$|V_{td}|$, $|V_{ts}|$ and rare kaon decays

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Summary. — We present a review of results on the CKM matrix elements V_{td} and V_{ts} from $B\overline{B}$ mixing and radiative penguin B decays. We discuss the prospects for measuring $|V_{td}|$ from the rare kaon decay $K^+ \to \pi^+\nu\overline{\nu}$ and we describe the NA62 proposal for an experiment which measures the $K^+ \to \pi^+\nu\overline{\nu}$ branching fraction with a statistical precision better than 10%.

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1. – Introduction

Second order charged weak processes involving box or penguin diagrams containing virtual quarks are sensitive to V_{td} and V_{ts}. The absolute value of the CKM matrix element V_{tq} $(q = d, s)$ can be determined by measuring the oscillation frequency ΔM_q in the $B_q^0\overline{B}_q^0$ mixing:

$$\Delta M_q = \frac{G_F^2}{6\pi^2} m_{B_q} \left(\hat{B}_{B_q} F_{B_q}^2\right) \eta_B m_W^2 S_0\left(x_t\right) |V_{tq}|^2 \tag{1}$$

Here, $\hat{B}_{B_q}$ is the bag parameter of the B meson, F_{B_q} is the weak B decay constant, η_B is a QCD correction and $S_0(x_t)$ is a slowly varying function of the top quark and W boson mass. To reduce the theoretical uncertainties coming from $\hat{B}_{B_q}$ and F_{B_q}, $|V_{td}/V_{ts}|$ is extracted from the ΔM_d over ΔM_s ratio. $|V_{td}/V_{ts}|$ can also be extracted by measuring $R = \mathcal{B}(B \to \rho/\omega\ \gamma\ )/\mathcal{B}(B \to K^*\gamma\ )$:

$$R \propto \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{m_B^2 - m_{\rho/\omega}^2}{m_B^2 - m_{K^*}^2}\right)^3 \frac{1}{\xi^2}\left(1 + \Delta R\right) \tag{2}$$

where ξ is a ratio of form factors and ΔR accounts for explicit $O(\alpha_s)$ corrections. Physics beyond the SM could create inconsistencies between the $|V_{td}/V_{ts}|$ measurement from radiative decays and $B\overline{B}$ mixing.

Table I. – $B_s^0\overline{B}_s^0$ *oscillation frequency* ΔM_s *measured by CDF (1*fb^{-1}*) and D0 (2.4*fb^{-1}*)*

CDF: World First Observation (5σ)	D0: Evidence (3σ)
$\Delta M_s = 17.77 \pm 0.10(\text{stat.}) \pm 0.07(\text{syst.})\ \text{ps}^{-1}$	$\Delta M_s = 18.53 \pm 0.93(\text{stat.}) \pm 0.30(\text{syst.})\ \text{ps}^{-1}$

2. – $B^0\overline{B}^0$ mixing

The experimental status of ΔM_d is dominated by the results from the B factories where two B_d^0 mesons with opposite flavor are produced in a coherent state from the $\Upsilon(4S)$ decay. The oscillation frequency is determined with an unbinned maximum likelihood fit that uses for each event the measured difference in decay times of the two B mesons.

Combining all published measurements and accounting for all identified correlations yields $\Delta M_d = 0.507 \pm 0.005\ \text{ps}^{-1}$ [1]. The ΔM_d result can be used to extract the magnitude of the CKM matrix element V_{td} within the SM: $|V_{td}| = (7.4 \pm 0.8)^{-3}$. The extraction is at present completely dominated by the uncertainty on the hadronic matrix element $f_{B_d}\sqrt{\hat{B}_{B_d}} = 244 \pm 11 \pm 24$ MeV [2] obtained from the lattice QCD calculations.

Unlike B factories, Tevatron can produce all B species and $b\overline{b}$ pairs do not evolve coherently. The proper time $t = \frac{m_B}{p}L$ is measured from the distance L between the production vertex and the B decay vertex, and from an estimate of the B momentum p. To extract information useful for the interpretation of B_s^0 oscillation searches a B_s^0 oscillation amplitude $\mathcal{A}$ is measured as a function of a fixed test value of Δm_s, using a maximum likelihood fit based on $\Gamma_s e^{-\Gamma_s t}(1 \pm \mathcal{A}\cos(\Delta m_s t))/2$. If Δm_s is equal to its true value, one expects $\mathcal{A} = 1$ within the total uncertainty $\sigma_{\mathcal{A}}$. However, if Δm_s is (far) below its true value, a measurement consistent with $\mathcal{A} = 0$ is expected. Table I summarizes the CDF and D0 results [3].

The information on $|V_{ts}|$ obtained, in the framework of the SM, from the CDF amplitude spectrum is hampered by the hadronic uncertainty, as in the B_d^0 case. However, several uncertainties cancel in the frequency ratio $\Delta m_s/\Delta m_d$:

$$\left|\frac{V_{ts}}{V_{td}}\right| = 0.2060 \pm 0.0007(\Delta m_s)^{+0.0081}_{-0.0060}(\Delta m_d + \text{theor}) \tag{3}$$

3. – $B \to V\gamma$ decays

The $B^+ \to \rho^+\gamma$, $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$ decays have been reconstructed by both *BABAR* and Belle [4]. Two almost uncorrelated kinematical variables are computed to separate the signal from backgrounds: the difference between the B candidate center of mass (CM) energy and the CM beam energy ΔE, and the "beam energy-substituted mass" m_{ES} obtained substituting the CM beam energy for the B candidate CM energy in the invariant mass espression. The signal content of the data is determined by a multidimensional unbinned maximum likelihood fit which is constructed individually for each of the three signal decay modes. The significance is computed as $\sqrt{2\Delta log\mathcal{L}}$, where $\Delta log\mathcal{L}$ is the log-likelihood difference between the best fit and the null-signal hypothesis.

By using the world average value of $\mathcal{B}(B \to K^*\gamma) = (41.8 \pm 1.7)\cdot 10^{-6}$, both *BABAR* and Belle have computed the $|V_{td}/V_{ts}|$ ratio from eq.(2). The results are shown in

TABLE II. – *Significance (Σ) in standard deviations including systematic errors, the isospin-averaged branching fraction ($\mathcal{B}$) and the $|V_{td}/V_{ts}|$ measured by BABAR and Belle.*

	Σ	$\mathcal{B}(B \to \rho/\omega\ \gamma) \cdot 10^6$	$\|V_{td}/V_{ts}\|$
BABAR	6.4	$1.25^{+0.25}_{-0.24} \pm 0.09$	$0.200^{+0.021}_{-0.020} \pm 0.015$
Belle	6.2	$1.14 \pm 0.20^{+0.10}_{-0.12}$	$0.195^{+0.020}_{-0.019} \pm 0.015$

Table II. The measurement of $|V_{td}/V_{ts}|$ is consistent with the one from $B\overline{B}$ mixing and gives a competitive constraint on the unitarity triangle (UT) [5].

4. – Rare kaon decays

The rare kaon decays $K \to \pi\nu\overline{\nu}$ are sensitive probes of the physics at high energy scales and allow in particular to access the CKM couplings of the top quark in a very clean way. We limit the discussion to $K^+ \to \pi^+\nu\overline{\nu}$ transitions. The SM prediction can be parameterized as follows [6]:

$$\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu}\ ) \approx 1.6 \times 10^{-5} \cdot |V_{cb}|^4 \cdot (\sigma\overline{\eta}^2 + (\rho_c - \overline{\rho})^2) \approx (8.0 \pm 1.1) \times 10^{-11} \quad (4)$$

where $\sigma = 1/(1 - \lambda^2/2)^2$ and $\rho_c \approx 1.4$.

The determination of $|V_{td}|$ is subject to various uncertainties:

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 0.39\frac{\sigma(P_c)}{P_c} \pm 0.70\frac{\sigma(\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu}))}{\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu})} \pm \frac{\sigma(|V_{cb}|)}{|V_{cb}|} \quad (5)$$

where the parameter P_c results from Z-penguin and electroweak box diagrams involving internal charm quark exchange and has been calculated at NNLO [7].

On the experimental side, the decay was studied with stopped kaons by E787 and its upgraded version, E949, leading to the publication of three candidates and to a first determination of the branching ratio: $\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu}\ ) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ [8]. The mean value is about twice larger than the SM prediction but, given the large errors, completely in agreement with it.

The proposal to measure the $K^+ \to \pi^+\nu\overline{\nu}$ decay at the CERN SPS with a 10% accuracy has been put forward by the NA62 collaboration [9]. The aim is to observe ~ 100 signal events in 2 years with a 10/1 signal to background ratio. This can be achieved by exploiting a decay in flight technique which allows a 10% signal acceptance and by using a beam line able to provide of the order of 10^{13} kaon decays. The signature of the $K^+ \to \pi^+\nu\overline{\nu}$ event is one reconstructed positive track in the downstream detector. Background rejection relies on the event kinematics, photon veto, and particle identification. The squared missing mass, m^2_{miss}, defined as the squared difference between the kaon and the pion candidate four-momentum allows a kinematical separation between the signal and almost 92% of the background. To reject the main sources of background, $K^+ \to \mu^+\nu_\mu$ and $K^+ \to \pi^+\pi^0$, we need a μ/π separation at 10^{-8} level and a photon veto at 10^{-5}, respectively. To further suppress the $K^+ \to \pi^+\pi^0$ background, the kaon beam is chosen to have 75GeV/c. Requiring the π^+ to have less than 35 GeV of energy, leaves at least

40 GeV of energy associated to the π^0. The drawback of a high energy kaon beam is that pions and protons cannot be efficiently separated from kaons. Although only about 6% of particles are kaons and only $\approx 10\%$ of the kaons decay in the useful decay volume, all particles (800MHz) have to be tracked and precisely timed. The main elements of the proposed detector are:

- A differential Cherenkov counter (CEDAR) placed on the incoming beam for positive kaon identification.
- Thin silicon micro-pixel detectors (GIGATRACKER) able to work at 1 GHz to measure the momentum of the incoming particle.
- A magnetic spectrometer (STRAW) measuring the direction and the momentum of the out-going pion.
- A gas ring imaging Cherenkov counter (RICH) providing pion/muon separation with a muon suppression factor of at least 10^{-2}. The RICH must also work as a timing detector for the downstream track with a requested time resolution of 100 ps.
- The NA48 hodoscope for fast timing and triggering (CHOD).
- A photon veto system composed by a set of ring-shaped anti-counters (LAV) for photons originating from kaon decays in the fiducial region from 10 mrad to 50 mrad, a high-performance liquid krypton calorimeter (LKr) for photons in the region between 1 and 10 mrad, and two calorimeters (IRC and SAC) to cover the region below 1 mrad.
- A hadron calorimeter (HAC) and a muon detector (MUD) able to identify muons with 10^{-5} inefficiency.

A preliminary analysis has been done based on simulation. We expect ~ 50 signal events per year of data taking with a background to signal ratio between 14% and 17%.

5. – Conclusions

We reviewed the status of the $|V_{td}|$ and $|V_{ts}|$ measurements and we discussed the prospects of a theoretically clean determination of $|V_{td}|$ from $K^+ \to \pi^+\nu\overline{\nu}$ decays. However, in the future, the role of the $K \to \pi\nu\overline{\nu}$ will shift towards the search for new physics.

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