QCD at low energy: mesonic sector

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Outline

Low energy hadronic physics

- laboratory for testing our understanding of nonperturbative physics with the help of two complementary approaches
  - effective field theory
  - lattice QCD
- important for the determination of standard model parameters (*e.g.* light quark masses, V_{us}, $V_{ts}^{*}V_{td}$ from $K \to \pi\nu\bar{\nu}$)
- in the LHC era still an important experimental activity (NA48, NA62, DIRAC)
- Focus of this talk: $\pi\pi$ scattering
  - isospin breaking corrections and experimental data
  - lattice results

Why is $\pi\pi$ scattering interesting

- the pions are the quasi-Goldstone bosons of spontaneous chiral symmetry breaking of QCD
- $m_{u,d}/\Lambda_{\mathrm{QCD}} \sim$ few percent $\Rightarrow$ precise predictions within the effective field theory method are possible
- $\pi\pi$ scattering is special also from the S-matrix point of view: at low energy dispersion relations and unitarity relate this amplitude only to itself, also in the crossed channels
- conversely, many other observables are influenced by the $\pi\pi$ interaction in intermediate or final states (*e.g.* $K \to 2\pi, 3\pi$, $\eta \to 3\pi$, $(g-2)_\mu$)
- recent progress in experiments and on the lattice allows a thorough test of our understanding of QCD at low energy

Low-energy theorem for $\pi\pi$ scattering

$$\mathcal{M}(\pi^0\pi^0 \to \pi^+\pi^-) \equiv A(s,t,u) = \text{isospin invariant amplitude}$$

Low energy theorem: $$A(s,t,u) = \frac{s - M^2}{F^2} + \mathcal{O}(p^4)$$ Weinberg 1966

$$M^2 = B(m_u + m_d) \qquad M_\pi^2 = M^2 + O(p^4), \quad F_\pi = F + O(p^2)$$

All physical amplitudes can be expressed in terms of $A(s,t,u)$

S wave projections

(I=0) $$t_0^0(s) = \frac{2s - M_\pi^2}{32\pi F_\pi^2} + O(p^4) \qquad a_0^0|_{O(p^2)} = \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16$$

(I=2) $$t_0^2(s) = \frac{2M_\pi^2 - s}{32\pi F_\pi^2} + O(p^4) \qquad a_0^2|_{O(p^2)} = \frac{-M_\pi^2}{16\pi F_\pi^2} = -0.045$$

Chiral predictions for a_0^0 and a_0^2

Quark mass dependence of M_π and F_π:

$$M_\pi^2 = M^2 \left(1 - \frac{M^2}{32\pi^2 F^2}\bar{\ell}_3 + O(p^4)\right)$$

$$M^2 \equiv -\frac{m_u + m_d}{F^2}\langle 0|\bar{q}q|0\rangle$$

Gell-Mann, Oakes, Renner (68)

$$F_\pi = F\left(1 + \frac{M^2}{16\pi^2 F^2}\bar{\ell}_4 + O(p^4)\right)$$

Phenomenological determinations (indirect):

$$\bar{\ell}_3 = 2.9 \pm 2.4$$

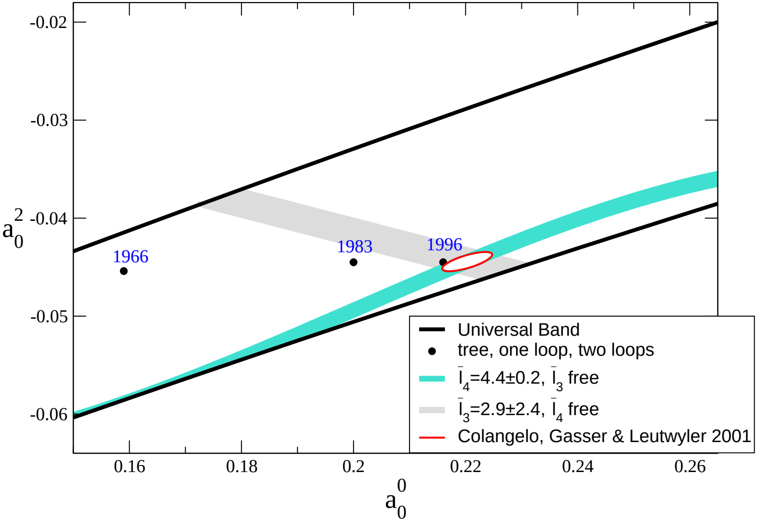
Gasser & Leutwyler (84)

$$\bar{\ell}_4 = 4.4 \pm 0.2$$

GC, Gasser & Leutwyler (01)

Lattice calculations determine these constants directly (cf. below)

Chiral predictions for a_0^0 and a_0^2



Calculation at $O(p^6)$ – numerical analysis

Scattering lengths have been calculated at NLO (Gasser & Leutwyler 84)
and at NNLO (Bijnens, GC, Ecker, Gasser & Sainio, 95)
Numerical prediction obtained matching the $O(p^6)$ chiral representation to the dispersive one (solution of the Roy equations):

GC, Gasser & Leutwyler (01)

$$
\begin{aligned}
a_0^0 &= \quad 0.220 \pm 0.001 + 0.009\Delta\ell_4 - 0.002\Delta\ell_3 \\
10 \cdot a_0^2 &= -0.444 \pm 0.003 - 0.01\Delta\ell_4 - 0.004\Delta\ell_3
\end{aligned}
$$

where $\bar{\ell}_4 = 4.4 + \Delta\ell_4$ $\quad \bar{\ell}_3 = 2.9 + \Delta\ell_3$

Adding errors in quadrature $[\Delta\ell_4 = 0.2, \Delta\ell_3 = 2.4]$

$$
\begin{aligned}
a_0^0 &= \quad 0.220 \pm 0.005 \\
10 \cdot a_0^2 &= -0.444 \pm 0.01 \\
a_0^0 - a_0^2 &= \quad 0.265 \pm 0.004
\end{aligned}
$$

Experiments on $\pi\pi$ scattering

$\pi\pi$ scattering at low energy can be measured in

- pionium decay
  the decay width is proportional to $(a_0 - a_2)^2$ DIRAC
- $K^\pm \to \pi^0\pi^0\pi^\pm$
  a cusp at $M_{\pi^0\pi^0} = 2M_{\pi^+}$ is proportional to $(a_0 - a_2)^2$ NA48
- K_{e4} decays
  the phase difference $\delta_0^0 - \delta_1^1$ between threshold and $s \sim M_K^2$ can be extracted by measuring certain angular distributions E865, NA48

All measurements have reached a remarkably high accuracy
$\Rightarrow$ necessary to take isospin breaking corrections into account

Pionium lifetime measurement

Master formula:

Deser, Goldberger, Baumann, Thirring 54

Gall, Gasser, Lyubovitskij, Rusetsky 99, 01

cf. also Sazdjian, and Gashi, Oades, Rasche, Woolcock

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^\star (a_0^0 - a_0^2)^2 (1 + \delta_1) + o(\delta^{9/2})$$

with $p^\star$ the modulus of the π^0 3-momentum and
$\delta_1 = (5.8 \pm 1.2) \times 10^{-2}$ isospin breaking corrections

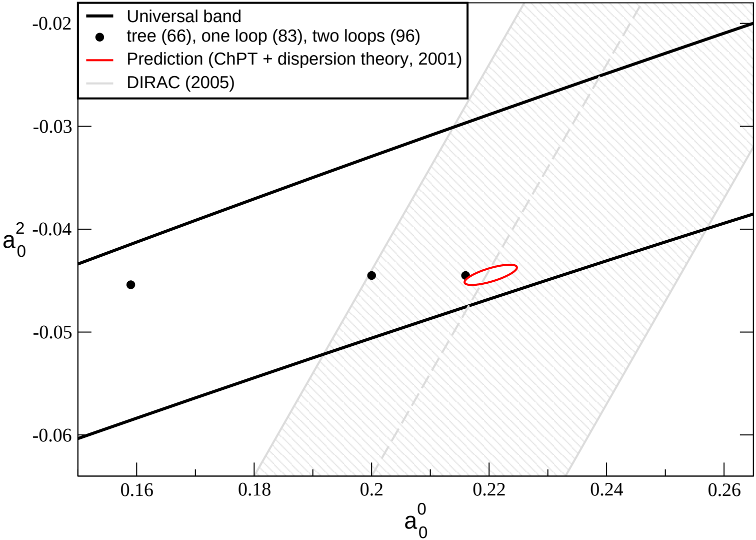
DIRAC (05):

$$\tau = 2.91\ {}^{+0.45}_{-0.38}\ (\text{stat})\ {}^{+0.19}_{-0.49}\ (\text{syst}) \times 10^{-15}\text{s}$$

which translates to

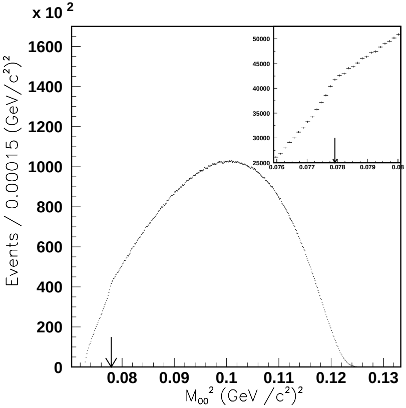
$$a_0^0 - a_0^2 = 0.264\ {}^{+0.033}_{-0.020}$$

Pionium lifetime measurement



Cusp in $K^\pm \longrightarrow \pi^0\pi^0\pi^\pm$

NA48 has made a high-statistics measurement of $K^\pm \longrightarrow \pi^0\pi^0\pi^\pm$ and clearly observed a cusp in the $\pi^0\pi^0$ spectrum at the $\pi^+\pi^-$ threshold



Cusp in $K^\pm \to \pi^0\pi^0\pi^\pm$

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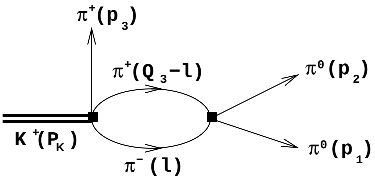
Theoretical interpretation: Cabibbo 04

Early history: Wigner (48), Budini, Fonda (61),... , Bernstein et al. (97), Meißner et al. (97)

Two-loop treatment: Cabibbo, Isidori 05, Gamiz, Prades, Scimemi 06, GC, Gasser, Kubis, Rusetsky 06

a $\pi^+\pi^-$ intermediate state is responsible for the cusp



Cusp in $K^\pm \to \pi^0\pi^0\pi^\pm$: results

NA48 published the first results of their analysis in 2006:

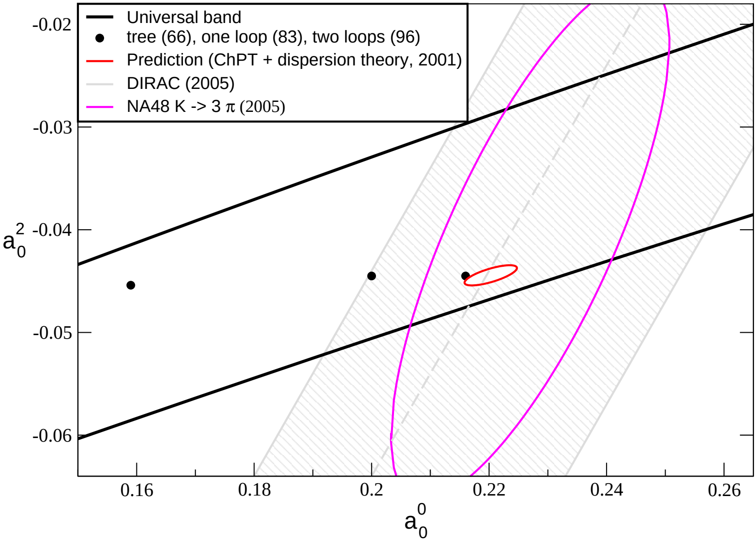
$$\begin{aligned} a_0^0 - a_0^2 &= 0.268 \pm 0.010(\text{stat}) \pm 0.004(\text{syst}) \pm 0.013(\text{ext}) \\ a_0^2 &= -0.041 \pm 0.022(\text{stat}) \pm 0.014(\text{syst}) \end{aligned}$$

Correlation coeff. $= -0.858$
Constraining a_0^0 and a_0^2 to respect the low-energy theorem which relates them to the scalar radius they get

$$a_0^0 = 0.220 \pm 0.006(\text{stat}) \pm 0.004(\text{syst}) \pm 0.011(\text{ext})$$

More statistics already analyzed – cf. talk by Bloch-Devaux

Cusp in $K^\pm \to \pi^0\pi^0\pi^\pm$: results



Extracting the $\pi\pi$ phases from K_{e4}

Isospin limit

$$\begin{aligned} V_\mu - A_\mu &\equiv \langle \pi(p_1)\pi(p_2)|\bar{s}\gamma_\mu(1-\gamma_5)u|K(p)\rangle \\ A_\mu &= \frac{-i}{M_K}\left[(p_1+p_2)_\mu F + (p_1-p_2)_\mu G + L_\mu R\right] \end{aligned}$$

$$F = f_s e^{i\delta_0^0} + f_p e^{i\delta_1^1}\cos\theta + D\text{-wave} \qquad G = g_p e^{i\delta_1^1} + D\text{-wave}$$

the phases δ_ℓ^I are those of $\pi\pi$ scattering (Watson's theorem)

Measuring certain angular distributions one can extract very cleanly the phase difference:

$$\delta_0^0(s) - \delta_1^1(s) \quad 2M_\pi < \sqrt{s} < M_K - m_l$$

Extracting the $\pi\pi$ phases from K_{e4}

Isospin breaking effects

radiative corrections and $m_u - m_d \neq 0$ break isospin

NA48 corrects two effects

- the effect of the Coulomb attraction between two slow pions is removed by applying the Gamow factor
- the effect of real emission of photons is taken into account with the help of the program PHOTOS Was et al.

The mass splitting between the charged and neutral pions and the difference $m_d - m_u$, however, are not corrected for

We (GC, Gasser and Rusetsky) proceed by assuming that

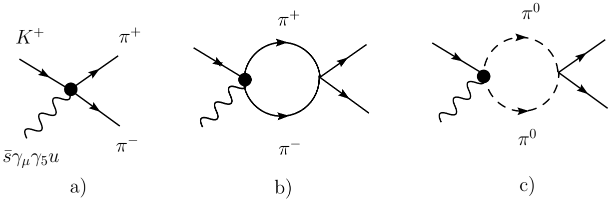
Full isospin breaking eff. = Coulomb factor×PHOTOS×mass effects

and discuss how the latter affect the measured phases

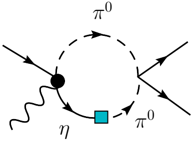
Isospin breaking effects in K_{e4}

GC, Gasser Rusetsky

Tree and one-loop diagrams in K_{e4}:



The different thresholds between b) and c) affect the phases



The $\pi^0 - \eta$ mixing $\propto (m_u - m_d)$ also modifies the phases

Isospin breaking effects in K_{e4}

GC, Gasser Rusetsky

Both effects add up to [$\delta =$measured phase]

$$\delta = \frac{1}{32\pi F^2}\left\{(4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2)\left(1 + \frac{3}{2R}\right)\sigma_0\right\} - \delta_1^1 + O(p^4)$$

where $\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2, \quad \sigma = \sqrt{1 - \frac{4M_\pi^2}{s}}, \quad R = \frac{m_s - \hat{m}}{m_d - m_u}$

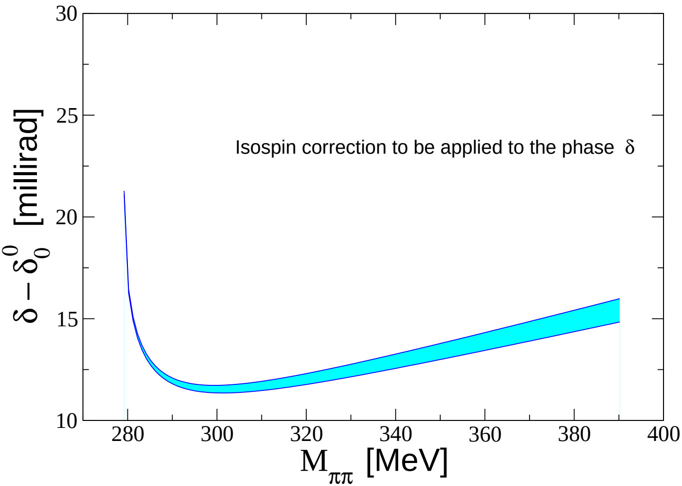
For the numerical analysis we use $R = 37 \pm 4$

Related work also by

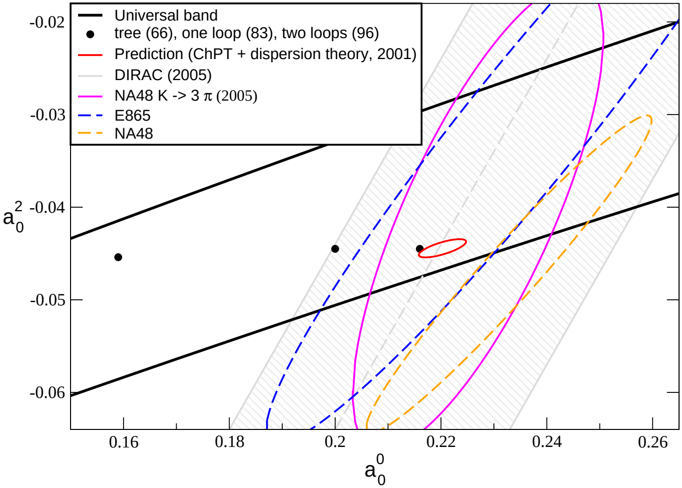
- Cuplov and Nehme, Nehme [CHPT calculation at $O(p^4)$ including isospin breking effects]
- Gevorkyan et al. 07 [Coulomb and real photon corrections $\Leftrightarrow$ double counting problems – approach not sound]
- Descotes-Genon and Knecht (in preparation)

Isospin breaking effects in K_{e4}

GC, Gasser Rusetsky



Effect of isospin breaking on the scattering lengths



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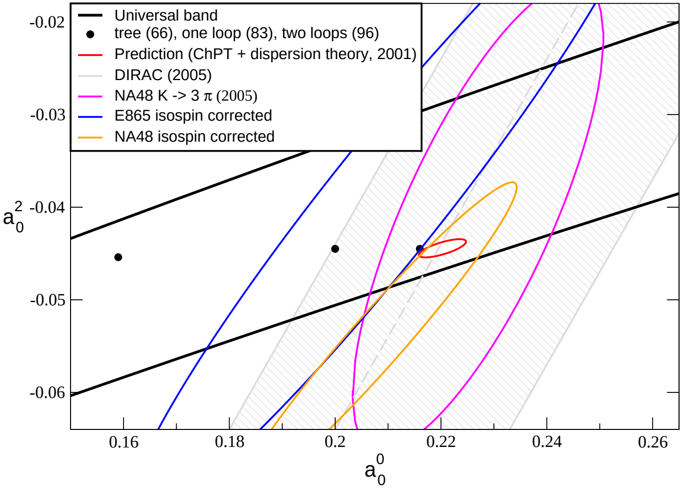


GC, Gasser Rusetsky

Effect of isospin breaking on the scattering lengths

Fits assuming the LET ($a_0^2(a_0^0)$)

Before...

$$a_0^0 = \begin{cases} 0.243 \pm 0.037 & \chi^2 = 2.2 \quad \text{Geneva-Saclay} \quad [\ 5 \text{ data}] \\ 0.218 \pm 0.013 & \chi^2 = 5.7 \quad \text{E865} \quad [\ 6 \text{ data}] \\ 0.245 \pm 0.007 & \chi^2 = 9.6 \quad \text{NA48} \quad [10 \text{ data}] \end{cases}$$

and after applying isospin breaking corrections

$$a_0^0 = \begin{cases} 0.222 \pm 0.040 & \chi^2 = 2.1 \quad \text{Geneva-Saclay} \\ 0.195 \pm 0.013 & \chi^2 = 6.6 \quad \text{E865} \\ 0.223 \pm 0.007 & \chi^2 = 11.5 \quad \text{NA48} \end{cases}$$

Averaging the latter three independent determinations yields

$$a_0^0 = 0.217 \pm 0.008_{\text{exp}} \pm 0.006_{\text{th}} \qquad [S = 1.3]$$

GC, Gasser Rusetsky, 08

new results with more statistics available, cf. talk by B. Bloch-Devaux

Lattice calculation of a_0^2 (NPLQCD coll.)

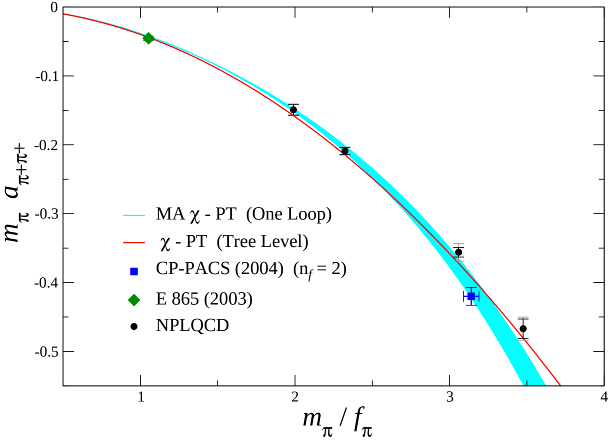


Figure from NPLQCD arXiv:0706.3026

$\bar{\ell}_3$ and $\bar{\ell}_4$

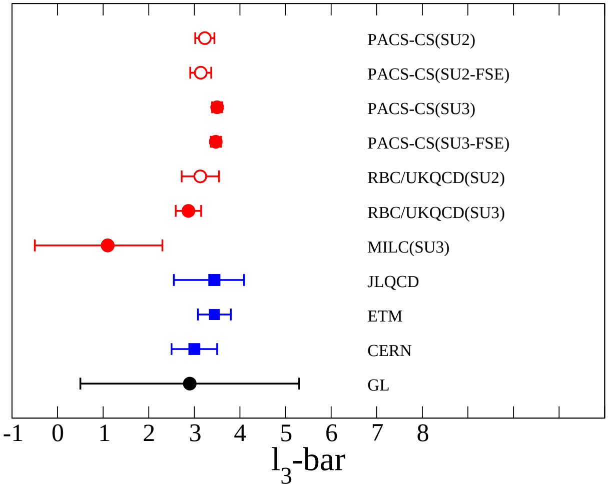


Figure from PACS-CS arXiv:0807.1661

$\bar{\ell}_3$ and $\bar{\ell}_4$

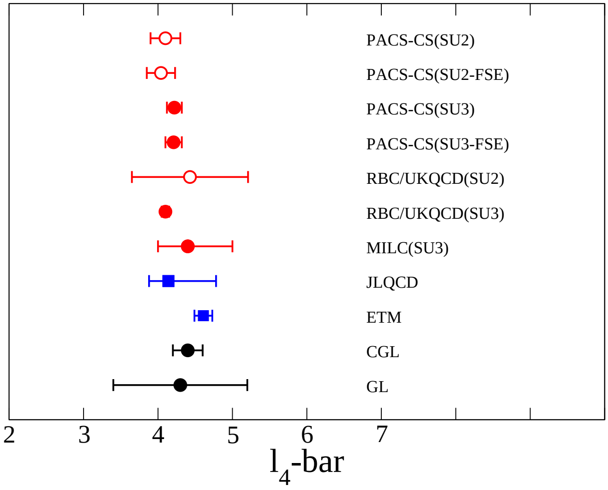
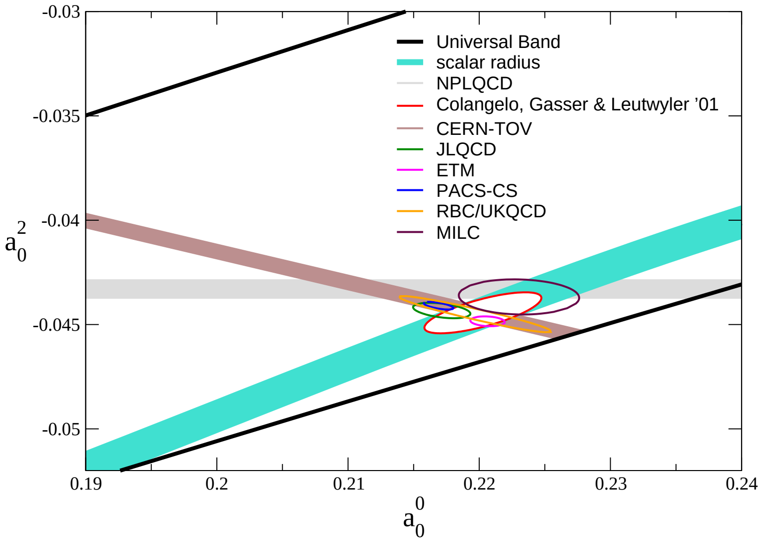


Figure from PACS-CS arXiv:0807.1661

Implications for the $\pi\pi$ scattering lengths



Summary

- The high precision in the prediction for the scattering lengths is obtained through a combined use of dispersive methods and chiral symmetry
- Experimental data are approaching the same level of accuracy and thereby test the underlying assumptions about the structure of the QCD vacuum
- With present data it is essential to apply radiative and other isospin breaking corrections and to do it within a controlled framework: NREFT and CHPT
- Today even the direct comparison to first principle QCD calculations is possible: recent lattice calculations of the $I = 2$ scattering length and of the quark mass dependence of F_π and M_π

Pionium lifetime measurement

Master formula:

Deser, Goldberger, Baumann, Thirring 54

Gall, Gasser, Lyubovitskij, Rusetsky 99, 01

cf. also Sazdjian, and Gashi, Oades, Rasche, Woolcock

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^\star (a_0^0 - a_0^2)^2 (1 + \delta_1) + o(\delta^{9/2})$$

with $p^\star$ the modulus of the π^0 3-momentum and
$\delta_1 = (5.8 \pm 1.2) \times 10^{-2}$ isospin breaking corrections

Formula derived in a nonrelativistic effective field theory:

- a_0^0 and a_0^2 are parameters of the theory (no assumptions about them)
- systematic expansion in $\delta = \alpha, (m_u - m_d)^2$
- CHPT used only in the estimate of δ_1
- theoretical framework under control

Gasser, Lyubovitskij, Rusetsky, Phys. Rep. 08

$\bar{\ell}_3$ and $\bar{\ell}_4$

group	ChPT	$\bar{\ell}_3$	$\bar{\ell}_4$
		$N_f = 2+1$	
PACS-CS	SU(2) no FV	3.23(21)	4.10(20)
	SU(2) FV	3.14(23)	4.04(19)
	SU(3) no FV	3.50(11)	4.22(10)
	SU(3) FV	3.47(11)	4.21(11)
RBC/UKQCD	SU(3)	2.87(28)	4.10(5)
	SU(2)	3.13(33)(24)	4.43(14)(77)
MILC	SU(3)	$1.1(6)\left(\begin{smallmatrix}+1.0\\-1.5\end{smallmatrix}\right)$	$4.4(4)\left(\begin{smallmatrix}+4\\-1\end{smallmatrix}\right)$
		$N_f = 2$	
JLQCD	SU(2)	$3.44(57)\left(\begin{smallmatrix}+0\\-68\end{smallmatrix}\right)\left(\begin{smallmatrix}+32\\-0\end{smallmatrix}\right)$	$4.14(26)\left(\begin{smallmatrix}+49\\-0\end{smallmatrix}\right)\left(\begin{smallmatrix}+32\\-0\end{smallmatrix}\right)$
ETM	SU(2)	3.44(8)(35)	4.61(4)(11)
CERN	SU(2)	3.0(5)(1)	—
		pheno	
CGL	SU(2)	2.9(2.4)	4.4(2)