Recent results from NA48/2 on Ke4 and K3π decays, interpretation in terms of $\pi\pi$ scattering lengths

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New results from the NA48/2 experiment at the CERN/SPS are presented. Large samples of Ke4 decays have been collected in 2003 and 2004 in the charged ($K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$) and neutral ($K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$) modes. Form factors and Branching fraction measurements are reported. The $\pi\pi$ scattering lengths a_0^0 and a_0^2 can be extracted from the charged Ke4 form factor δ and from the K3π cusp in the M_{00}^2 distribution.

1. Introduction

Charged Ke4 data are of particular interest as they give access to the $\pi\pi$ phase shift difference $\delta = \delta_0^0 - \delta_1^1$ in absence of any other hadron. The measured variation of the phase shift with the invariant mass $M_{\pi\pi}$ near threshold can be related to the scattering lengths a_0^0 and a_0^2 using dispersion relations and data at intermediate energies [1–3]. More constrained predictions using two-loop Chiral perturbation theory [2] can be compared to the experimental measurement. The neutral Ke4 mode cannot address the same question as the two $\pi^{0\prime}s$ cannot exhibit a phase difference. However, the measurement of the only form factor in the neutral mode can be compared to the same one measured in the charged Ke4 mode. The rescattering of two charged pions from $K^{\pm} \to \pi^+\pi^-\pi^{\pm}$ decays can explain the cusp observed in the $M_{\pi^0\pi^0}$ distribution of the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ decays at $M_{00} = 2m_{\pi^+}$ and give another measurement of the a_0^0 and a_0^2 scattering lengths.

2. Experimental setup

Beam and detector

Two simultaneous K^+ and K^- beams were produced by 400 GeV protons from the CERN/SPS, impinging on a beryllium target. The beams were then deflected in a front-end achromat to select momenta in the range (60 $\pm$ 3) GeV/c and focused $\sim$ 200 m downstream at the first spectrometer chamber. A schematic view of the beam line can be found in [4]. The decay volume is a 114 m long vacuum tank between the final collimator in the beam line and the first spectrometer chamber of the detector.

The NA48/2 detector main components are :

- A magnetic spectrometer consisting of a dipole magnet surrounded by two sets of drift chambers. The momentum of charged decay products is measured with a relative precision of $\sim 1\%$ for 10 GeV/c tracks.

- A $27x_0$ liquid krypton calorimeter used to measure electromagnetic deposits and identify electrons through their E/p ratio. The energy and position resolutions are $\sim 1\%$ and ~ 1.5 mm (resp.) for 10 GeV showers.

Data Taking and statistics

The event statistics collected during years 2003-2004 are given below. The analysis is completed for some of the decay modes. Preliminary results are quoted otherwise.

decay mode	events collected	analyzed
$K^{\pm} \to \pi^+\pi^-\pi^{\pm}$	$\sim 4 \cdot 10^9$	*all*
$K^{\pm} \to \pi^0\pi^0\pi^{\pm}$	$\sim 100 \cdot 10^6$	$23 \cdot 10^6$
$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$	$\sim 1 \cdot 10^6$	$0.37 \cdot 10^6$
$K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$	$\sim 4 \cdot 10^4$	*all*

3. Charged Ke4 decays analysis

Data selection

Part of the 2003 data was selected for three charged tracks topologies, requiring two opposite sign pions, one electron ($E/p > 0.9$) and missing energy and p_t (neutrino). The background contamination was estimated from "wrong sign" events ($\pi^{\pm}\pi^{\pm}e^{\mp}\nu$), which can only be background as the decay would violate the $\Delta S = \Delta Q$ rule.

The background sources are $K^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$ decays with subsequent $\pi \to e\nu$ decay or a pion misidentified as an electron and $K^{\pm} \to \pi^{\pm}\pi^{0}(\pi^{0})$ decays with subsequent Dalitz decay of a π^0 with an electron misidentifed as a pion and photon(s) undetected. The relative level of background to signal is $\sim$ 0.5% and has been cross-checked using Monte Carlo simulated events.

Formalism

The Ke4 decay is fully described by the five kinematic Cabibbo-Maksymowicz variables [5]: two invariant masses $M_{\pi\pi}$ and $M_{e\nu}$ and three angles θ_π, θ_e and Φ described in Figure 1.

Two axial (F,G) and one vector (H) form factors contribute to the transition amplitude and can be developed in partial wave expansion of s, p, d waves [6].

$$\begin{aligned} F &= F_s\, e^{i\delta_s} + F_p\, e^{i\delta_p} cos\theta_\pi + d\ wave.. \\ G &= G_p\, e^{i\delta_g} + d\ wave.. \\ H &= H_p\, e^{i\delta_h} + d\ wave.. \end{aligned}$$

Neglecting d wave terms and assuming the same phase for F_p, G_p, H_p, only five form factors are left, which are expanded further [7] in powers of $q^2 = ({M_{\pi\pi}}^2/4m_\pi^2 - 1.)$.

$$\begin{aligned} F_s &= f_s + f_s' q^2 + f_s'' q^4 \\ F_p &= f_p + f_p' q^2 + .. \\ G_p &= g_p + g_p' q^2 + .. \\ H_p &= h_p + h_p' q^2 + .. \\ \delta(q^2) &= \delta_s - \delta_p \end{aligned}$$

Form Factor measurements

From the data sample, a total of 15000 equi-populated bins are defined (ten along $M_{\pi\pi}$, five along $M_{e\nu}$, five along $cos\theta_\pi$, five along $cos\theta_e$ and twelve along Φ). Ten independent five-parameter fits are performed, one for each bin in $M_{\pi\pi}$. Then a second fit determines the variation of each form factor with q^2 as shown in Figures 2 and 3.

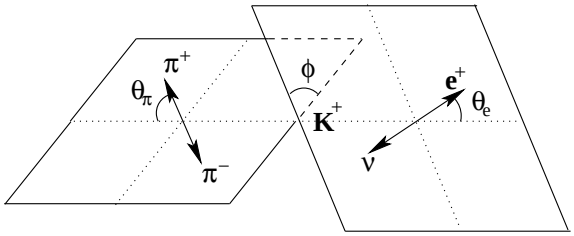

Figure 1. Topology of the charged Ke4 decay.

In the case of the phase δ, the variation was fitted using a one parameter function, corresponding to the center line of the Universal Band [2]. A rather loose constraint between the $\pi\pi$ scattering lengths a_0^0 and a_0^2 is then imposed.

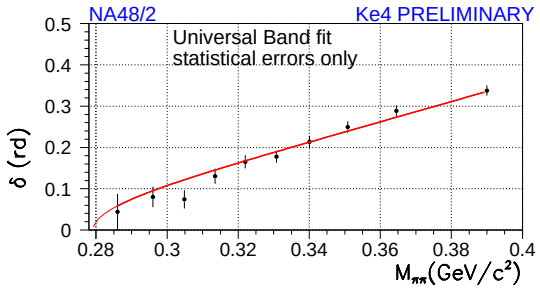

Figure 2. Variation of the phase difference with $M_{\pi\pi}$ as measured in 10 independent bins.

Results

Systematic uncertainties were assessed comparing two independent analyses using different event selections, Kaon reconstructions, detector corrections and fit methods. Uncertainty on the acceptance control, background level, electron identification, radiative corrections and possible unknown $M_{e\nu}$ dependence of the form factors has been investigated. The results are given relative to f_s, as absolute values can only be obtained

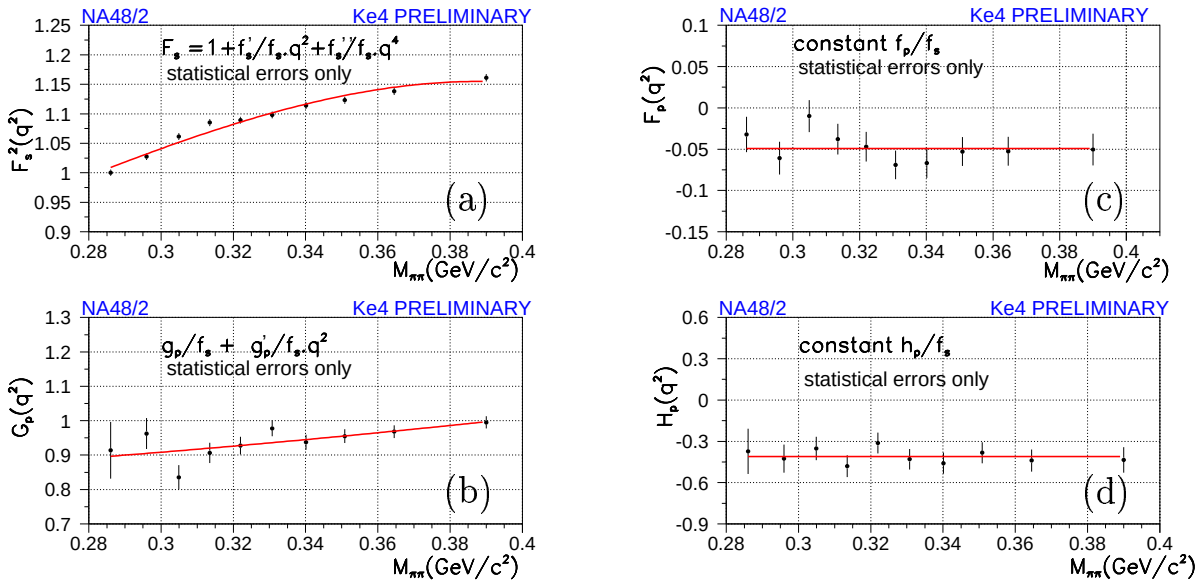

Figure 3. Variation of the form factor with $M_{\pi\pi}$ as measured in 10 independent bins. The line is the result of (a): a quadratic fit in q^2 for F_s, (b): a linear fit in q^2 for G_p and (c),(d): constant fits for F_p and H_p .

through the measurement of the decay rate (to be completed):

$$
\begin{array}{rrll}
f_s'/f_s = & 0.169 & \pm 0.009_{\rm stat} & \pm 0.034_{\rm syst} \\
f_s''/f_s = & -0.091 & \pm 0.009_{\rm stat} & \pm 0.031_{\rm syst} \\
f_p/f_s = & -0.047 & \pm 0.006_{\rm stat} & \pm 0.008_{\rm syst} \\
g_p/f_s = & 0.891 & \pm 0.019_{\rm stat} & \pm 0.020_{\rm syst} \\
g_p'/f_s = & 0.111 & \pm 0.031_{\rm stat} & \pm 0.032_{\rm syst} \\
h_p/f_s = & -0.411 & \pm 0.027_{\rm stat} & \pm 0.038_{\rm syst} \\
a_0^0 = & 0.256 & \pm 0.008_{\rm stat} & \pm 0.007_{\rm syst} \\
 & & \pm 0.018_{theo} &
\end{array}
$$

A comparison with a_0^0 values obtained by previous Ke4 experiments and using the Universal band center line can be found below:

experiment	data sample	a_0^0
Ge − Sa [8]	$\sim$ 30000 K^+	0.253 ± 0.037
E865 [9]	$\sim$ 400000 K^+	0.229 ± 0.015
NA48/2	$\sim$ 370000 $K^\pm$	0.256 ± 0.011

The distributions of the five kinematic variables (Cabibbo-Maksymowicz variables) are shown in Figure 4. The background contribution can hardly be seen on the linear scale. The distributions of simulated events using the fitted values of the form factors are superimposed as histograms.

Summary

The extracted value of a_0^0 is compatible with previous measurements. The improved sensitivity already achieved by NA48/2, due to sizeable acceptance at high $M_{\pi\pi}$ and low background, should allow more stringent tests of Chiral Perturbation Theory.

4. Neutral Ke4 decays analysis

Data selection

From 2003 (2004) data, $\sim$ 10000 (30000) events were selected requiring one charged track (with electron identification), two neutral pions (reconstructed from four $\gamma's$ in the calorimeter) and missing energy and p_t (neutrino). The background contamination was estimated from data by reversing some of the selection requirements. It is mainly due to $K^\pm \to \pi^0\pi^0\pi^\pm$ events where the $\pi^\pm$ is misidentified as an electron. Another contribution comes from radiative K_{e3} decays and

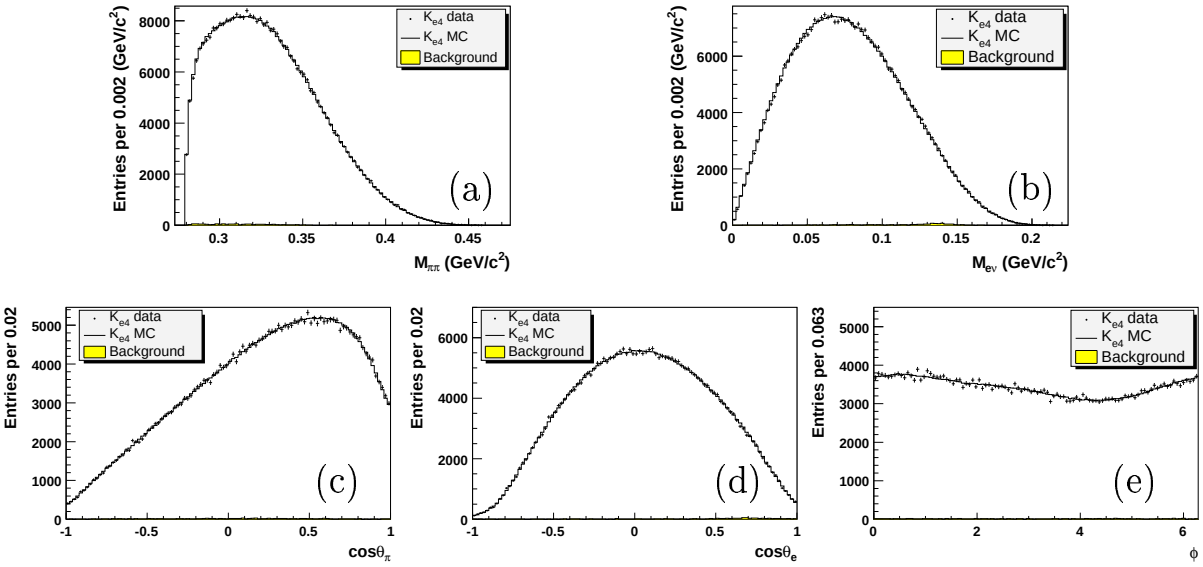

Figure 4. Distributions of the five kinematic variables for K^+ and K^- decays: $M_{\pi\pi}$ (a), $M_{e\nu}$ (b), $cos\theta_\pi$ (c), $cos\theta_e$ (d) and Φ (e). The full symbols correspond to the data, the shaded area to the background (hardly seen) and the histograms to the simulation.

an accidental extra photon, the two photons faking a π^0. The relative background to signal level was 3% in 2003 and 2% in 2004. The reconstructed Kaon mass is shown in Figure 5.

Branching fraction and Form factor measurement

The branching fraction is measured from 2003 data only. The preliminary value obtained, normalized to $K^\pm \to \pi^0\pi^0\pi^\pm$ is:

$$BR(K^{00}_{e4}) = (2.587 \pm 0.026_{\rm stat} \pm 0.019_{\rm syst} \pm 0.029_{ext}) \cdot 10^{-5}$$

The external error comes from the uncertainty on the normalization mode branching ratio.

As there are two identical π^0's in the decay, only one form factor is needed to describe the decay in the s-wave, expanded in powers of q^2 as in the charged mode:

$$F_s = f_s + f'_s q^2 + f''_s q^4$$

This form factor and it's q^2 dependence has been measured from the full statistics :

$$f'_s/f_s = 0.129 \pm 0.036_{\rm stat} \pm 0.020_{\rm syst}$$
$$f''_s/f_s = -0.040 \pm 0.034_{\rm stat} \pm 0.020_{\rm syst}$$

While measured with a smaller precision than in the charged Ke4 mode, the values are in good agreement as shown in Figure 6 where the same correlation has been used for systematic and statistical errors.

Summary

An improved measurement of the Branching fraction of the neutral Ke4 decay mode has been presented with a relative error of 1.7%. This result is compatible with previous measurement but more precise by a factor ~ 8 [10]. The form factor has been measured with a relative precision of 30%, and its value is in good agreement with the one measured in the charged Ke4 mode.

5. Cusp in $K^\pm \to \pi^0\pi^0\pi^\pm$ decays

Preliminary results using part of the data statistics and presented last year at this conference have been already published [11]. However,

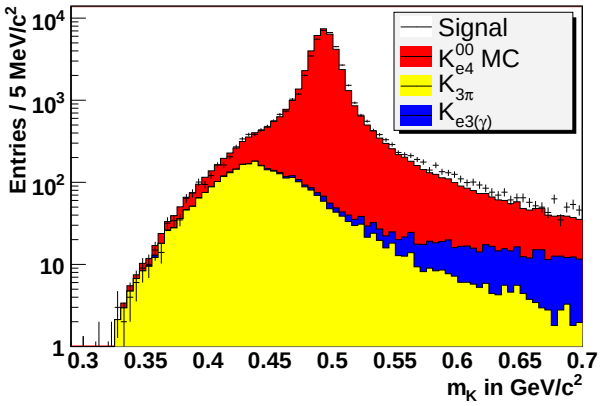

Figure 5. Invariant mass distribution of the reconstructed Kaon in the $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$ mode (K^{00}_{e4})

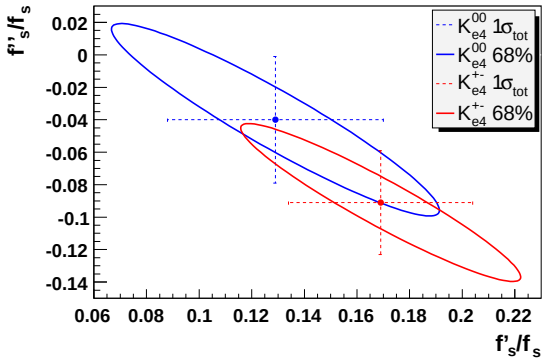

Figure 6. Correlation between of f'_s/f_s and f''_s/f_s the for the $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$ (K^{+-}_{e4}) and $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$ (K^{00}_{e4}) modes

some improvements in the analysis deserve to be mentionned.

Method

The invariant mass squared distribution of the two π^0's, M^2_{00}, shows a sudden change of slope at $M^2_{00} = 2m^2_{\pi^+}$ as seen in Figure 7. This effect has been explained and computed by a theoretical rescattering model [12] using the Dalitz plot formulation:

$$M_{\pm 00} \propto \quad 1 + \tfrac{1}{2}g_0 u + \tfrac{1}{2}h' u^2 + \tfrac{1}{2}k' v^2$$

where k' was set to zero, as suggested by [13]. A fit of the single mass distribution allows the simultaneous determination of g_0, h', a^2_0 and $(a^0_0 - a^2_0)$.

Going to a 2D-fit, would imply to use both squared invariant masses M^2_{00} and $M^2_{\pm 0}$. An alternative choice is M^2_{00} and $cos\theta$, where θ is the angle between the charged $\pi^{\pm}$ and the $\pi^{0\prime}s$ direction in their rest frame. Figure 8 shows data and simulation $cos\theta$ distributions for three values of k'. The 2D fit to the Dalitz plot allows a simultaneous measurement of all 5 parameters.

Results

The following results were obtained, assuming a zero value for k':

$$\begin{aligned}
g_0 &= \quad 0.645 \quad \pm 0.004_{\rm stat} \quad \pm 0.009_{\rm syst} \\
h' &= -0.047 \quad \pm 0.012_{\rm stat} \quad \pm 0.011_{\rm syst} \\
a^2_0 &= -0.041 \quad \pm 0.022_{\rm stat} \quad \pm 0.014_{\rm syst} \\
a^0_0 - a^2_0 &= \quad 0.268 \quad \pm 0.010_{\rm stat} \quad \pm 0.004_{\rm syst} \\
&\qquad\qquad\quad \pm 0.013_{ext}
\end{aligned}$$

The systematic uncertainty includes acceptance control and trigger efficiency. The external error corresponds to an estimate of the effect of the missing higher order terms and radiative corrections in the rescattering model.

When fitting also the k' parameter in the 2D-fit, it is measured significantly away from zero:

$$k' = \quad 0.0097 \quad \pm 0.0003_{\rm stat} \quad \pm 0.0008_{\rm syst}$$

The corresponding changes for g_0 and h' are $\sim 2\%$ (resp. 25%) but negligible for the scattering lengths.

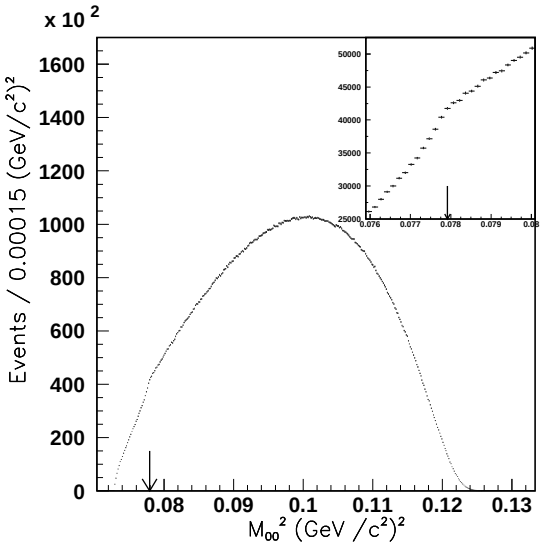

Figure 7. Invariant mass squared of the two $\pi^{0\prime}s$, The insert is an enlargement of the narrow region around $M_{00}^2 = 4m_{\pi^+}^2$ (arrow).

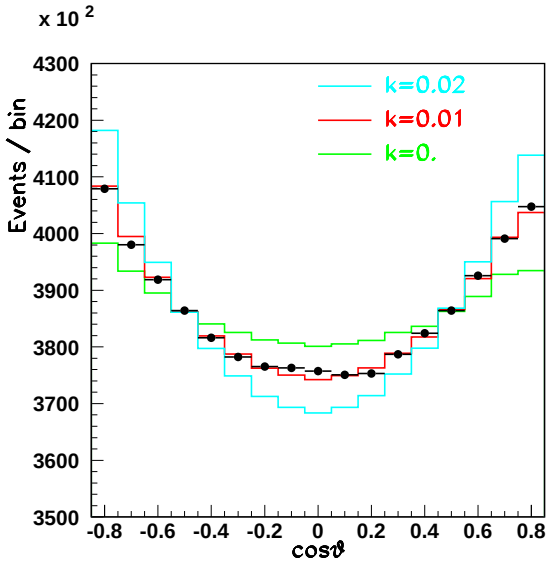

Figure 8. Distribution of the cosine of the angle between the $\pi^{\pm}$ and the $\pi^{0\prime}s$ direction in their rest frame for data (full symbols) and simulated events (histograms). The simulation is shown for 3 values of k' parameter.

6. Conclusion

Using part of the data recorded in 2003 and 2004, NA48/2 has improved measurements of the Ke4 Form factors in the charged and neutral modes. A new measurement of the neutral Ke4 Branching fraction has been achieved. First evidence for a non-zero v^2 term in the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ Dalitz plot has been observed. A preliminary value for the $\pi\pi$ scattering length a_0^0 has been obtained with 3% statistical and 3% systematic precision in the very conservative approach of the Universal Band. A joint study of the Ke4 and Cusp analyses will provide stringent constraints in the (a_0^0, a_0^2) plane.

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