



Recent results from NA48/2 on  $K_{e4}$  and  $K \rightarrow \pi^{\pm}\pi^0\pi^0$  decays  
Interpretation in terms of  $\pi\pi$  scattering lengths



Brigitte Bloch-Devauz  
CEA Saclay, DAPNIA-SPP

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On behalf of the NA48/2 collaboration:  
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz,  
Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

- The NA48/2 experiment : goals, beams, detector, performances
- Ke4 charged decays ( $K^\pm \rightarrow e^\pm \nu \pi^+ \pi^-$ ): formalism, event selection, form factors
- Ke4 neutral decays ( $K^\pm \rightarrow e^\pm \nu \pi^0 \pi^0$ ): event selection, form factors, Branching fraction
- K3pi neutral decays ( $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ ): the "cusp" effect, form factors
- Interpretation in terms of  $\pi \pi$  scattering lengths
- Conclusion



## The primary goals :

Search for **CP-violating charge asymmetries** ( $K^+ K^-$ ) in  $K^\pm \rightarrow 3 \pi$  decays

Two measurements : "charged"  $\pi^\pm \pi^+ \pi^-$  and "neutral"  $\pi^\pm \pi^0 \pi^0$  asymmetries

both modes with large **BR's of  $(2-5) 10^{-2}$**

( presented by R. Arcidiacono)

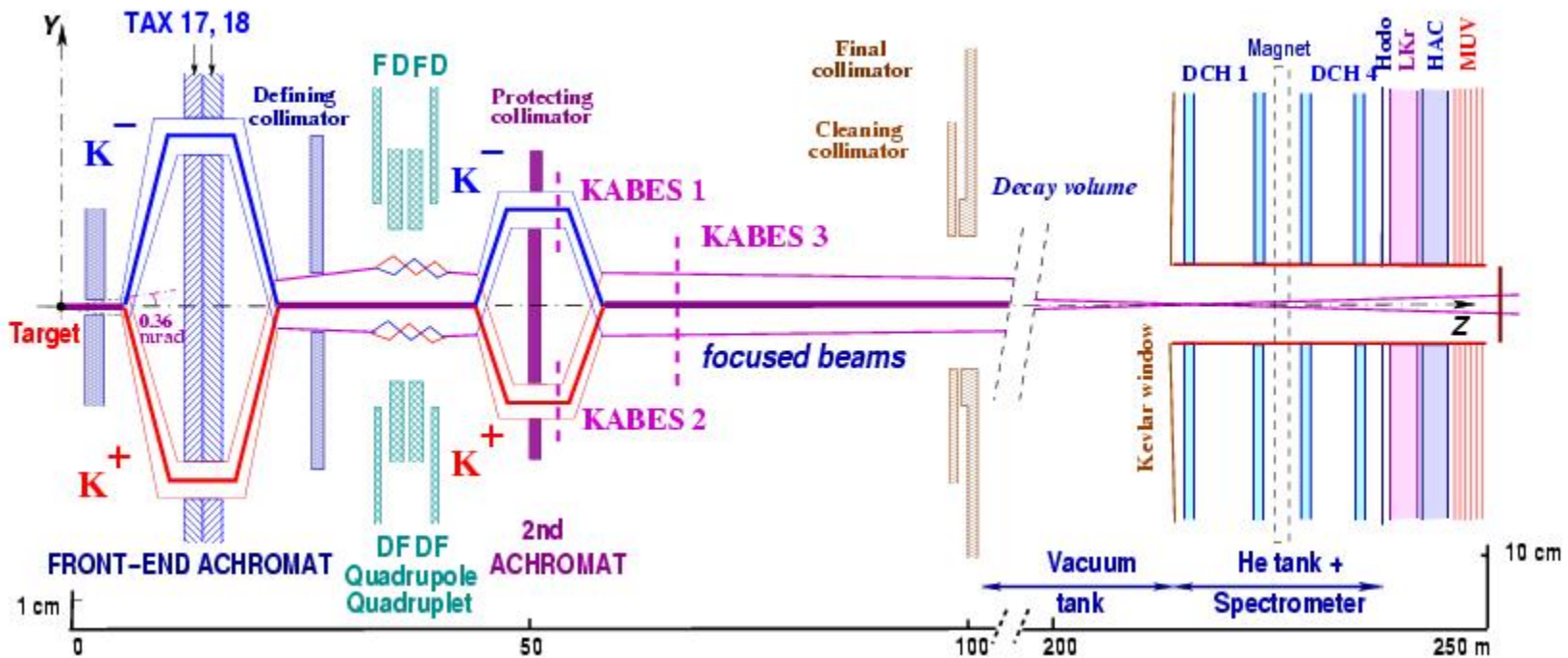
and also

Study of **rare decays like Ke4** in the "charged"  $\pi^+ \pi^- e^\pm \nu$  and "neutral"  
 $\pi^0 \pi^0 e^\pm \nu$  final states

both modes with small **BR's of few  $10^{-5}$**

ChPT predictions exists for low energy  $\pi\pi$  interaction (qq condensate)

Simultaneous  $K^+/K^-$  beams :  $(60 \pm 3) \text{ GeV}/c$



2003 Run ~50 days

2004 Run ~60 days

**Total statistics :** $\sim 4.10^9 \pi^\pm \pi^+ \pi^-$  decays $\sim 1.10^8 \pi^\pm \pi^0 \pi^0$  decays $\sim 1.10^6 \pi^+ \pi^- e^\pm \nu$  decays $\sim 4.10^4 \pi^0 \pi^0 e^\pm \nu$  decays**Preliminary results based on**charged  $K_{e4}$  370 000 decays

(30 days in 2003)

neutral  $K_{e4}$  37 000 decays

(full statistics of 2003-2004)

neutral  $K_{3\pi}$  23 000 000 decays

(30 days in 2003)

Most important components for  $K_{e4}/K_{3\pi}$  analysis :

**Magnetic spectrometer** : 4 high-resolution DCH's

$$\Delta p/p = (1.0 \oplus 0.044 p)\% \quad (p \text{ in } \text{GeV}/c)$$

→ Very good resolution for **charged invariant masses** (Kaon)

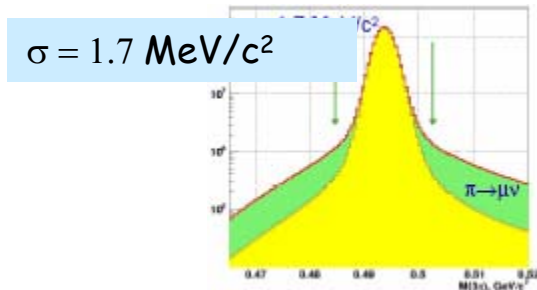
**LKr electromagnetic calorimeter** : quasi-homogenous and high granularity

$$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\% \quad (E \text{ in } \text{GeV})$$

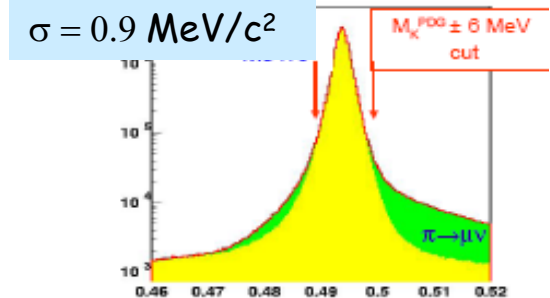
$$\sigma_x = \sigma_y \sim 1.5 \text{ mm for } E=10 \text{ GeV}$$

→ Very good resolution for **neutral invariant masses** ( $\pi^0$ )

→ E/p measurement for **e/ $\pi$  discrimination**

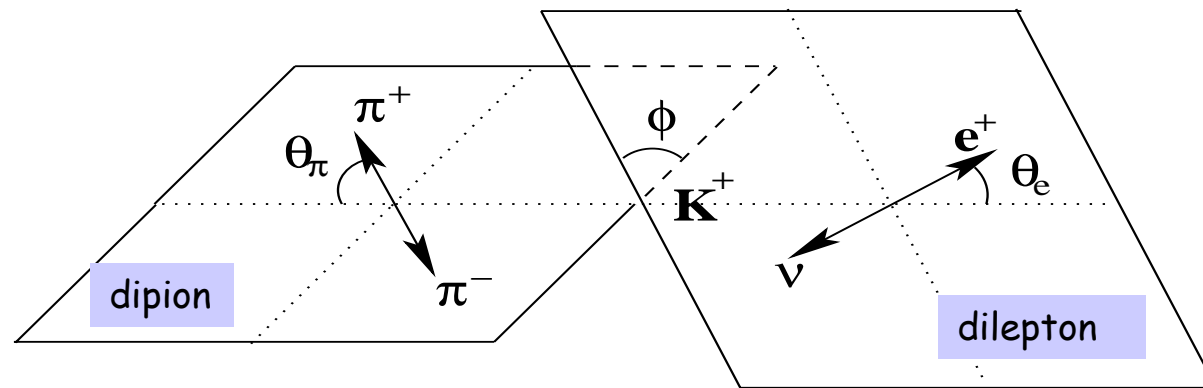


$(\pi^\pm \pi^+\pi^-)$  mass  $\text{GeV}/c^2$



$(\pi^\pm \pi^0 \pi^0)$  mass  $\text{GeV}/c^2$

The Ke4 decay are described using 5 kinematic variables ( defined by Cabibbo-Maksymowicz):  $S_\pi (M_{\pi\pi}^2)$ ,  $S_e (M_{e\nu}^2)$ ,  $\cos\theta_\pi$ ,  $\cos\theta_e$  and  $\phi$ .



The **form factors** which appear in the decay rate can be determined from a fit to the experimental data distribution of the 5 variables provided the binning is small enough.

Several formulations of the form factors appear in the literature, we have considered two of them, proposed by **Pais and Treiman** (Phys.Rev. 168 (1968)) and **Amoros and Bijmans** (J.Phys. G25 (1999)) which can be related.



Using a partial wave expansion:

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi + \text{d-wave term...}$$

$$G = G_p e^{i\delta_g} + \text{d-wave term...}$$

$$H = H_p e^{i\delta_h} + \text{d-wave term...}$$

Keeping only s and p waves ( $S_\pi$  is small in Ke4), rotating phases by  $\delta_p$  and assuming  $(\delta_g - \delta_p) = 0$  and  $(\delta_h - \delta_p) = 0$ , only 5 form factors are left:

$$F_s \quad F_p \quad G_p \quad H_p \quad \text{and} \quad \delta = \delta_s - \delta_p$$

developing in powers of  $q^2$  ( $q^2 = (S_\pi / 4m_\pi^2 - 1)$ ),  $S_e \dots$

$$F_s = f_s + f'_s q^2 + f''_s q^4 + f_e \left( S_e / 4m_\pi^2 \right) + \dots$$

$$F_p = f_p + f'_p q^2 + \dots$$

$$G_p = g_p + g'_p q^2 + \dots$$

$$H_p = h_p + h'_p q^2 + \dots$$





## Ke4 charged decays: event selection and background rejection



**Signal**  $\pi^+\pi^-\pi^\pm e^\pm \nu$  **Topology** : 3 charged tracks ,two opposite sign pions, one electron (LKr info E/p), some missing energy and  $p_T$  (neutrino)

**Background** : main sources

$\pi^\pm \pi^+ \pi^-$  decay +  $\pi \rightarrow e \nu$  decay (dominates with same topology as signal)  
+  $\pi$  misidentified as e

$\pi^\pm \pi^0(\pi^0)$  decay +  $\pi^0$  Dalitz decay ( $e^+e^-\gamma$ ) with e misidentified as  $\pi$  and  $\gamma$  (s)  
undetected

**Control from data sample** : **Wrong Sign** events have the same total charge but  $e^-$  and  $\pi^+ \pi^+$  for  $K^+$  decays ( $e^+$  and  $\pi^- \pi^-$  for  $K^-$  decays). Depending on the process, background events appear in **Right Sign** events with the **same rate** as in WS events or **twice the rate**

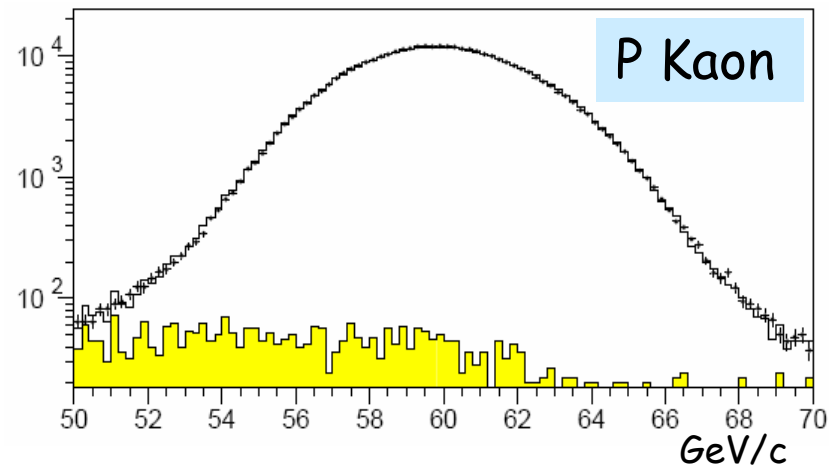
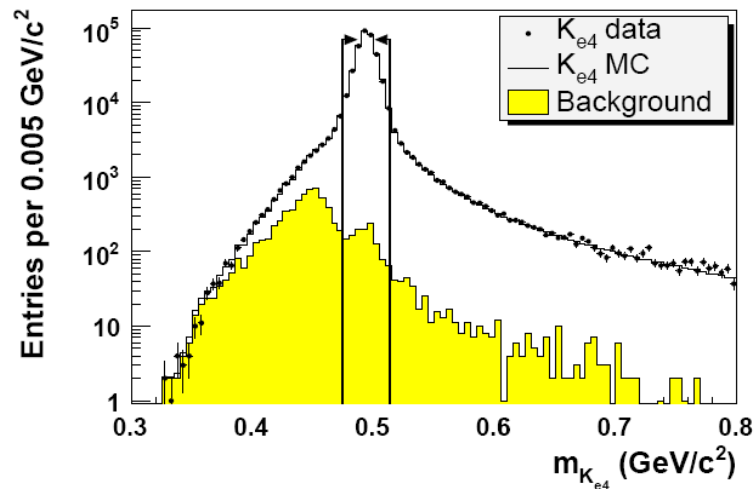
Total background level can be kept at  $\sim 0.5\%$  relative level

## Ke4 charged decays : event reconstruction

Reconstruction of the *C.M.* variables : Two options

- use  $\nu$  constrain to solve energy-momentum conservation equations and get  $P_K$
- assume a 60 GeV/c Kaon , assign the missing  $p_T$  to the  $\nu$  and compute the mass of the system

Then boost particles to the Kaon rest frame and dipion/dilepton rest frames to get the angular variables.





Using **equal population bins** in the 5-dimension space of the C.M. variables, ( $M_{\pi\pi}$ ,  $M_{e\nu}$ ,  $\cos\theta_{\pi}$ ,  $\cos\theta_e$  and  $\phi$ ) one defines a grid of

**10x5x5x5x12=15000 boxes.**

The set of form factor values are used to minimize the  $T^2$ , a log-likelihood estimator well suited for small numbers of **data events/bin  $N_j$**  and taking into account the statistics of the simulation =  **$M_j$  simulated events/bin** and  **$R_j$  expected events/bin.**

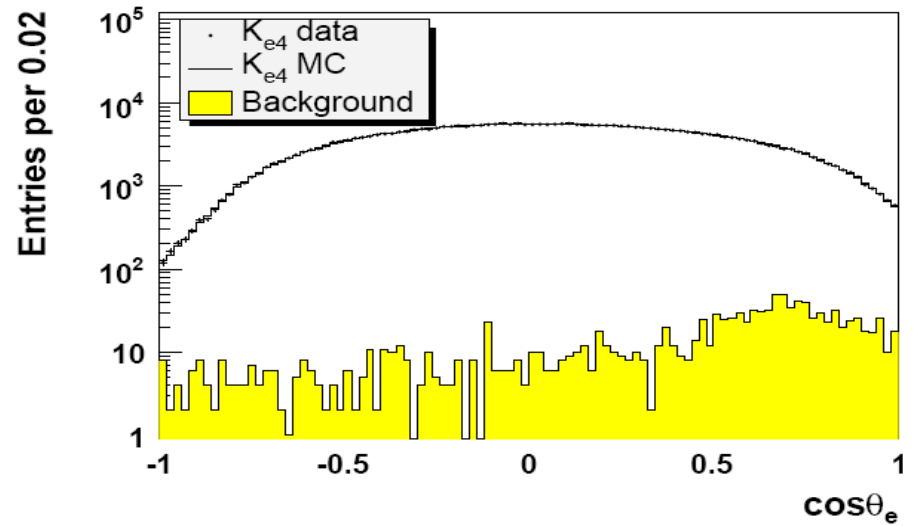
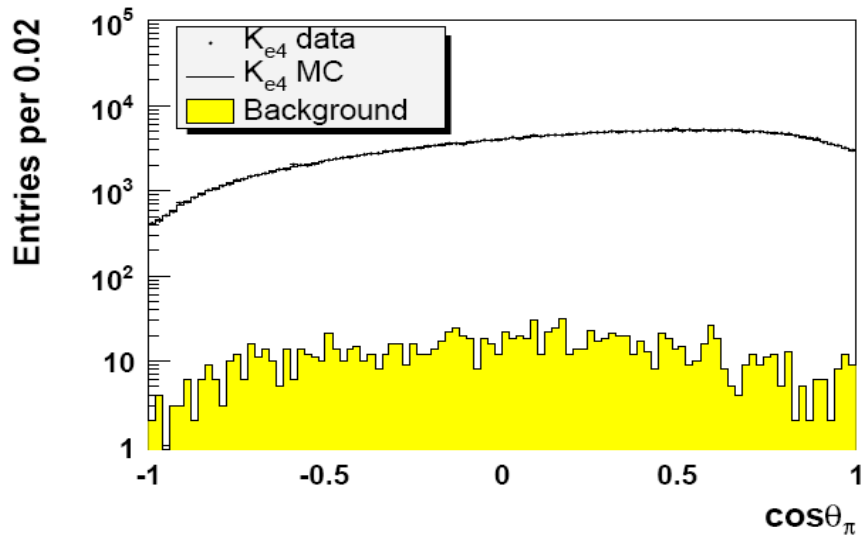
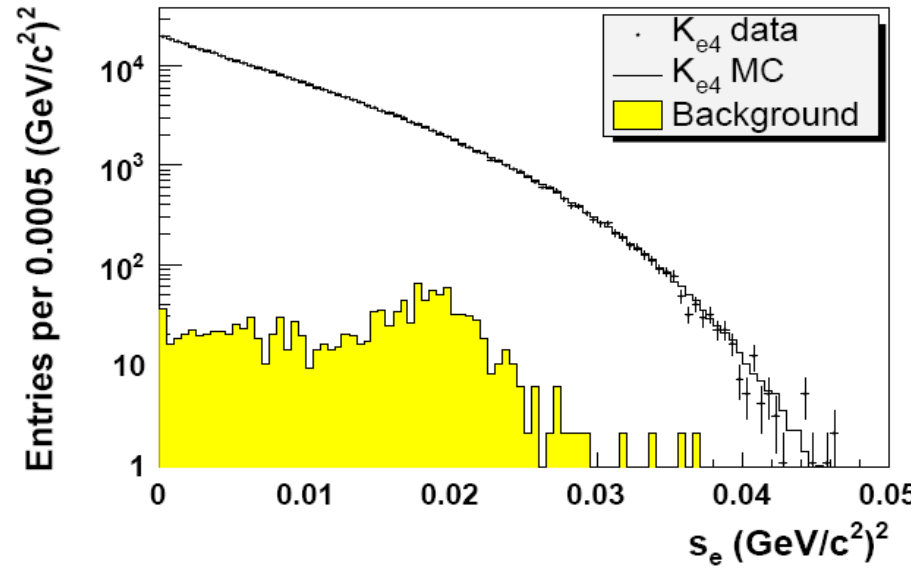
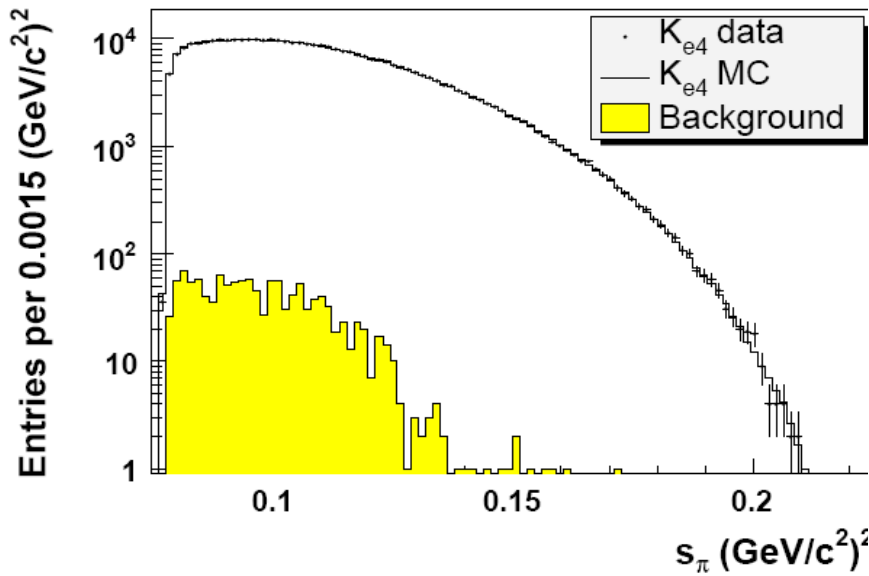
$$T^2 = 2 \sum_{j=1}^S \left\{ N_j \text{Log} \left[ \frac{N_j}{R_j} \left( 1 - \frac{1}{M_j + 1} \right) \right] + (N_j + M_j + 1) \text{Log} \left[ \frac{1 + \frac{R_j}{M_j}}{1 + \frac{N_j}{M_j + 1}} \right] \right\}$$

K<sup>+</sup> sample (235000 events)      16 events/bin

K<sup>-</sup> sample (135000 events)      9 events/bin

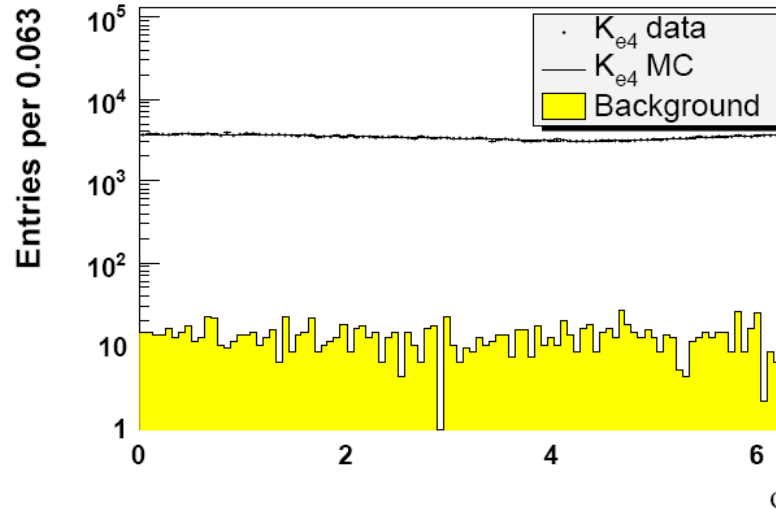
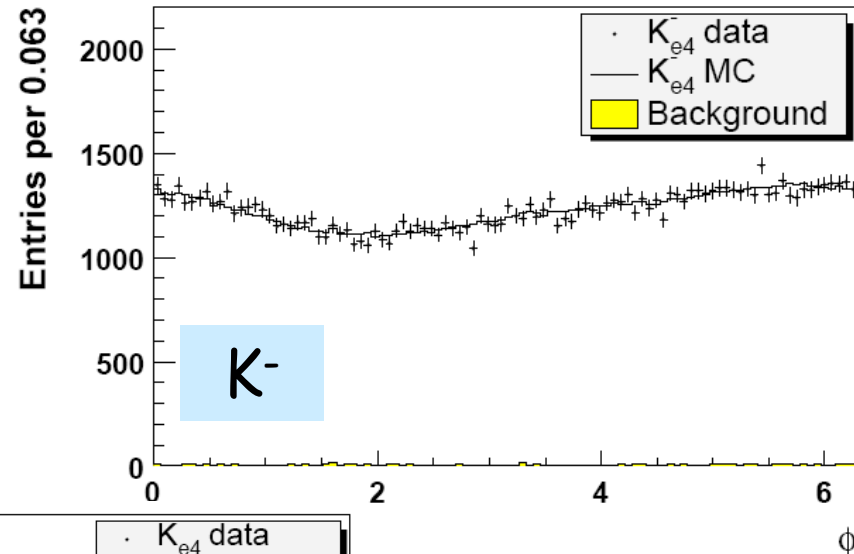
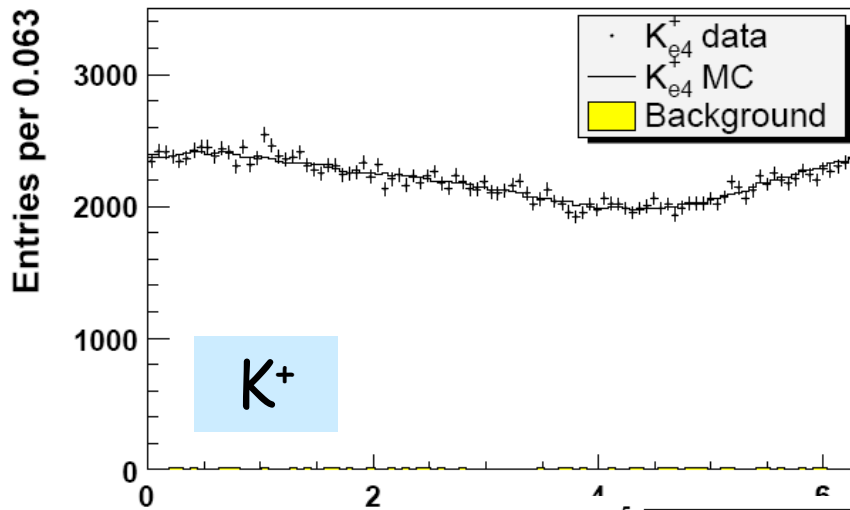


# Ke4 charged decays : 4 C.M. distributions (log scale)



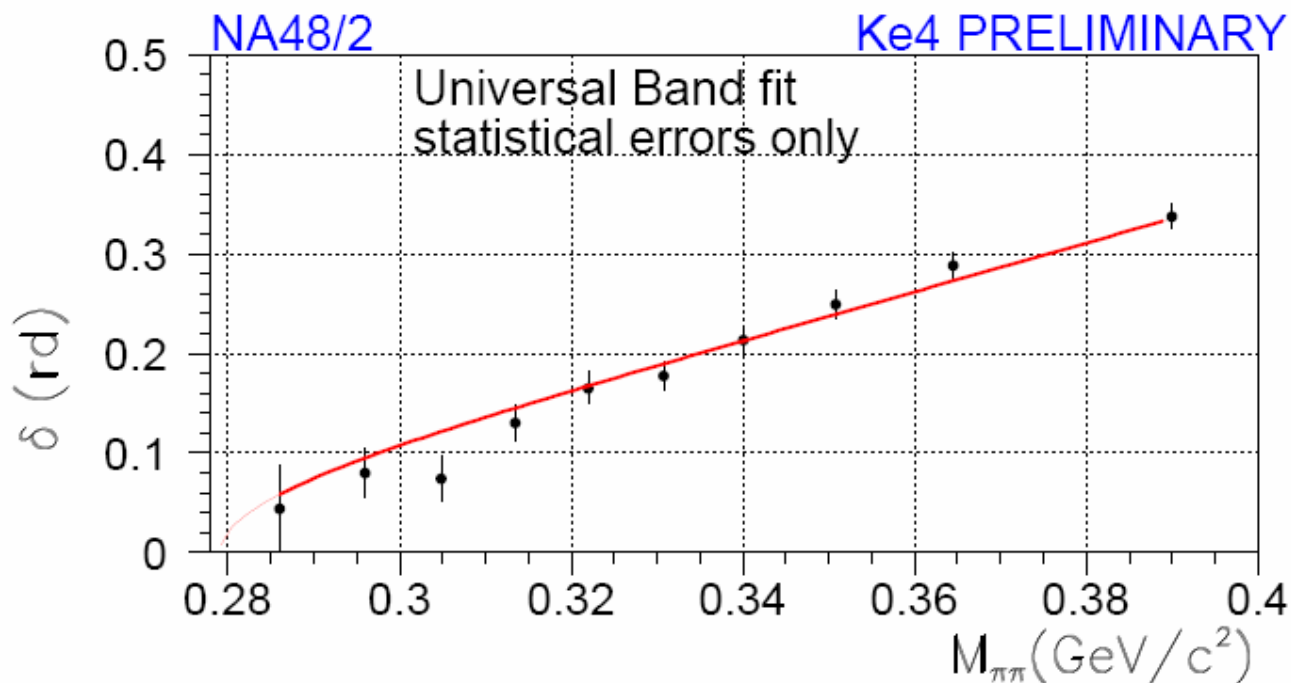
# Ke4 charged decays : the 5th distribution $\phi$

CP symmetry :  $(K^+) \phi$  distribution is opposite of  $(K^-) \phi$  distribution

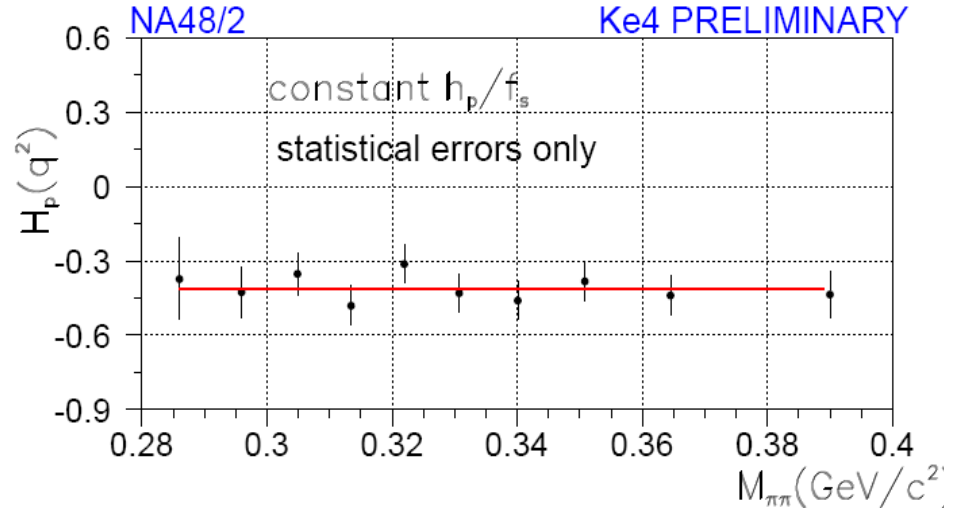
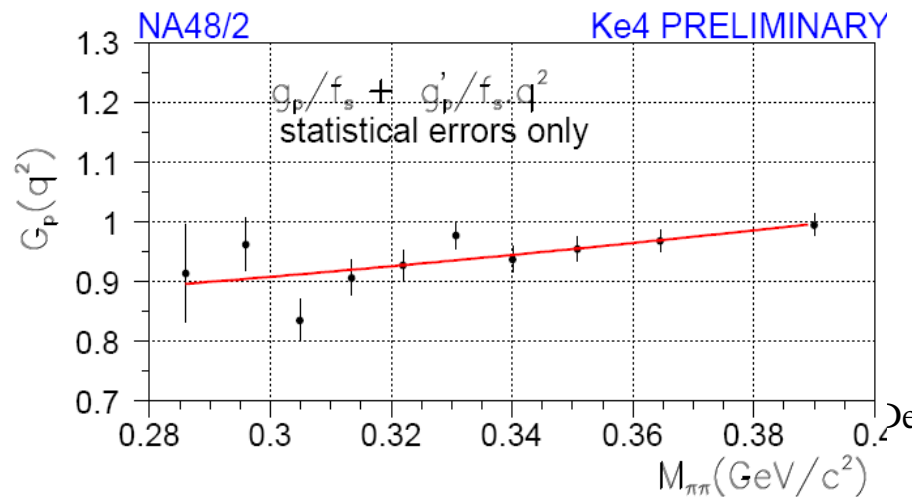
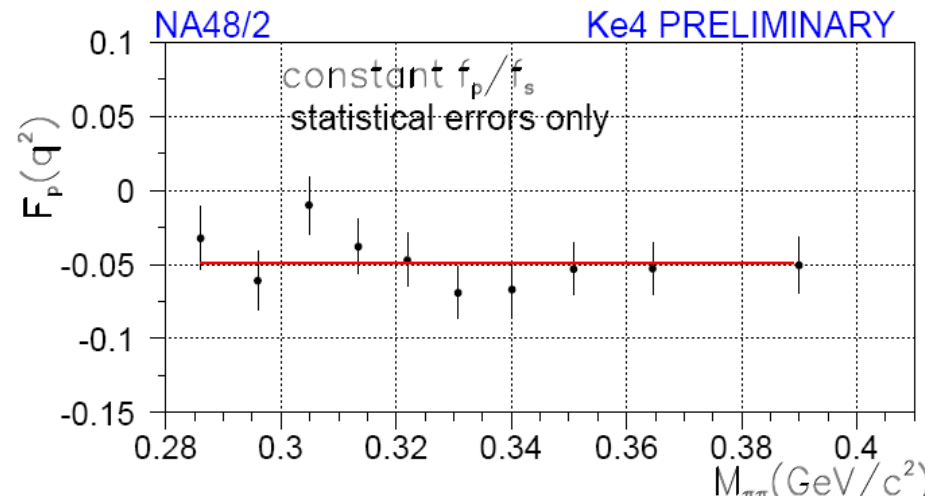
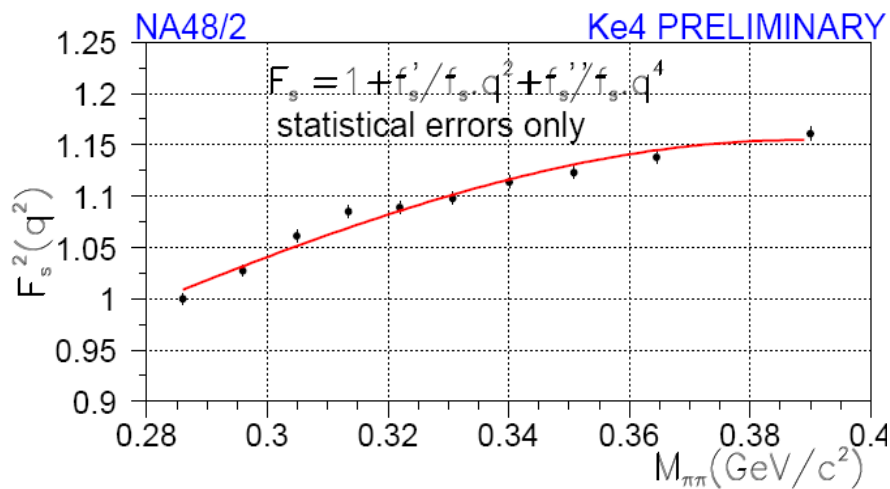


$K^+(\phi) + K^-(-\phi)$   
 Note the log scale to see the background!

- **Ten independent fits**, one in each  $M_{\pi\pi}$  bin, assuming  $\sim$ constant form factors over each bin.
- **Use the Universal Band** parameterization to extract one parameter  $a_0^0$  with  $a_0^2 = f(a_0^0)$  (numerical solution of Roy equations as Phys. Rep.353 (2001)) from  $\delta = (\delta_0^0 - \delta_1^1)$  distribution



- Not giving (yet) the overall normalization from the Branching fraction, one can quote **relative form factors** and their **variations with  $q^2, q^4$**  ( $q^2 = (S_\pi - 4m_\pi^2)/4m_\pi^2$ )
- Se dependence measurement consistent with 0.





## Form Factors $K^+$ and $K^-$ combined

Snapshot on systematic checks :

- Two independent analyses with slightly different **selections**, different Kaon **reconstruction**, **detector corrections** and **fit methods**. Residual differences quoted
- Stability of **acceptance versus time**: variation of simulated beam conditions
- **Background level** control : estimated both with data (varying cuts) and MC (absolute normalization of various sources).
- **Electron identification** : variation of the  $e-\pi$  rejection efficiency
- **Radiative corrections** precision : PHOTOS generator used, fraction of full effect quoted
- **Neglected  $S_e$  dependence** : full effect from test simulated event sample quoted

$$f_s' / f_s = 0.169 \pm 0.009 \pm 0.034$$

$$f_s'' / f_s = -0.091 \pm 0.009 \pm 0.031$$

$$f_p / f_s = -0.047 \pm 0.006 \pm 0.008$$

$$g_p / f_s = 0.891 \pm 0.019 \pm 0.020$$

$$g_p' / f_s = 0.111 \pm 0.031 \pm 0.032$$

$$h_p / f_s = -0.411 \pm 0.027 \pm 0.038$$

$$a_0^0 = 0.256 \pm 0.008 \pm 0.007$$

**$\pm 0.018$  Theory (Universal Band)**

Corresponding to  $a_0^2 = -0.031 \pm 0.002 \pm 0.002$   
 $\pm 0.009$  Theory



**Signal**  $\pi^0 \pi^0 e^\pm \nu$  **Topology** : 1 charged track , 2  $\pi^0$ s (reconstructed from 4 $\gamma$ 's in LKr), 1 electron (LKr info E/p), some missing energy and  $p_T$  (neutrino)

2003 data : ~10000 signal events (with ~3% background events)

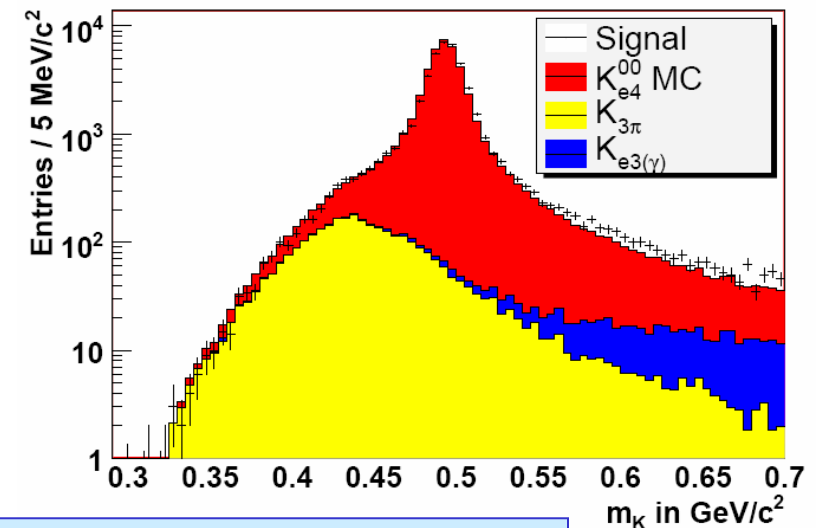
2004 data : ~30000 signal events (with ~2% background events)

**Background** : main sources estimated from data

$\pi^\pm \pi^0 \pi^0$  decay +  $\pi$  misidentified as  $e$  (dominant)

$\pi^0 e^\pm \nu \gamma$  decay + accidental  $\gamma$

Systematics from acceptance, trigger, electron id



Branching fraction (2003 data, K+ and K-)

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5}$$

( $\pi^\pm \pi^0 \pi^0$ ) as normalization

# Ke4 neutral decays : Form Factor



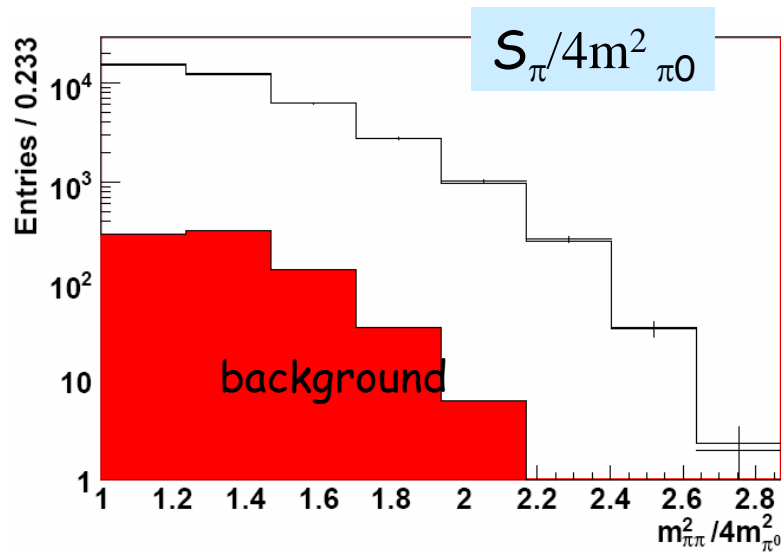
Two identical  $\pi^0 \rightarrow$  only **ONE form factor F** (no p wave)

$$F_s = f_s + f'_s q^2 + f''_s q^4 + f_e \left( S_e / 4m_{\pi}^2 \right) + ..$$

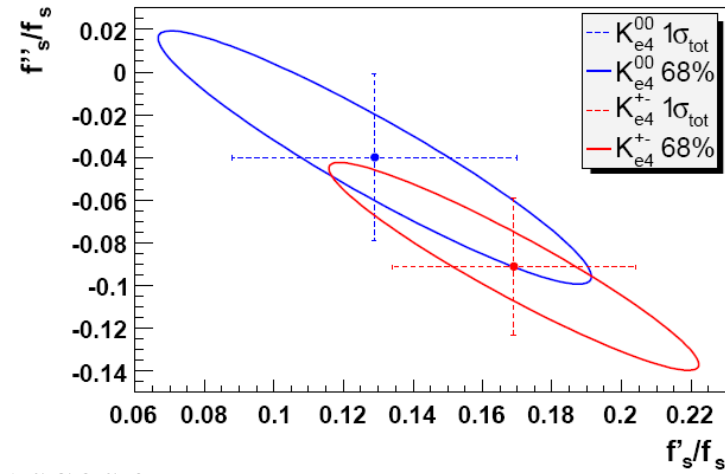
Form factors (2003 +2004 data, K+ and K-)  
 Se dependence measurement consistent with 0.

$$f'_s / f_s = 0.129 \pm 0.036_{stat} \pm 0.020_{syst}$$

$$f''_s / f_s = -0.040 \pm 0.034_{stat} \pm 0.020_{syst}$$



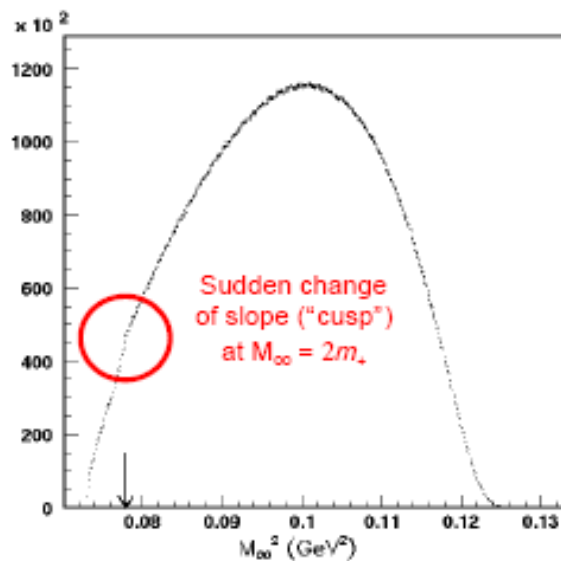
## Ke4 charged and neutral FF consistent



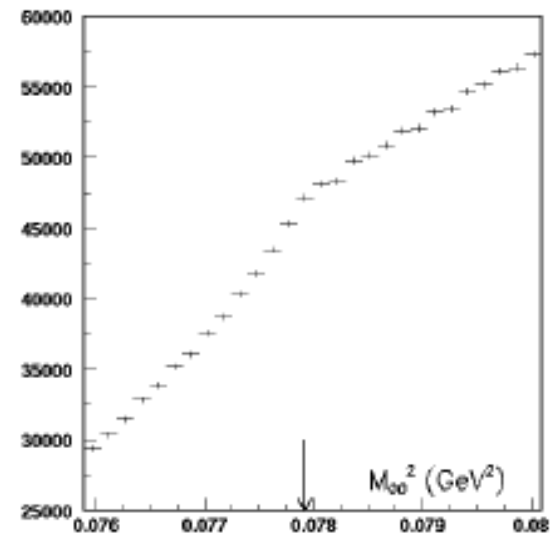
# The cusp effect in $\pi^\pm \pi^0 \pi^0$ decays

Result published in Phys. Lett. B633 (2006)

Reminder: from  $23 \cdot 10^6$  ( $\pi^\pm \pi^0 \pi^0$ ) decays, the  $M_{00}^2$  shows a sudden change of slope at  $M_{00} = 2m_{\pi^\pm}$



"Zoom" on the cusp region





**Principle:** 1-dimension fit to the  $M_{00}^2$  distribution based on the improved rescattering model of Cabibbo-Isidori (JHEP 0503 (2005))

In the Dalitz plot,  $g_0$  and  $h'$  are free parameters while **parameter  $k$  is set to 0.**

$$M_0 = A_0 \left( 1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 + \frac{1}{2} k v^2 \right)$$

Form Factors as in Phys.Lett.B633(2006)

$$g_0 = 0.645 \pm 0.004_{\text{stat}} \pm 0.009_{\text{syst}}$$

$$h' = -0.047 \pm 0.012_{\text{stat}} \pm 0.011_{\text{syst}}$$

$$a_2 = -0.041 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}}$$

$$(a_0 - a_2) = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}}$$

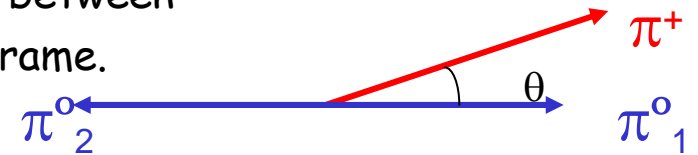
if the correlation between  $a_0$  and  $a_2$  predicted by ChPT is taken into account,

$$(a_0 - a_2) = 0.264 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}}$$



Going to a 2D fit would imply to use  $M^2_{00}$  and  $M^2_{+0}$  variables.

An alternate choice is  $M^2_{00}$  and  $\cos\theta$  where  $\theta$  is the angle between the charged  $\pi$  and the direction of the  $\pi^0$ 's in their rest frame.

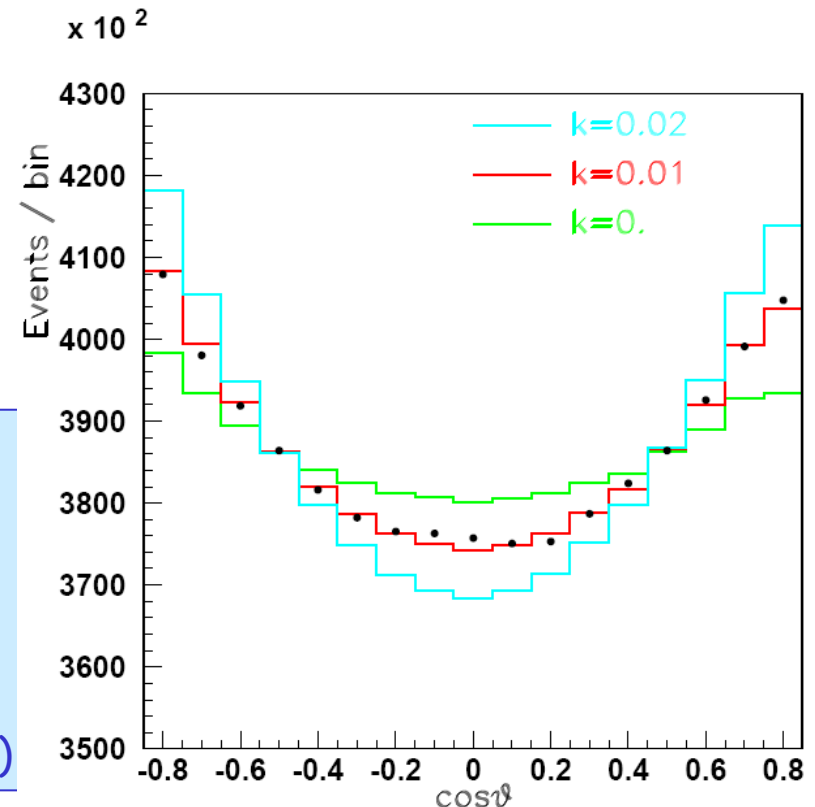


Use a modified matrix element :

$$M_0 = A_0 \left( 1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 + \frac{1}{2} k' v^2 \right)$$

re-fit in  $M^2_{00}$  range  $[0.082, 0.097] (GeV/c^2)^2$

no incidence on previous  $(a_0 - a_2)$  result.



Preliminary result (2003 data, K= and K-)

$k' = 0.0097 \pm 0.0003_{\text{stat}} \pm 0.0008_{\text{syst}}$

Note: -the new meaning of  $g_0$  and  $h'$   
 -the different meaning  $(h', k')$  wrt PDG  $(h, k)$

Comparison with previous published Ke4 results : **same framework needed !**

- CERN/PS Geneva-Saclay  $\sim 30000$  decays ( $K^+$ ) (Phys. Rev. D15 (1977))
- BNL E865  $\sim 400\,000$  decays ( $K^+$ ) (Phys. Rev. Lett. 87 (2001), Phys. Rev. D67 (2003))
- CERN/SPS NA48/2 : prelim. result from  $\sim 370\,000$  decays ( $K^+/-$ )

use the **same** Universal Band function (stat. + syst. errors added):

CERN/PS Geneva-Saclay

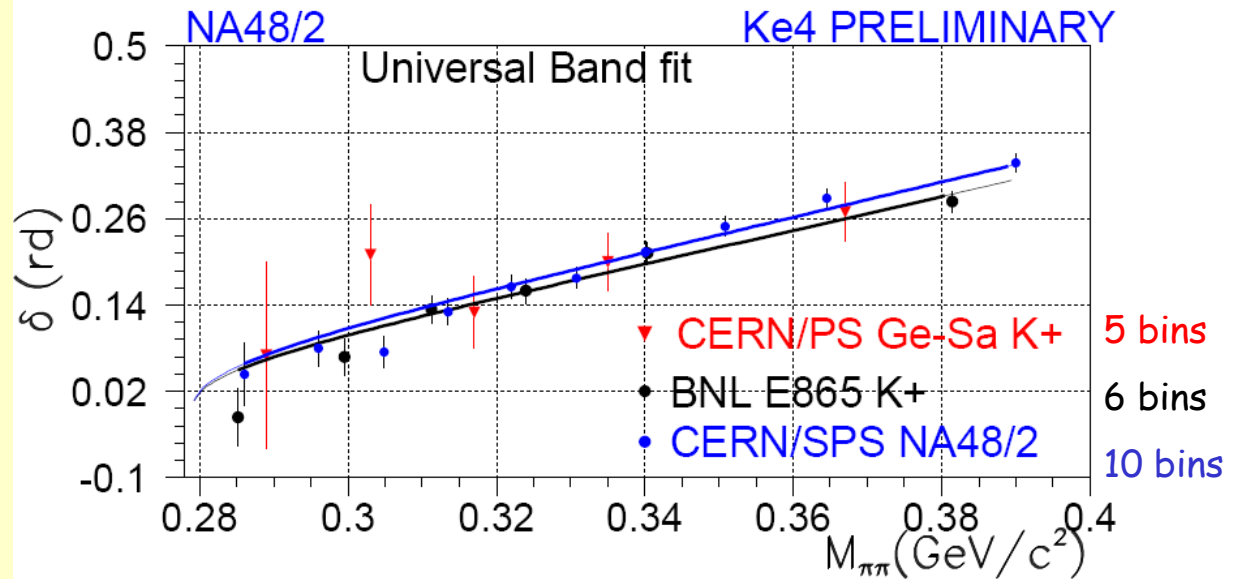
$$a_0^0 = 0.253 \pm 0.040$$

BNL E865

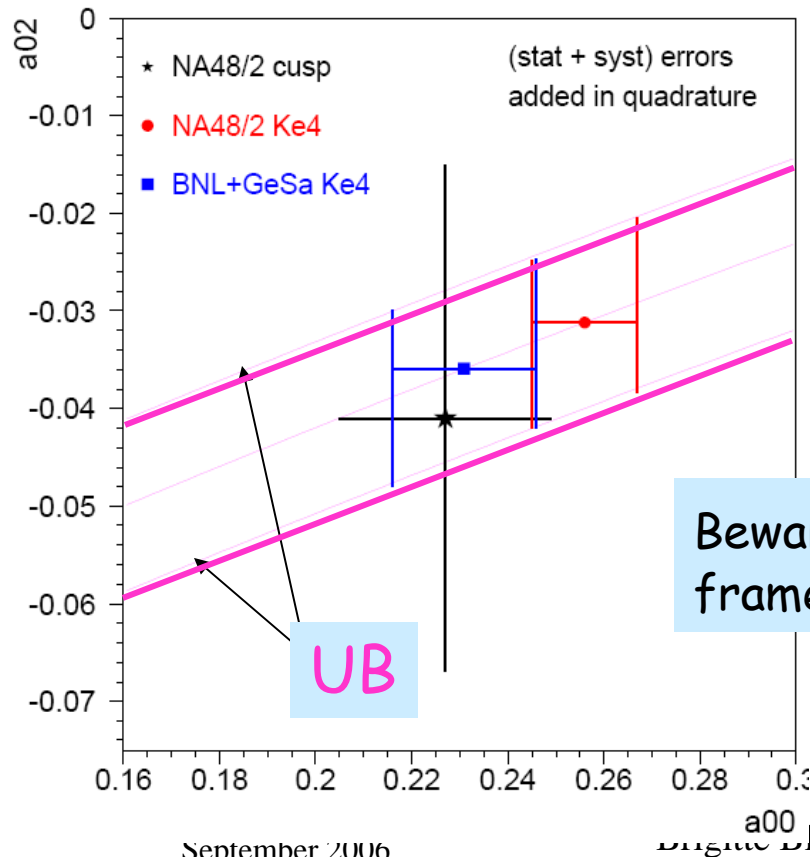
$$a_0^0 = 0.229 \pm 0.015$$

CERN/SPS NA48/2

$$a_0^0 = 0.256 \pm 0.011$$



NA48/2 uses 2 independent channels to measure  $\pi-\pi$  scattering lengths: form factors from the  $Ke4$  decays and the *cusp* effect in  $\pi^\pm \pi^0 \pi^0$  decays.



They **maybe** compared together and to other experimental measurements **if** in the **same theoretical framework**

Beware of different theoretical frameworks... work in progress !

## Ke4

Using partial data samples recorded in 2003-2004, Na48/2 has improved measurements of the **Ke4 form factors** in the charged and neutral modes (5 to 30% stat. precision).

$$\text{BR}(\text{Ke}4^{00}) = (2.587 \pm 0.026_{\text{stat}} \pm 0.019_{\text{syst}} \pm 0.029_{\text{theo}}) \cdot 10^{-5}$$

(10 times better than current PDG)

Using a conservative theoretical approach, a **preliminary** value of  $a_0^0$  is obtained with **3% precision (both stat. and syst.)**.

$$a_{00} = 0.256 \pm 0.008_{\text{stat}} \pm 0.007_{\text{syst}} \pm 0.018_{\text{theo}} \quad (\text{Universal Band})$$

## Cusp

New measurements of Matrix element and  $\pi\pi$  scattering length in  $K3\pi$  decays

$$a_{00} - a_{20} = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{theo}}$$

First evidence for a non-zero  $k'$  term

$$k' = 0.0097 \pm 0.0003_{\text{stat}} \pm 0.0008_{\text{syst}}$$

A joint study of Ke4 and Cusp analyses will provide stringent constraints in the  **$(a_0^0, a_0^2)$  plane** (with help from theorists!)