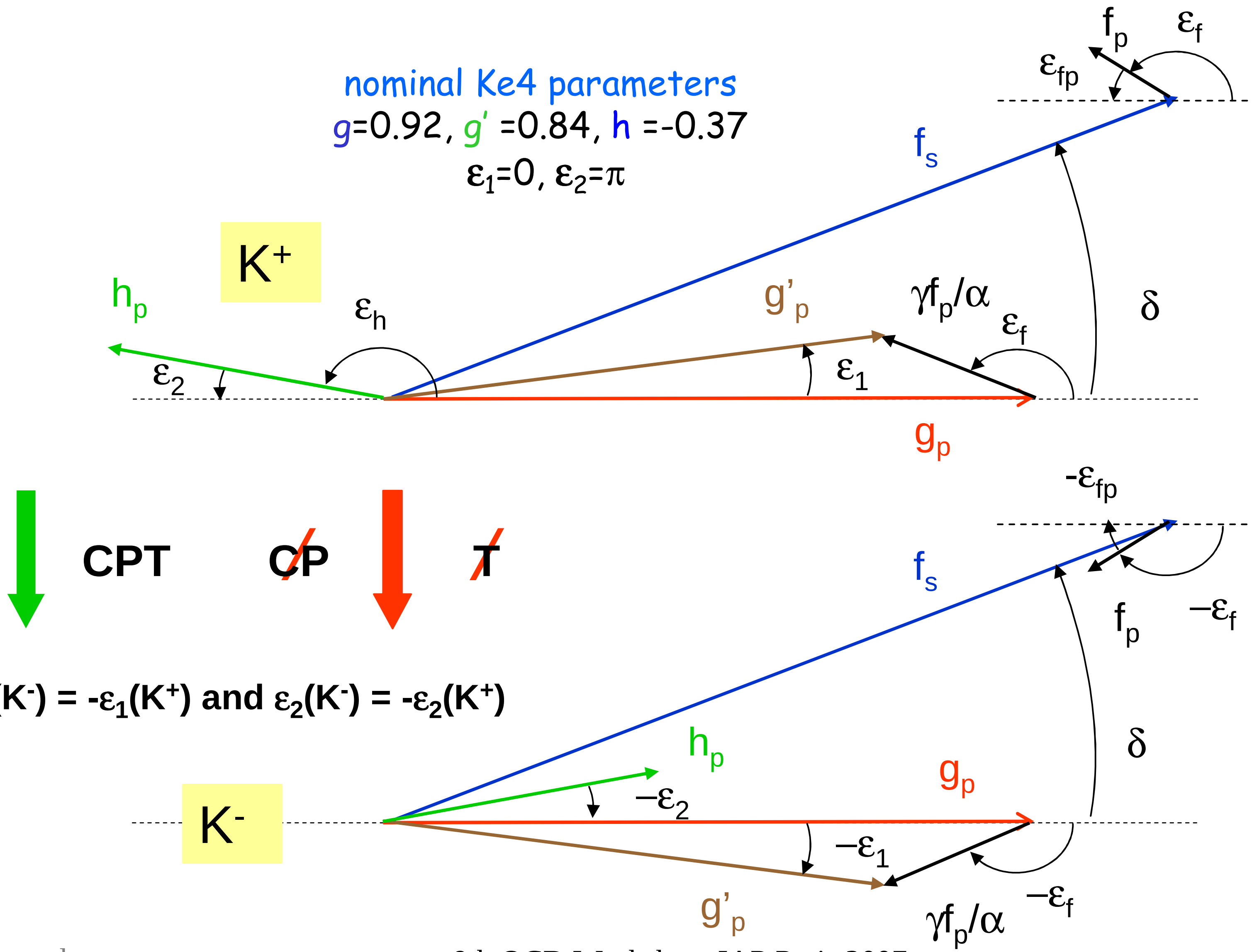
$\pi^{+}\pi^{-}$ resonant structure
with experimental resolution

Ke4 matrix elements

JiBi	Partial rate for $K^+ \to \pi^+ \pi^- e^+ \nu_e$	Partial rate for $K^- \to \pi^+ \pi^- e^- \nu_e$	
1	$\mathcal{D}5 = f_s^2\gamma^2 \sin^2\theta_e$	$\bar{\mathcal{D}}5 = f_s^2\gamma^2 \sin^2\theta_e$	
2	$+g^2\beta^2 \sin^2\theta_\pi (1-\sin^2\theta_e \cos^2\phi)$	$+g^2\beta^2 \sin^2\theta_\pi (1-\sin^2\theta_e \cos^2\phi)$	
3	$+g'^2\alpha^2 \cos^2\theta_\pi \sin^2\theta_e$	$+g'^2\alpha^2 \cos^2\theta_\pi \sin^2\theta_e$	
4	$+h^2\beta^2\gamma^2 \sin^2\theta_\pi (1 - \sin^2\theta_e \sin^2\phi)$	$+\bar{h}^2\beta^2\gamma^2 \sin^2\theta_\pi (1 - \sin^2\theta_e \sin^2\phi)$	
5	$+2f_s g'\alpha\gamma\cos(\delta-\varepsilon_1)\cos\theta_\pi \sin^2\theta_e$	$+2f_s g'\alpha\gamma\cos(\delta-\bar{\varepsilon}_1)\cos\theta_\pi \sin^2\theta_e$	ϕ even
6	$+2f_s g\beta\gamma\cos\delta\sin\theta_\pi \sin\theta_e \cos\theta_e \cos\phi$	$+2f_s g\beta\gamma\cos\delta\sin\theta_\pi \sin\theta_e \cos\theta_e \cos\phi$	
7	$-2f_s h\beta\gamma^2\cos(\delta-\varepsilon_2)\sin\theta_\pi \sin\theta_e \cos\phi$	$-2f_s \bar{h}\beta\gamma^2\cos(\delta-\bar{\varepsilon}_2)\sin\theta_\pi \sin\theta_e \cos\phi$	
8	$+2gg'\alpha\beta\cos\varepsilon_1 \cos\theta_\pi\sin\theta_\pi\sin\theta_e\cos\theta_e \cos\phi$	$+2gg'\alpha\beta\cos\bar{\varepsilon}_1 \cos\theta_\pi\sin\theta_\pi\sin\theta_e\cos\theta_e \cos\phi$	
9	$-2g'h\alpha\beta\gamma\cos(\varepsilon_2-\varepsilon_1)\cos\theta_\pi \sin\theta_\pi \sin\theta_e \cos\phi$	$-2g'\bar{h}\alpha\beta\gamma\cos(\bar{\varepsilon}_2-\bar{\varepsilon}_1)\cos\theta_\pi \sin\theta_\pi \sin\theta_e \cos\phi$	
10	$-2gh\beta^2\gamma\cos(\varepsilon_2)\sin^2\theta_\pi \cos\theta_e$	$-2g\bar{h}\beta^2\gamma\cos\bar{\varepsilon}_2\sin^2\theta_\pi \cos\theta_e$	
11	$+2f_s g\beta\gamma\sin\delta\sin\theta_\pi \sin\theta_e \sin\phi$	$-2f_s g\beta\gamma\sin(\delta)\sin\theta_\pi \sin\theta_e \sin\phi$	
12	$-2f_s h\beta\gamma^2\sin(\delta-\varepsilon_2)\sin\theta_\pi \sin\theta_e \cos\theta_e \sin\phi$	$+2f_s \bar{h}\beta\gamma^2\sin(\delta-\bar{\varepsilon}_2)\sin\theta_\pi \sin\theta_e \cos\theta_e \sin\phi$	
13	$+2gg'\alpha\beta\sin\varepsilon_1 \cos\theta_\pi \sin\theta_\pi \sin\theta_e \sin\phi$	$-2gg'\alpha\beta\sin\bar{\varepsilon}_1 \cos\theta_\pi \sin\theta_\pi \sin\theta_e \sin\phi$	ϕ odd
14	$-2g'h\alpha\beta\gamma\sin(\varepsilon_1-\varepsilon_2)\cos\theta_\pi \sin\theta_\pi \sin\theta_e \cos\theta_e \sin\phi$	$+2g'\bar{h}\alpha\beta\gamma\sin(\bar{\varepsilon}_1-\bar{\varepsilon}_2)\cos\theta_\pi \sin\theta_\pi \sin\theta_e \cos\theta_e \sin\phi$	
15	$-2gh\beta^2\gamma\sin\varepsilon_2 \sin^2\theta_\pi \sin^2\theta_e \sin\phi\cos\phi$	$+g\bar{h}\beta^2\gamma\sin\bar{\varepsilon}_2 \sin^2\theta_\pi \sin^2\theta_e \sin\phi\cos\phi$	

$K^+ \to K^-$: $\theta_e \to \pi-\theta_e$, $\phi \to \pi+\phi$, $\bar{h} = -h$, $\varepsilon_i \to \bar{\varepsilon}_i$

K_{e4} amplitudes and phases with T and CP violation



$\varepsilon_1(K^-) = -\varepsilon_1(K^+)$ and $\varepsilon_2(K^-) = -\varepsilon_2(K^+)$