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On behalf of the NA48/2 collaboration:

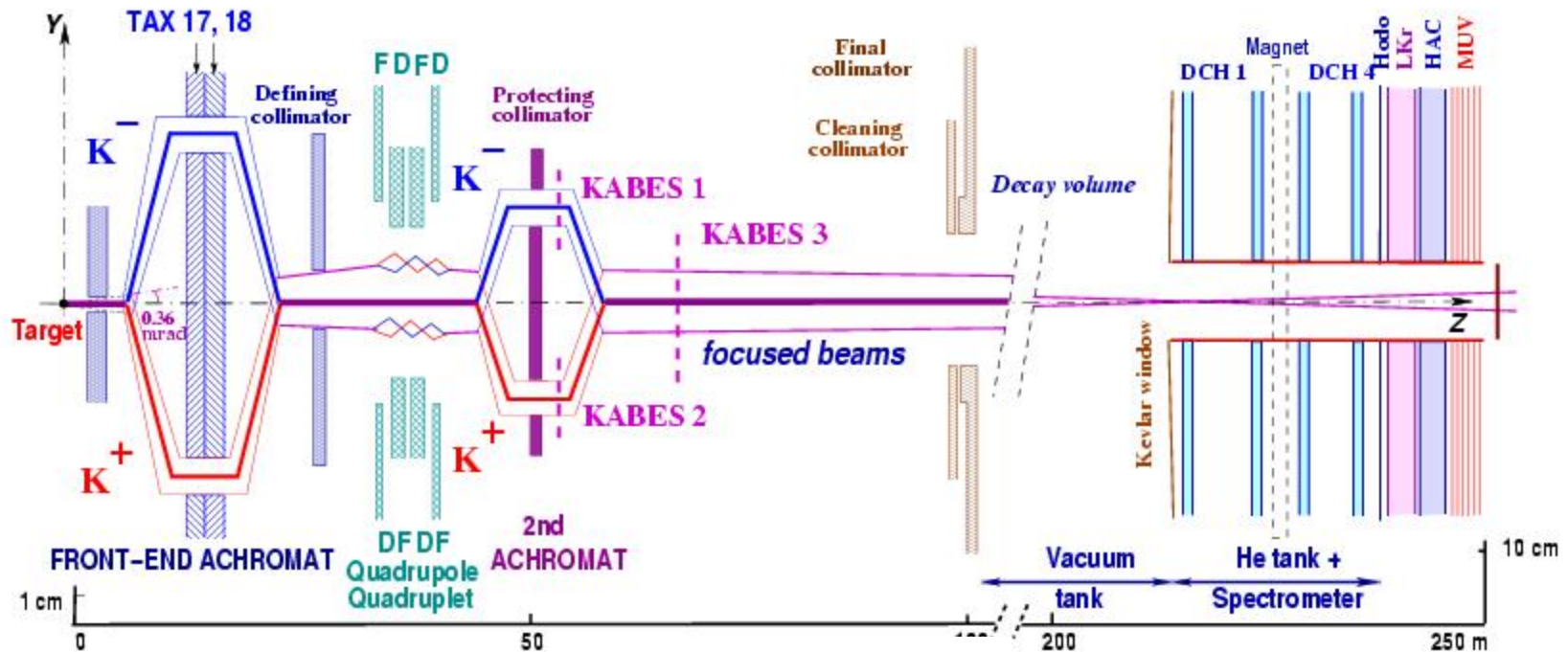
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze,
Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

outline

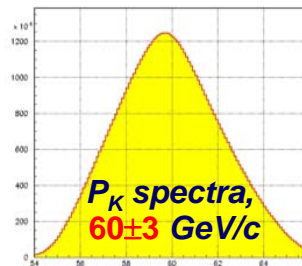
- Introduction to NA48/2
- QCD tests from study of Kaon decays
- Ke4 decays ($K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$) :
Form Factors and $\pi\pi$ scattering lengths
- K3 π decays ($K^\pm \rightarrow \pi^0\pi^0\pi^\pm$) : the "cusp" effect
Dalitz plot parameters and $\pi\pi$ scattering lengths
- Summary

The NA48/2 experiment at the CERN-SPS : primarily designed for CP violating charge asymmetries studies in $K^3\pi$ decays

2003 run: ~ 50 days + 2004 run: ~ 60 days



Simultaneous K^+ and K^- beams:
large charge symmetrization of
experimental conditions



Beams coincide within ~1mm
all along the 114m decay volume
flux ratio $K^+/K^- \sim 1.8$

The NA48/2 experiment: detector and performances

Magnetic spectrometer :

4 high-resolution DCH's + dipole magnet

$$\Delta p/p = (1.0 \oplus 0.044 p)\% \quad (p \text{ in } \text{GeV}/c)$$

Very good resolution for charged invariant masses: $\sigma (M_{3\pi^\pm}) = 1.7 \text{ MeV}/c^2$

Hodoscope for charged fast trigger

$$\sigma\tau = 150 \text{ ps}$$

LKr electromagnetic calorimeter :

quasi-homogenous and high granularity

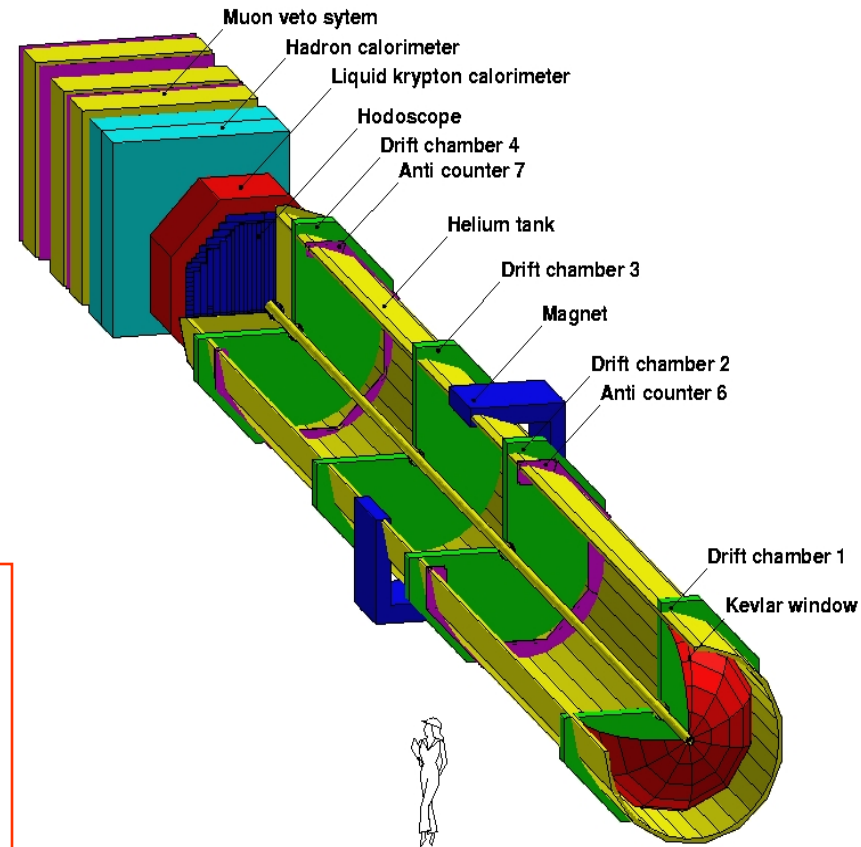
$$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\% \quad (E \text{ in } \text{GeV})$$

$$\sigma_x = \sigma_y \sim 1.5 \text{ mm for } E=10 \text{ GeV}$$

Very good resolution for neutrals ($\pi^0 \rightarrow \gamma\gamma$)

$$\sigma (M_{\pi\pi^0\pi^0}) = 1.4 \text{ MeV}/c^2$$

E/p ratio used for e/π discrimination



Kaon decays : what can be learned on QCD @ Low Energy ?

Hadronic decay modes into 3 pions:

- large Br's : $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ (1.7 %) and $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$ (5.6 %),

60 Millions events now analyzed (PRL B633 (2006) partial sample)

- three pions $\rightarrow \pi^0 \pi^0$ system + nearby hadron
- accessible $M_{\pi\pi}$ range from threshold to $(M_K - M_\pi)$

Semileptonic decay mode Ke4:

- small Br's : $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ ($4.1 \cdot 10^{-5}$),

1.1 Million events now analyzed (EPJC 54 (2008) partial sample)

- only two $\pi^+ \pi^-$ pions, very clean environment
- accessible $M_{\pi\pi}$ range from threshold to $(M_K - M_e) = M_K$

Two different but complementary approaches to $\pi\pi$ scattering near threshold \rightarrow extract s-wave scattering lengths (a_0, a_2) for Isospin $I = 0$ and $I = 2$

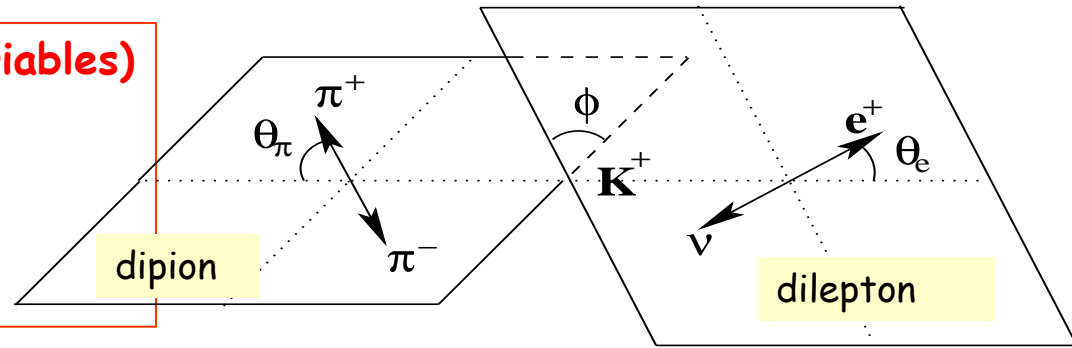
Ke4 decays : formalism

Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

$$S_\pi (M^2_{\pi\pi}), S_e (M^2_{e\nu}),$$

$$\cos\theta_\pi, \cos\theta_e \text{ and } \phi.$$



Partial Wave expansion of the amplitude

(Pais-Treiman 1968) into s and p waves

F, G = 2 Axial Form Factors

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi$$

$$G = G_p e^{i\delta_g}$$

H = 1 Vector Form Factor

$$H = H_p e^{i\delta_h}$$

Map the five-dimensional space of the Ca.Ma. variables with 4 Form factors and one phase shift, assuming identical phases for the p-wave Form Factors F_p, G_p, H_p :

The fit parameters are :

$$F_s, F_p, G_p, H_p \text{ and } \delta = \delta_s - \delta_p$$

Ke4 decays: event selection and background rejection

Signal ($\pi^+\pi^-\ e^\pm\nu$) topology :

- 3 charged tracks and a good vertex
- two opposite sign pions,
- 1 electron (LKr info $E/p \sim 1$),
- some missing energy and p_T (ν)
- reconstruct PK (missing ν hypothesis)

Background main sources :

- $K^\pm \rightarrow \pi^+\pi^-\pi^\pm$ (dominant)
 - ↳ $e\nu$ or misidentified as e
- $K^\pm \rightarrow \pi^0(\pi^0)\pi^\pm$
 - ↳ $(e+e-\gamma) + 1e$ misidentified as π and γ (s) undetected

Control sample from data ($\Delta S = \Delta Q$ valid)

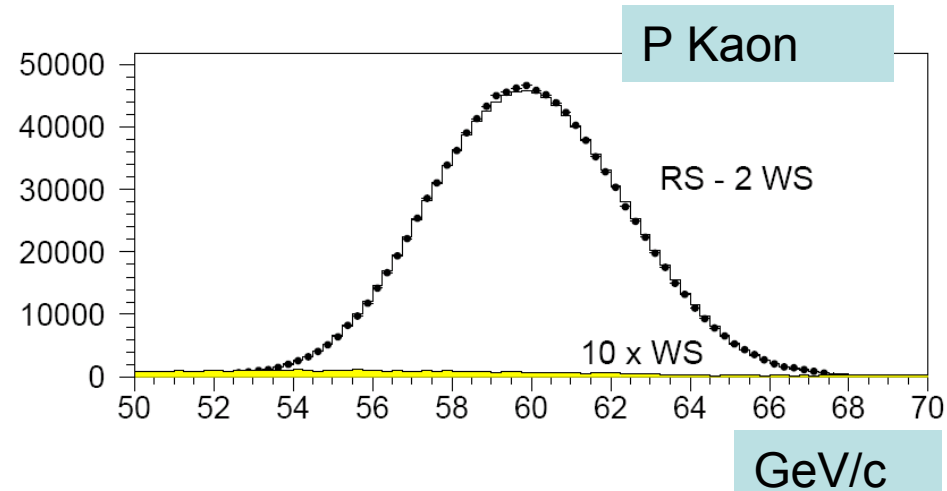
$K^\pm \rightarrow \pi^\pm\pi^\pm e^\mp\nu$ "Wrong Sign" events

- total charge (± 1) as Right Sign events
- electron charge opposite to total charge

Rate (RS/WS) events:

2 if coming from $K3\pi$

1 if coming from $K2\pi(\pi^0)$



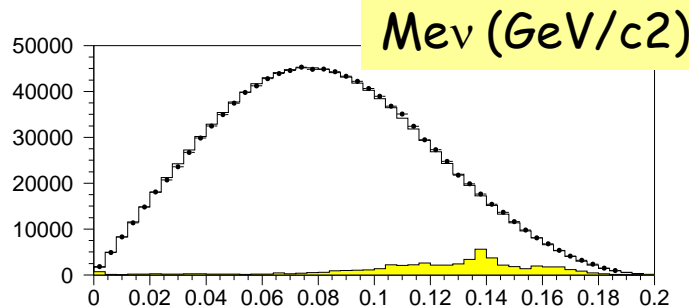
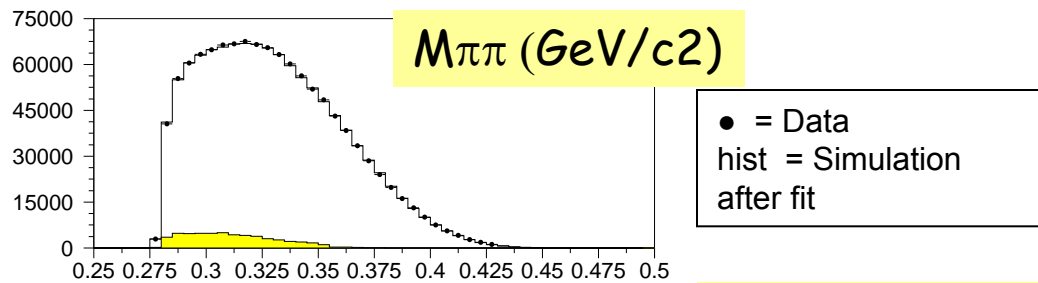
Total background level can be kept at $\sim 0.5\%$ relative level estimated from WS events rate and checked from MC simulation

Ke4 decays: fitting procedure and Form Factors

Total statistics 1.1 Millions Ke4 decays distributed over a grid of **15000 iso populated** boxes = $10(M_{\pi\pi}) \times 5(\text{MeV}) \times 5(\cos\theta_{\pi}) \times 5(\cos\theta_e) \times 12(\Phi)$

Fits are repeated in each $M_{\pi\pi}$ bin for K^+ and K^- separately without any assumption of Form Factor dependence with q^2 ($S_{\pi}/4m_{\pi}^2 - 1$) and $Se/4m_{\pi}^2$ using a large (~30 Millions) simulated sample to account for acceptance and experimental conditions.

Taylor expansion of Form Factors used to characterize bin to bin variation with q^2 , q^4 and Se (valid in the isospin symmetry limit)



Preliminary

	value	stat error	syst error(2003)
$f_s^{\prime\prime}/f_s$	0.158	± 0.007	± 0.006
f_s^{\prime}/f_s	-0.078	± 0.007	± 0.007
f_e^{\prime}/f_s	0.067	± 0.006	± 0.009
f_p/f_s	-0.049	± 0.003	± 0.004
constant			
g_p/f_s	0.869	± 0.010	± 0.012
g_p^{\prime}/f_s	0.087	± 0.017	± 0.015
h_p/f_s	-0.402	± 0.014	± 0.008
constant			

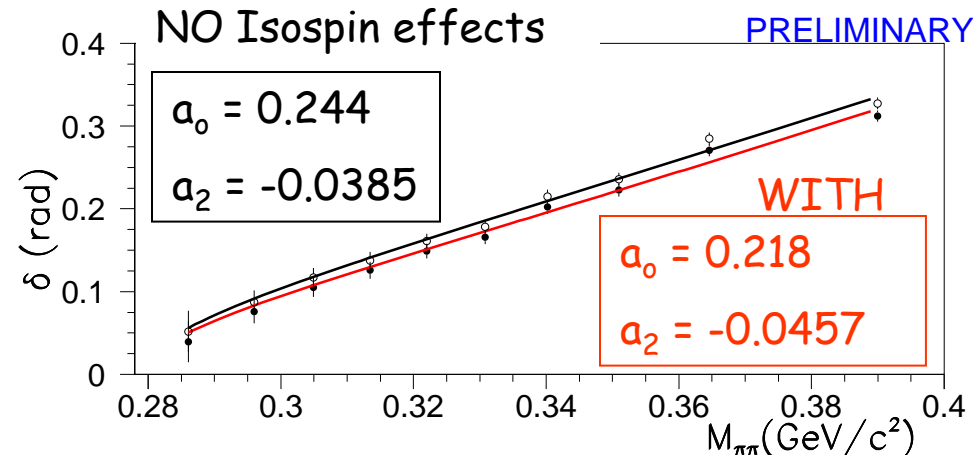
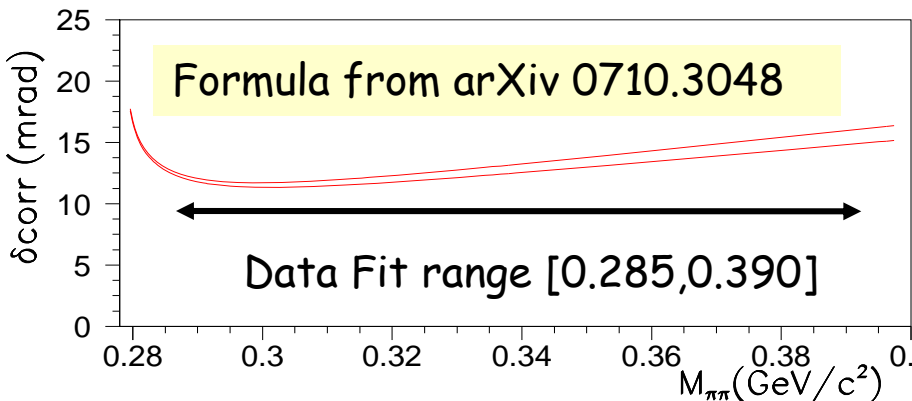
Ke4 decays: from phase shifts to scattering lengths (a_0, a_2)

$\pi\pi$ phases at threshold can be predicted from data above 0.8 GeV using **Roy equations** (unitarity, analyticity and crossing symmetries) and 2 subtraction constants **a_0 and a_2**

Numerical solutions have been developed (ACGL Phys.Rep.353(2001), DFGS EPJ C24(2002)) valid in the **Isospin symmetry limit**, broken in the experimental world. (**Universal Band**)

Radiative effects: included in the simulation, **Mass effects:** recently computed as a correction to the measurements, even larger than current experimental precision! (CGR hep-ph/0811.0775, DK in progress)

Induces a large **$2 \sigma_{\text{exp}}$ change** on (a_0, a_2) values from a 2p fit:
 $\sigma(a_0): \pm 0.013$ (stat) ± 0.007 (sys)
 $\sigma(a_2): \pm 0.0084$ (stat) ± 0.0041 (sys)



Ke4 decays: comparison with theoretical predictions

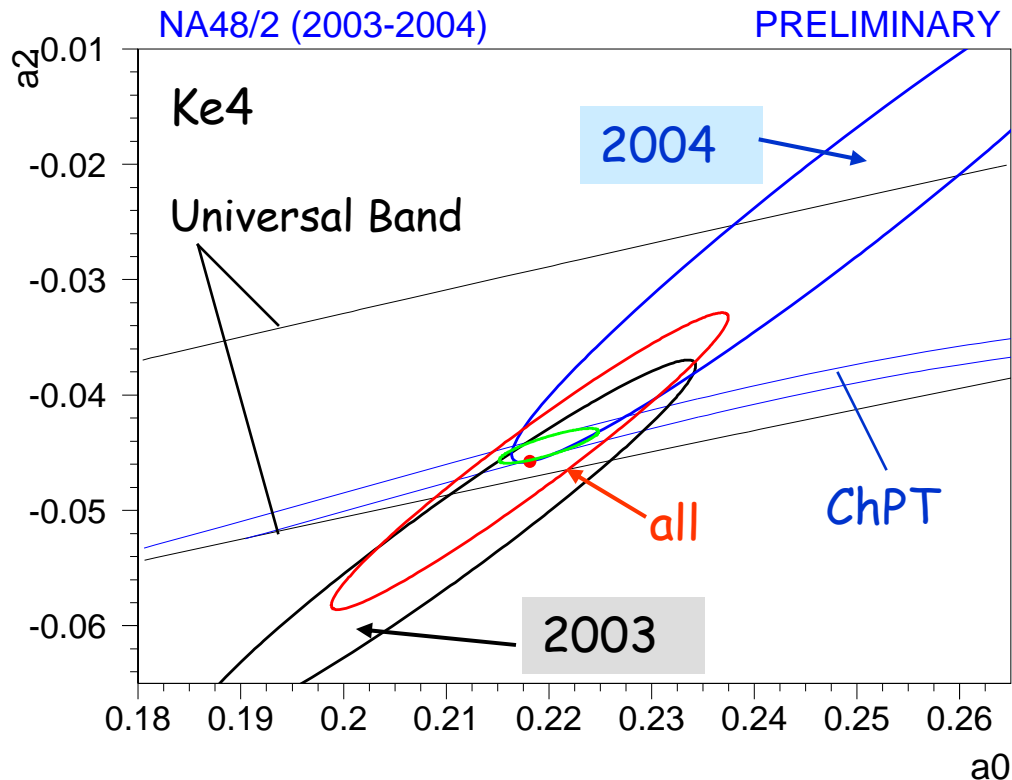
Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001)):

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0008$$



a_0 ChPT 1p fit	0.220 ± 0.005 stat ± 0.002 syst* ± 0.006 theo **
a_0 free	0.218 ± 0.013 stat ± 0.007 syst* ± 0.017 theo**
a_2 free 2p fit	-0.0457 ± 0.0084 stat ± 0.0041 syst* ± 0.0030 theo**



*systematics from 2003 data

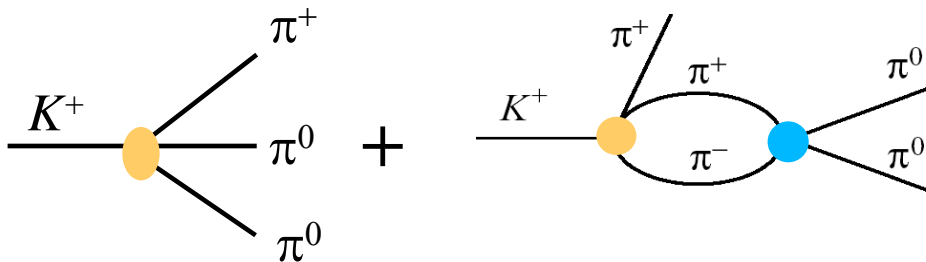
**Theory error evaluated from control of the isospin corrections, inputs to Roy equation numerical solutions ([CGR arXiv:0811.0775](https://arxiv.org/abs/0811.0775))

Cusp effect : first observation and interpretation (Cabibbo PRL93(2004))

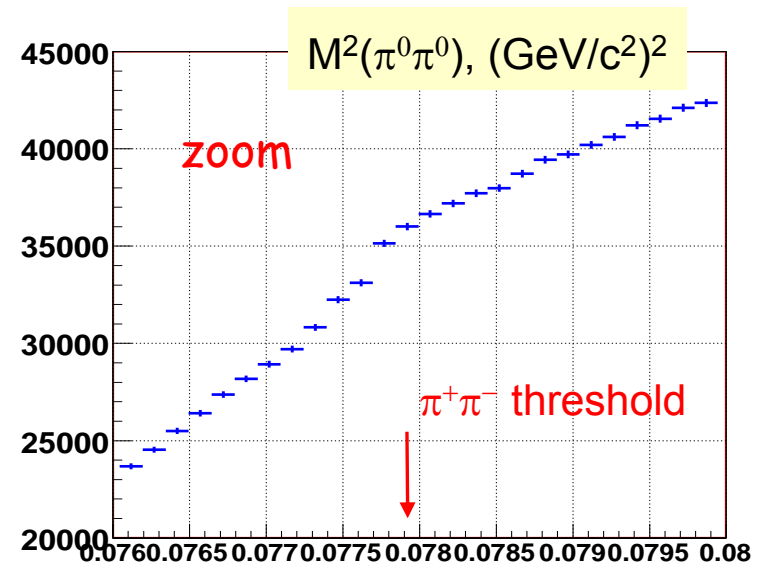
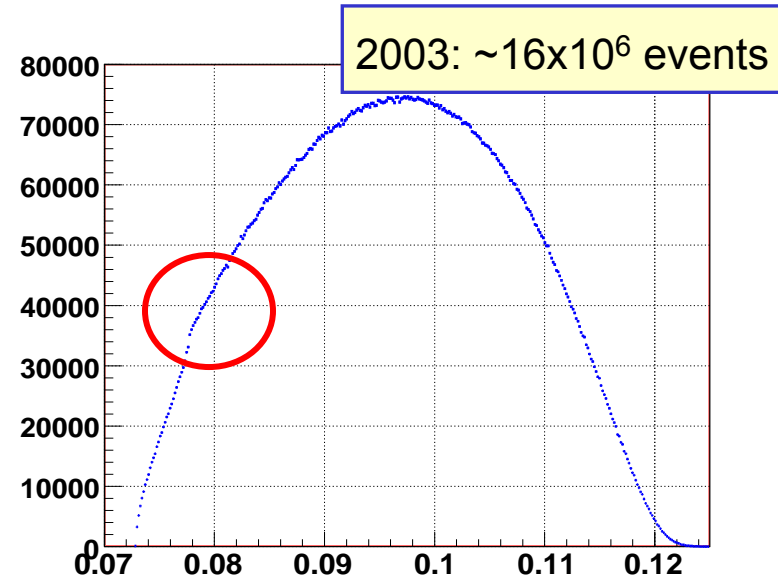
In $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ decay, the matrix element is usually described as a polynomial expansion using the **Dalitz Plot variables u and v**

First observation of a cusp structure was made with 16 M events collected in 2003 thanks to the very good mass resolution.

The structure at $\pi^+ \pi^-$ threshold was interpreted as due to the $\pi\pi$ rescattering in the $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$ final state



increased statistics with 44 M more data from 2004



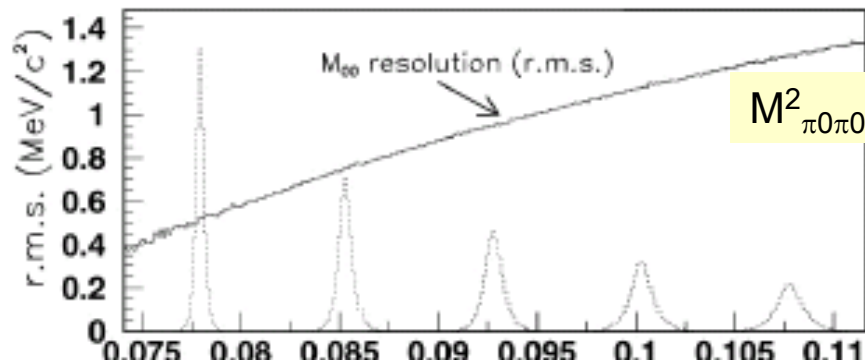
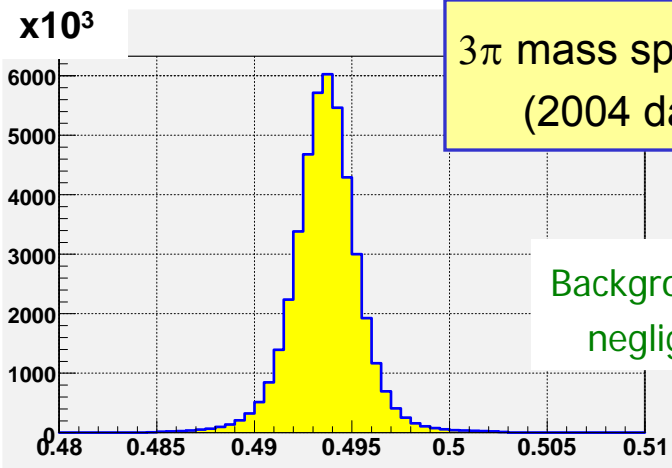
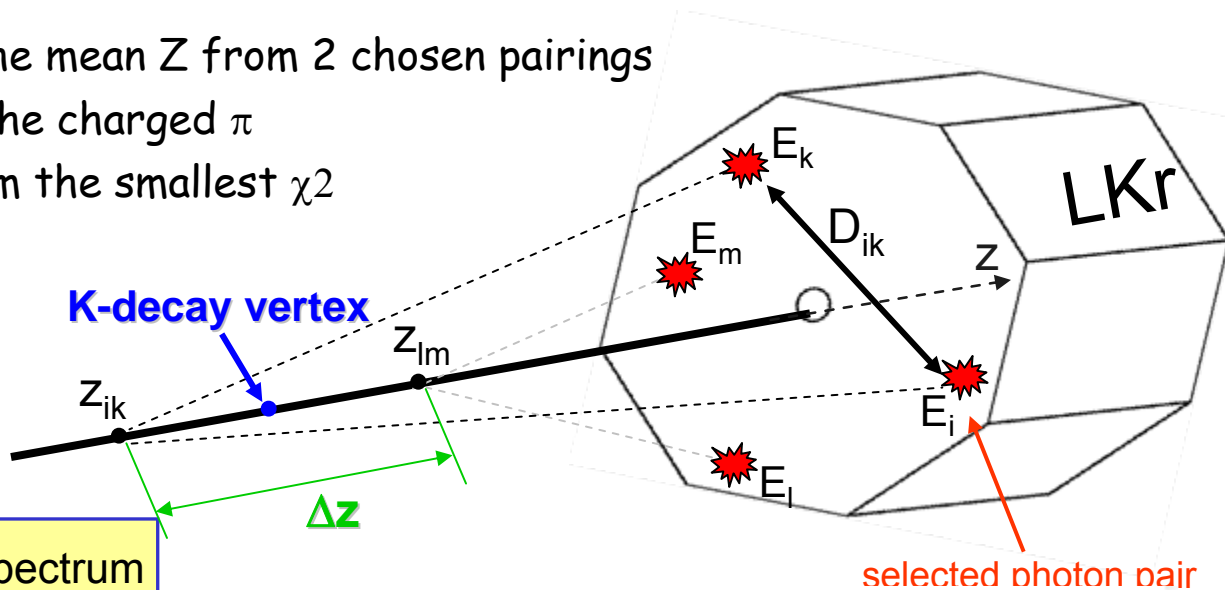
CUSP effect: $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ event selection

- Each photon pair (i,k) defines a decay vertex under the assumption of $\pi^0 \rightarrow \gamma\gamma$ decay

$$Z_{ik}^2 \cong E_i E_k D_{ik}^2 / m_{\pi^0}^2$$

- Neutral vertex defined as the mean Z from 2 chosen pairings
- Reconstruct K mass adding the charged π
- Chose the best 2 γ pairs from the smallest χ^2

$$\chi^2 = \left(\frac{\Delta Z}{\sigma_Z} \right)^2 + \left(\frac{\Delta m_K}{\sigma_{m_K}} \right)^2$$



CUSP effect: fitting procedure : two approaches

Cabibbo-Isodori [JHEP 0503\(2005\)](#)

Without re-scattering, M_0 parameterization (as PDG)

$$M_0 = A_0(1 + g_0 u/2 + h_0' u^2/2 + k_0' v^2/2)$$

- First order term M_1 describes loop diagrams

$$M_1 = -\frac{2}{3} (a_0 - a_2) m_{\pi^+} M_+ \sqrt{1 - M_{00}^2 / 2m_{\pi^+}^2}$$

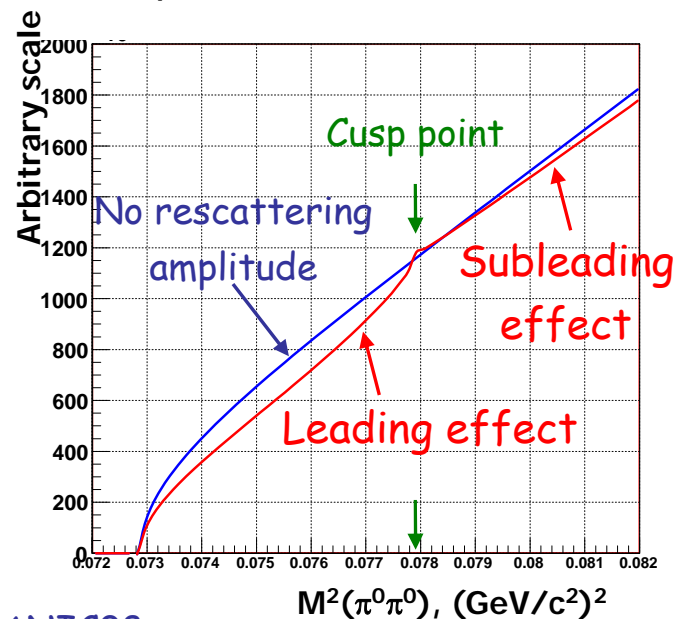
above threshold $|M|^2 = |M_0|^2 + |M_1|^2$

below threshold $|M|^2 = |M_0|^2 + |M_1|^2 + 2 M_0 M_1$

- Second order effects included
- Radiative corrections not (yet) included

Bern-Bonn Effective field theory
[CGKR PLB638 \(2006\)](#), [BFGKR arXiv:0807.0515](#)

- **electromagnetic effects** included in the amplitudes
- **two-loop formulation** different from CI introduces different correlations between parameters



Cusp : experimental fitting procedure

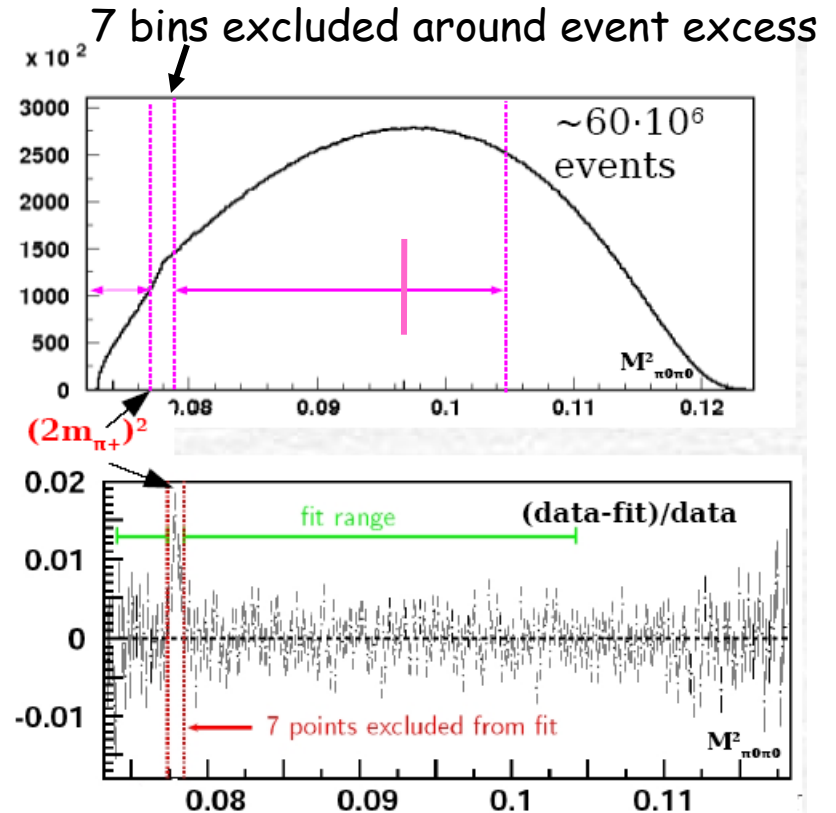
Fit the $M_{\pi^0\pi^0}^2$ projection using the detector response matrix R_{ij} obtained from a Monte-Carlo simulation and 4 physics parameters (g, h, a_0, a_2) for both approaches

the constant k'_0 (v-dependent term) is fixed to the value recently measured by a 2d fit of the $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ Dalitz plot
 $k'_0 = 0.0095 \pm 0.0002 \text{ stat} \pm 0.0005 \text{ syst}$

The M_+ amplitude for $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

$$M_+ = A_+(1 + g_+ u/2 + h'_+ u^2/2 + k'_+ v^2/2)$$

- fixed from NA48/2 measurement PLB649(2007) for CI
- fitted in a simultaneous fit of both Dalitz plots for CGKR



Event excess around the $M(\pi^+\pi^-)$ threshold can be explained as Pionium decay to $\pi^0\pi^0$ (Silagadze, JETP Lett.60 (1994))
 $R = \Gamma(K^\pm \rightarrow \pi^\pm A_{2\pi}) / \Gamma(K^\pm \rightarrow \pi^\pm \pi^+\pi^-)$
 $= (1.8 \pm 0.3) \times 10^{-5}$
 while the prediction is $R = 0.8 \times 10^{-5}$

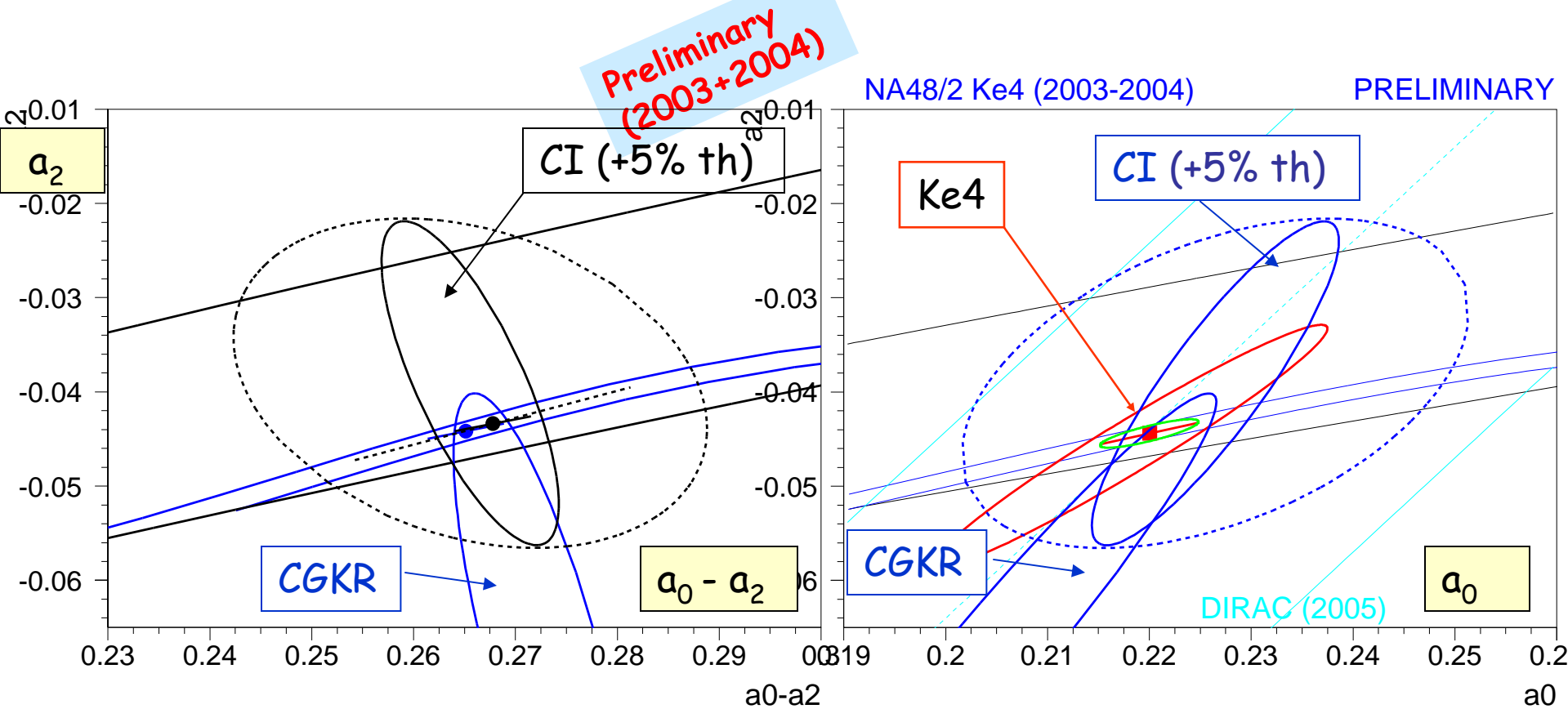
Cusp: scattering lengths results

Preliminary
(2003+2004)

Using ChPT constraint	Note : ext is mainly due to $R=A_+/A_0=1.975\pm 0.015$ th(CI) ~5% probably pessimistic (under evaluation)
CI model	$a_0 - a_2 = 0.268 \pm 0.003_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}} \pm 0.013_{\text{th}}$
CGKR model	$a_0 - a_2 = 0.266 \pm 0.003_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}}$

a_2 free	Note : correlations between a_2 and other parameters are larger in CGKR model
CI model	$a_0 - a_2 = 0.266 \pm 0.005_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}} \pm 0.013_{\text{th}}$ $a_2 = -0.039 \pm 0.009_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.002_{\text{ext}}$
CGKR model	$a_0 - a_2 = 0.273 \pm 0.005_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}}$ $a_2 = -0.065 \pm 0.015_{\text{stat}} \pm 0.010_{\text{syst}} \pm 0.002_{\text{ext}}$

Cusp and Ke4 : scattering lengths results



Two statistically independent measurements by NA48/2

Large overlap in the (a_2, a_0) plane

Impressive agreement with ChPT predictions (green ellipse)

Summary and Comparison with other experimental measurements

Ke4 : apply **isospin corrections** to published phase points of all experiments and perform a_0 **ChPT fit**

Note : E865 number dominated by highest energy data point, otherwise compatible

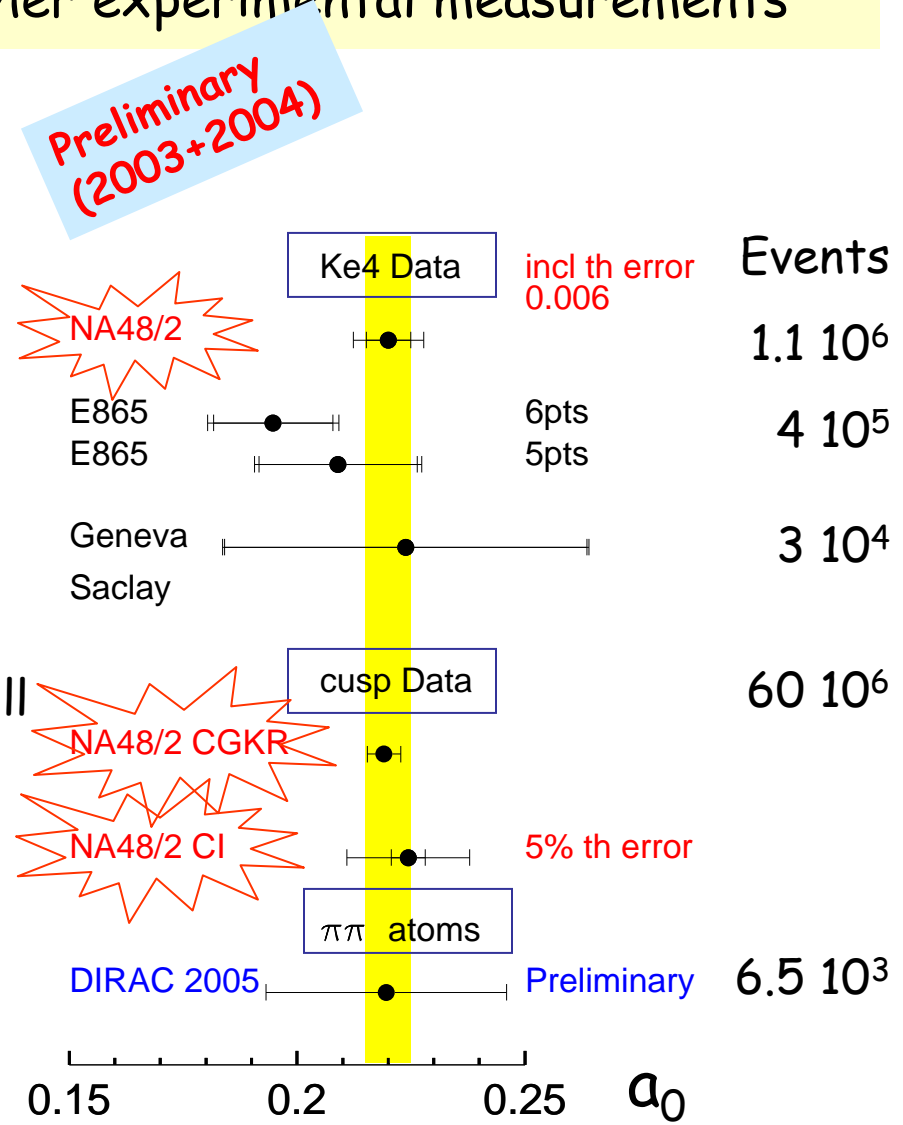
Cusp : $(a_0 - a_2)$ ChPT fit with 2 models

$\pi\pi$ atoms DIRAC: $|a_0 - a_2|$ errors from PLB619 (2005), use ChPT constraint (still being revisited + more Data analyzed)

Yellow band is ChPT prediction

$$a_0 = 0.220 \pm 0.005$$

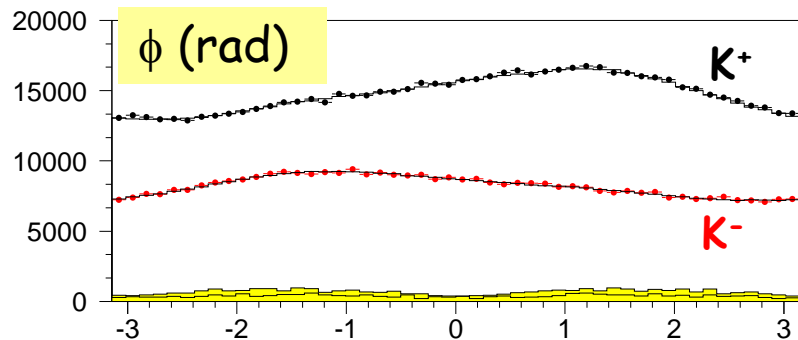
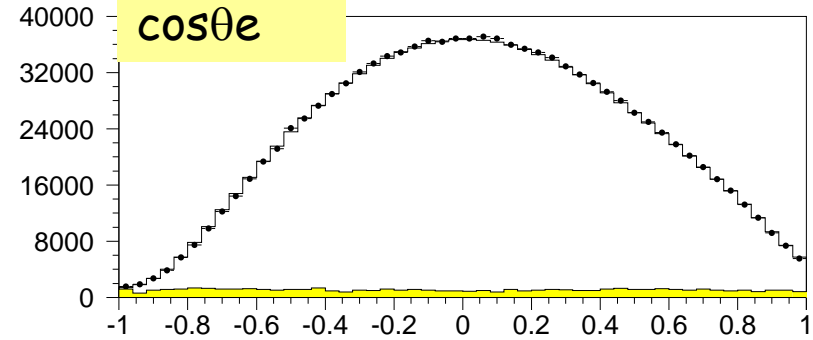
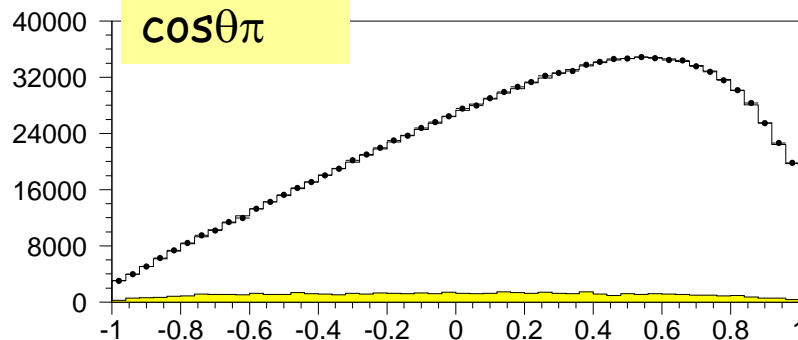
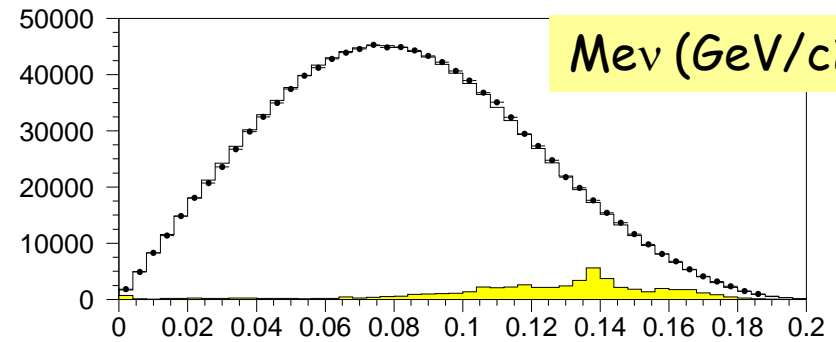
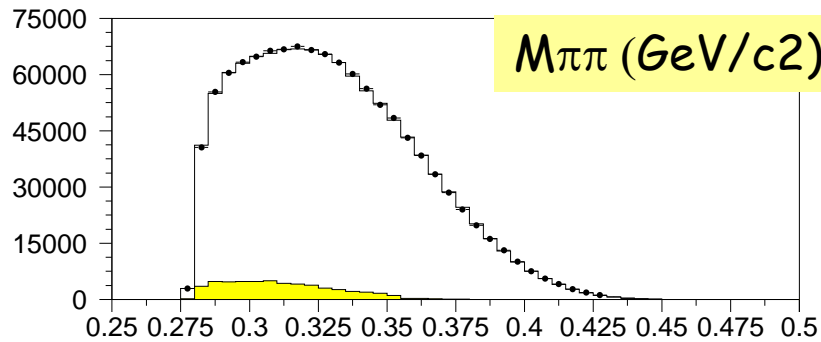
NA48/2 experimental precision now at the same level !



Final publications coming soon, fruitful collaboration with theory groups

spares

Ke4 decays : Data/MC comparison after fit

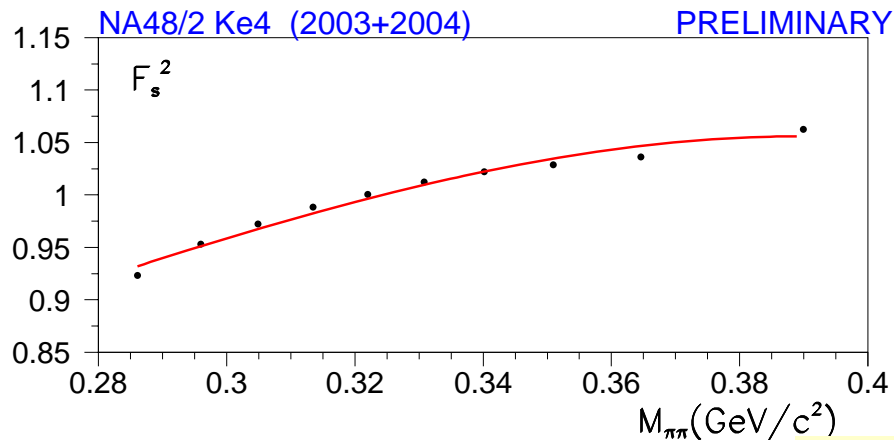


	= Data
	= Simulation after fit
	= WS Background (x 10 to be visible)

CP symmetry : (K^+) ϕ distribution is opposite of (K^-) ϕ distribution

Ke4 decays : getting F,G,H form factors and phase shift

- **Ten independent fits**, one in each $M_{\pi\pi}$ bin, assuming \sim constant form factors over each box. This allows a **model independent** analysis.
- Without the overall normalization (Branching fraction), one can quote **relative form factors** and their **variations with q^2, q^4** ($q^2 = (S_{\pi}/4m_{\pi}^2 - 1)$ and $S_{\pi}/4m_{\pi}^2$)
- F_s^2 is obtained from the relative bin to bin normalization Data/MC after fit
- if projected along Mev, a residual variation is observed.
- a 2-dimension fit of the normalization is performed to get the 3 slopes



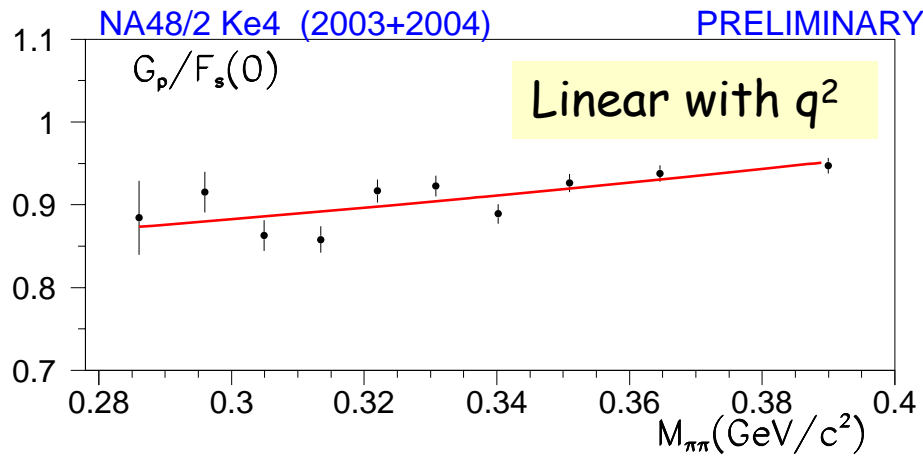
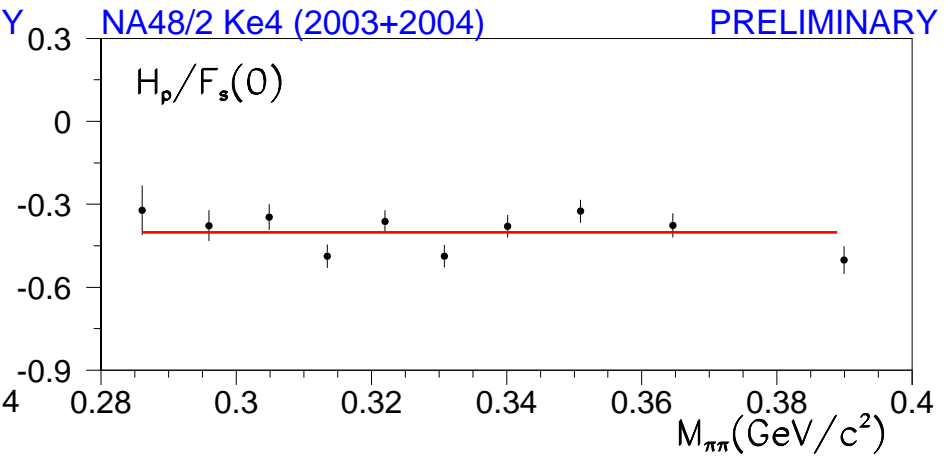
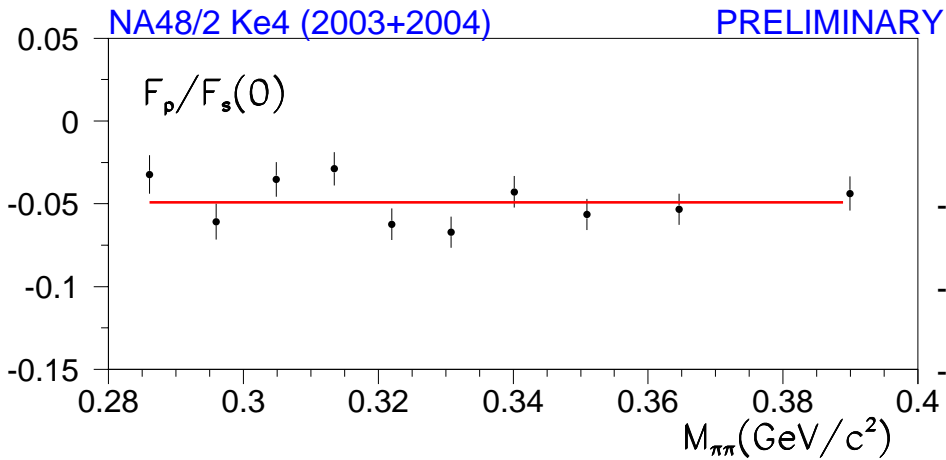
The 3 slopes are correlated

$$F_s^2 \propto (1 + f's q^2 + f''s q^4 + f'e S_{\pi}/4m_{\pi}^2)^2$$

	$f''s$	$f'e$
$f's$	-0.95	0.08
$f''s$		0.02

Other parameterizations could be easily tried if the Taylor expansion does not apply..

Getting F_p , G_p , H_p

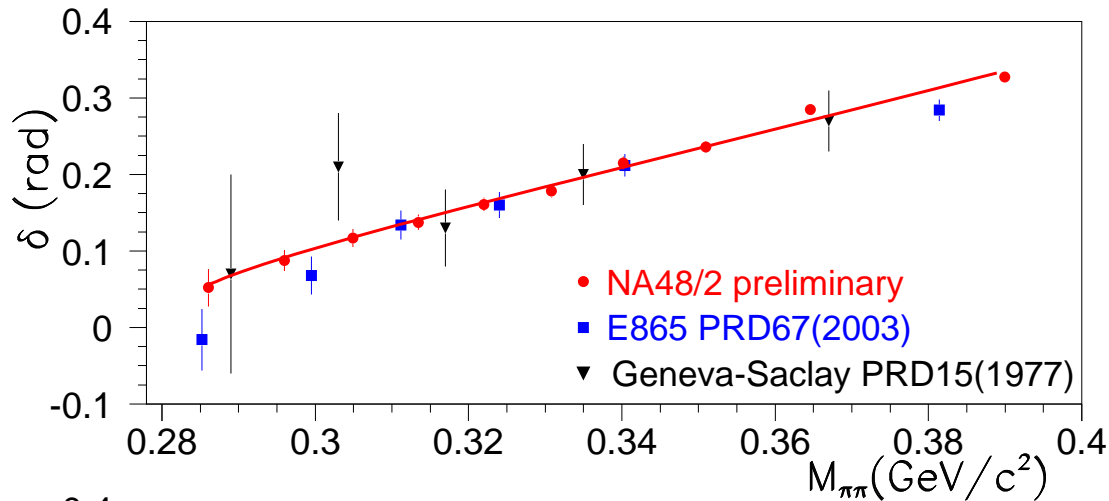


Correlation

$$g'_p$$

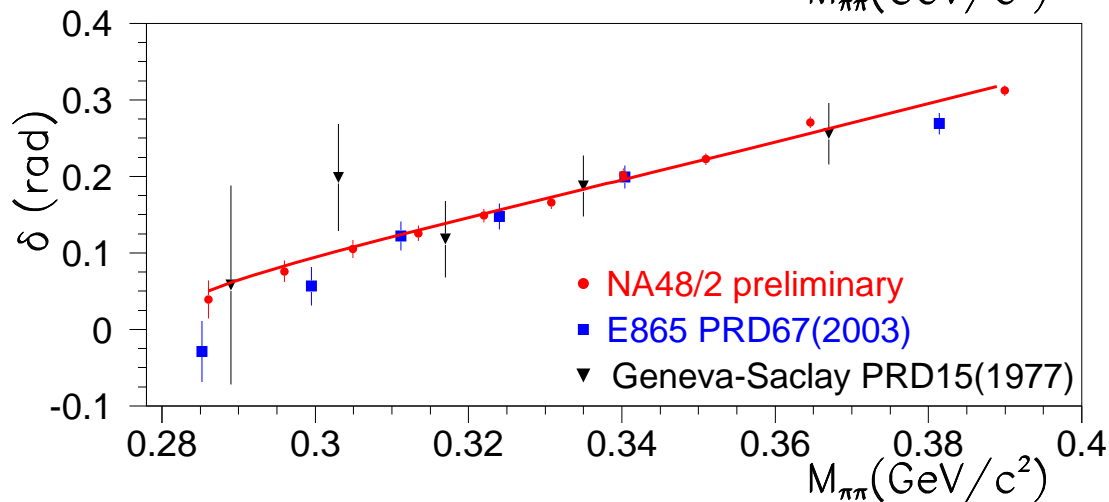
$$g_p(0) \quad -0.914$$

Comparison of Ke4 phase experimental measurements



Phase points
without isospin
corrections

Line from a 2p fit to
NA48 data alone



All Phase points
corrected for isospin
mass effects

K3π : measurement of the k' term

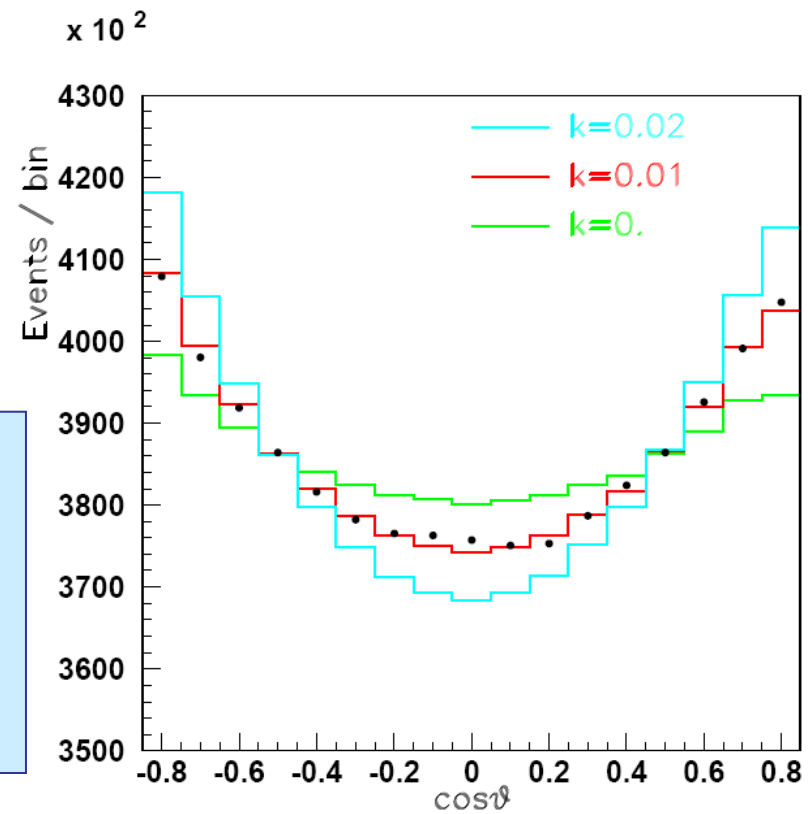
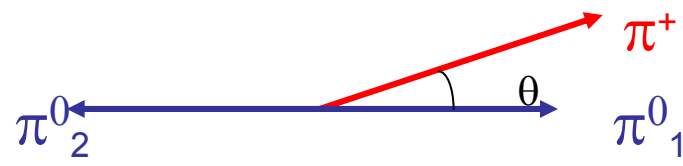
Going to a 2D fit would imply to use (M_{00}^2, M_{+0}^2) variables. An alternate choice is $(M_{00}^2, \cos\theta)$ where θ is the angle between the charged π and the direction of the π^0 's in their rest frame.

Use a modified matrix element :

$$M_0 = A_0 \left(1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 + \frac{1}{2} k' v^2 \right)$$

re-fit in M_{00}^2 range [0.082, 0.097] $(\text{GeV}/c^2)^2$

no incidence on previous $(\mathbf{a}_0 - \mathbf{a}_2)$ result.



Preliminary result (2003+2004 data, K⁺ and K⁻)

$k' = 0.0095 \pm 0.0002_{\text{stat}} \pm 0.0005_{\text{syst}}$

Note: the different meaning (g_0, h', k') wrt PDG (g_0, h, k)

Cusp: Dalitz plot slopes result

Reminder: $M(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$

$M_0 \sim (1 + g_0 u/2 + h'_0 u^2/2 + k'_0 v^2/2)$ but ...

$|M_0|^2$ (PDG) $\sim (1 + g u + h u^2 + k v^2)$ so $g_0 \approx g$, $h'_0 \approx h - g^2/4$, $k'_0 \approx k$

- k'_0 is extracted from a 2-dimensional fit

- Other parameters are fitted including a fixed k'_0 value

$$k'_0 = 0.0095 \pm 0.0002_{\text{stat.}} \pm 0.0005_{\text{syst}}$$

g_0

h'_0

Preliminary
(2003+2004)

	g_0	h'_0
CI model	$0.652 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$	$-0.039 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$
CGKR model	$0.621 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$	$-0.049 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$
ChPT fit	CI $a_0 - a_2 = 0.268$ $a_0 = 0.2244$ $a_2 = -0.0434$	GCKR $a_0 - a_2 = 0.266$ $a_0 = 0.2191$ $a_2 = -0.0470$