Low energy dynamics and the $\pi\pi$ scattering lengths from the NA48/2 experiment at CERN

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The observation of a cusp-like structure in the M_{00} invariant mass distribution in the decay $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ by the NA48/2 experiment at CERN has given a renewed interest in the $\pi\pi$ scattering lengths. We report the result of a study of a partial sample of 2.3×10^{7} decays, the observation of an anomaly in the $\pi^{0}\pi^{0}$ mass spectrum in the region around $M_{00} = m_{+}$, where m_{+} is the mass of the charged pion, and the subsequent interpretation in terms of the $\pi\pi$ scattering lengths. This anomaly, never observed in previous experiments, has been interpreted as an effect due to the final state charge exchange scattering process $\pi^{+}\pi^{-} \to \pi^{0}\pi^{0}$ in $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$. From the distribution we can determine precisely $(a_0 - a_2)$, the difference between the $\pi\pi$ scattering lengths in the isospin $I = 0$ and $I = 2$ states.

1. Introduction

The NA48/2 experiment at CERN has been designed with the main purpose of searching for direct CP violation in the decays of $K^{\pm}$ in three pions. The experiment has collected data in 2003 and 2004, using simultaneous and focused K^{+} and K^{-} beams of 60 GeV/c momentum, for a total of 4×10^{9} fully reconstructed $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and 0.1×10^{9} $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$. The result reported is based on a partial sample of the 2003 data and amounts to $2.3 \times 10^{7} decays$.

2. Beam and detector

Two simultaneous focused kaon beams of opposite charge, with a central momentum of 60 GeV/c and a momentum band of ±3.8% are produced by a 400 GeV proton beam impinging on a 40 cm Be target. The decay volume is a 114 m long vacuum tank; the $\pi^{\pm}\pi^{0}\pi^{0}$ final state is reconstructed by a magnetic spectrometer and a liquid krypton calorimeter (LKr). Charged particles are measured by the magnetic spectrometer, consisting of four drift chambers and a dipole magnet located between the second and the third chamber; the momentum resolution is $\sigma(p)/p = 1.02\% \oplus 0.044\% p$ (p in GeV/c). The magnetic spectrometer is followed by a scintillator hodoscope consisting of two planes segmented into horizontal and vertical strips and arranged in four quadrants (charged hodoscope). The $\pi^{0} \to \gamma\gamma$ decays are reconstructed using the LKr, an almost homogeneous ionization chamber with an active volume of $\sim 10\ m^{3}$ and a 27 X_0 thickness; the energy resolution is $\sigma(E)/E = 0.032/\sqrt{E} \oplus 0.09/E \oplus 0.0042$ (E in GeV). The space resolution for single electromagnetic shower can be parametrized as $\sigma_x = \sigma_y = 0.42/\sqrt{E} \oplus 0.06$ cm (E in GeV). At a depth of $\sim 9.5X_0$ inside the active volume of the calorimeter, a hodoscope consisting of a plane of scintillating fibres is installed (neutral hodoscope); the signals from the four quadrants are used to give a fast trigger.

A more detailed description of the beams and detector can be found elsewhere [1].

3. Trigger and event selection

The $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays are selected by a two level trigger. A signal in at least one quadrant of the charged hodoscope in coincidence with an energy deposition in the calorimeter consistent with at least two photons is required by the first level trigger. The second level is a fast on-

*on behalf of the NA48/2 Collaboration: Cambridge-CERN-Chicago-Dubna-Edinburgh-Ferrara-Firenze-Main-Northwestern-Perugia-Pisa-Saclay-Siegen-Torino-Wien line processor that reconstructs the momentum of charged particles and calculates the missing mass under the assumption that the particle is a $\pi^{\pm}$ originating from the decay of a 60 GeV $K^{\pm}$ travelling along the beam axis. The main $\pi^{\pm}\pi^{0}$ background is rejected by the requirement that the missing mass is not consistent with the π^{0} mass. For further analysis selection criteria on tracks and clusters are applied: at least one charged track with momentum above 5 GeV/c and at least four energy clusters in the calorimeter, each consistent with a photon and above an energy threshold of 3 GeV are required. Additional cuts on time consistency between tracks and photons and on the distance between any two photons and each photon and the impact point of the track on the LKr are applied in order to ensure full containment of the electromagnetic showers. For each possible pair of photons we assume that it originates from $\pi^{0} \to \gamma\gamma$ decay and we calculate the distance D_{ik} between the π^{0} decay vertex and the LKr:

$$D_{ik} = \frac{\sqrt{E_i E_k[(x_i - x_k)^2 + (y_i - y_k)^2]}}{m_{\pi^0}}$$

where E_i, E_k are the energies of the two photons and x, y their impact point coordinates on LKr. The two photon pairs with the smallest D_{ik} difference are selected as the best combination consistent with two π^{0} mesons from $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decay and the arithmetic average of the two D_{ik} values is used as the decay vertex position. The final event selection requires that the reconstructed $\pi^{\pm}\pi^{0}\pi^{0}$ invariant mass differs from the nominal $K^{\pm}$ mass quoted in the PDG by at most $\pm$ 6 MeV. This requirement is satisfied by 2.287×10^{7} events. The fraction of events with wrong photon pairing in this sample is $\sim 0.25\%$ as estimated by a high statistics Monte Carlo simulation.

4. The cusp effect

The invariant mass of the $\pi^{0}\pi^{0}$ system has been investigated in order to study the formation of pionium atoms in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays. Thanks to the high statistics, the very good M_{00}^{2} resolution, due the excellent intrinsic energy and spatial resolution of the LKr calorimeter, and the proper M_{00} reconstruction strategy, the data revealed a structure in the region $M_{00} = m_{+}$, where m_{+} is the mass of the charged pion. Fig.4 (upper part) shows the M_{00}^{2} mass distribution; a sudden change of slope near $M_{00}^{2} = m_{+}^{2} = 0.07792(\mathrm{GeV/c^2})^2$ is clearly visible. Fig.1 (lower part) is an enlargement of the region around $M_{00}^{2} = m_{+}^{2}$. Such an anomaly has not been observed in previous experiments.

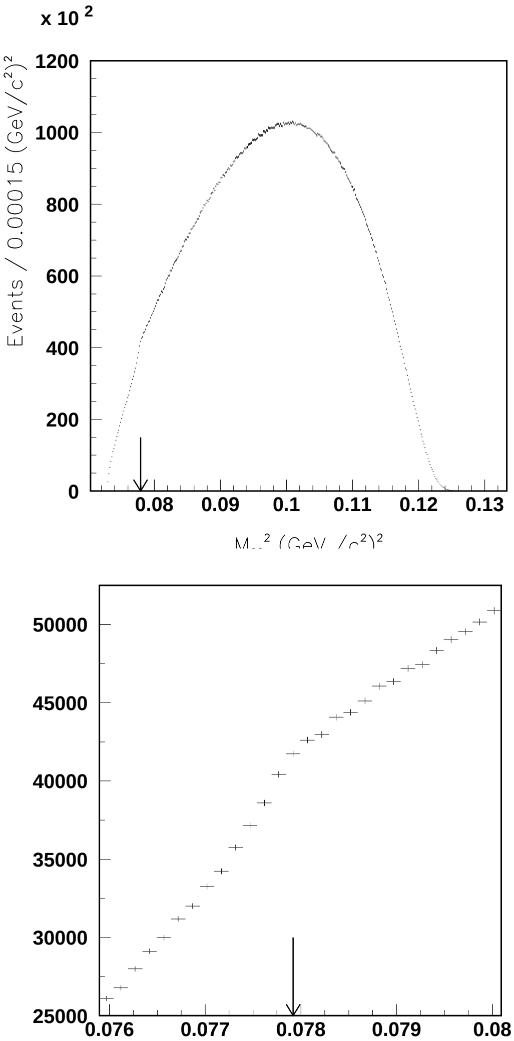

Figure 1. Distribution of the $\pi^{0}\pi^{0}$ invariant mass squared, M_{00}^{2}, (upper part). Enlargement of the region around $M_{00}^{2} = m_{+}^{2}$, indicated by the arrow (lower part).

5. Theoretical interpretation

The observed sudden change of slope suggests the presence of a threshold "cusp" effect from the decay $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ contributing to the $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ amplitude through the charge exchange process $\pi^{+}\pi^{-} \rightarrow \pi^{0}\pi^{0}$. The presence of a cusp at $M^2_{00} = m^2_{+}$ in $\pi^0\pi^0$ elastic scattering due to the effect of virtual $\pi^+\pi^-$ loops has been discussed first by Meissner et al. [2]. This contribution is directly proportional to $(a_0 - a_2)$ and displays a characteristic behavior when the $\pi^0\pi^0$ mass is in the vicinity of the $\pi^+\pi^-$ threshold, where it goes from dispersive to (dominantly) absorptive. Cabibbo recently computed [3] the $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ amplitude taking into account the 1-loop diagram shown in Fig. 2.

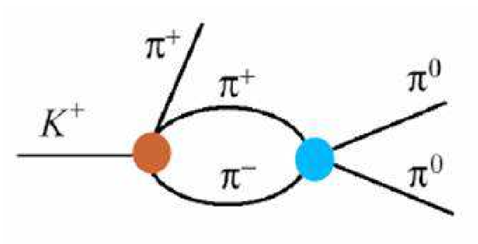

Figure 2. The $\pi\pi$ rescattering diagram.

In the Cabibbo theory, the $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ decay amplitude is given by the sum of two terms: the "unperturbed" amplitude $\mathcal{M}_0$ *à* la PDG plus a new term, $\mathcal{M}_1$, proportional to the I=0 and I=2 S-wave $\pi\pi$ scattering lengths (in the limit of exact isospin symmetry). The new term changes from real to imaginary at $M_{00} = m_{+}$. The destructive interference of $\mathcal{M}_0$ and $\mathcal{M}_1$ in the total amplitude causes the cusp and the apparent lack of events below the threshold. More recently Cabibbo and Isidori [4] have computed $\mathcal{O}(a_i^2)$ corrections to $K \rightarrow 3\pi$ amplitudes including one-loop (other rescattering processes) and two-loop diagrams. In the limit of exact isospin symmetry the amplitude depends on five S-wave scattering lengths, that can be expressed as a linear combination of a_0 and a_2. This model has been used to extract $(a_0 - a_2)$ from the $\pi^0\pi^0$ invariant mass distribution with high precision.

6. Determination of $(a_0 - a_2)$

The experimental M^2_{00} spectrum has been fitted using different theoretical models. In the first attempt to fit the data the unperturbed amplitude $\mathcal{M}_0$ has been used:

$$\mathcal{M}_0 = 1 + \frac{1}{2} g_0 u$$

where $u = (s_3 - s_0)/m^2_{+}$ is the Lorentz-invariant variable, $s_i = (P_K - P_i)^2$ $(i = 1, 2, 3)$, P_K and P_i are the 4-mumentum vectors of the initial kaon and of the three outgoing pions (i=3 corresponds to the $\pi^{\pm}$), respectively; $s_0 = (s_1 + s_2 + s_3)/3$ and $M^2_{00} = s_3$. The free parameters of the fit are g_0 and an overall normalization constant. Because of the anomaly at $M_{00} = m_{+}$ it is impossible to find a reasonable fit to the data. However fits with acceptable χ^2 values are obtained if the lower edge of the fit interval is raised few bins above $M_{00} = m_{+}$. The quality of this fit is displayed in Fig. 3, where the quantity Δ=(data-fit)/data is plotted as a function of M^2_{00}; it can be seen that in the region $M^2_{00} < (2m_{+})^2$ the data fall below the prediction based on the same parameters obtained from the fit region.

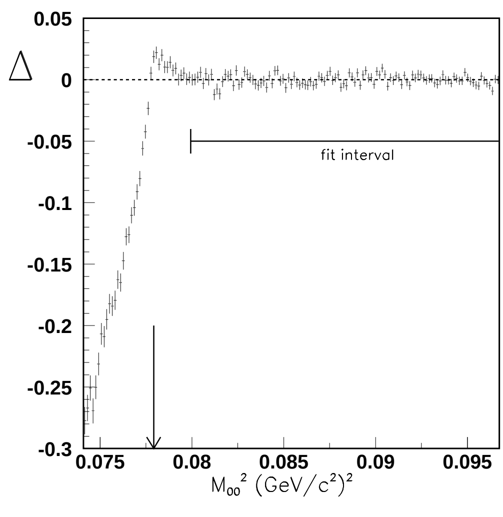

Figure 3. Δ=(data-fit)/data versus M^2_{00}.

In order to investigate this deficit, several checks against instrumental effects have been performed. We find no evidence for either resolution effects or acceptance non-linearities in the cusp region. In addition, variation in shape of photon energy distribution across the cusp agrees with MC prediction without cusp and no difference is observed between K^+ and K^- nor between data taken with opposite directions of the magnetic field. We conclude that the deficit of events in the data in the region $\mathrm{M}_{00}^2 < \mathrm{m}_+^2$ is due to a real physical effect.

Using the Cabibbo one-loop model, a fit to the M_{00}^2 distribution in the interval $0.074 < \mathrm{M}_{00}^2 < 0.097$ $(GeV/c^2)^2$ has been performed; the result, shown in Fig. 4a, gives $\chi^2 = 420.1$ for 148 degrees of freedom. One can see that this model provides a much better but still unsatisfactory description of the data. In particular the data points are systematically above the fit in the region near $\mathrm{M}_{00}^2 = \mathrm{m}_+^2$. A significant improvement is obtained when the Cabibbo-Isidori model is used. This fit has five free parameters: $(a_0 - a_2)\mathrm{m}_+$, $a_2\mathrm{m}_+$, the linear slope g_0, the quadratic slope h and an overall normalization constant. The quality of the fit ($\chi^2 = 154.8$ for 146 degrees of freedom) is shown in Fig. 4b. A better fit ($\chi^2 = 149.1$ for 145 degrees of freedom, see Fig. 4c) is obtained by adding to the model a term describing the expected formation of the $\pi^+\pi^-$ atom ("pionium"), decaying to $\pi^0\pi^0$ at $\mathrm{M}_{00} = \mathrm{m}_+$. The best fit value for the rate $K^\pm \rightarrow \pi^\pm +$ pionium decay, normalized to the $\mathrm{K}^\pm \rightarrow \pi^\pm\pi^+\pi^-$ dacay rate, is $(1.6 \pm 0.66) \times 10^{-5}$, in reasonable agreement with the predicted value $\sim 0.8 \times 10^{-5}$ [5]. Finally, since the rescattering model of ref. [4] does not include radiative corrections, we prefer to exclude from the final fit a region of seven consecutive bins centered at $\mathrm{M}_{00} = \mathrm{m}_+$. The quality of this fit ($\chi^2 = 145.5$ for 139 degrees of freedom) is shown in Fig. 4d. Taking into account all systematic and external uncertainties, the final result, taken as the arithmetic average of two independent analysis [9], is:

$$(a_0 - a_2)\mathrm{m}_+ = 0.268 \pm 0.010(\mathrm{stat}) \pm 0.004(\mathrm{syst}) \pm 0.013(ext)$$

$$a_2\mathrm{m}_+ = -0.041 \pm 0.022(\mathrm{stat}) \pm 0.014(\mathrm{syst})$$

The external error is an additional theoretical error of $\pm 5\%$ on $(a_0 - a_2)$ estimated in ref. [4] as the result of neglecting higher order terms in the rescattering model. This uncertainties have no significant effect on $a_2\mathrm{m}_+$. The two statistical errors from the fit are strongly correlated, with a correlation coefficient of -0.858. Performing the fit with constraints imposed on a_0 and a_2 by analyticity and chiral symmetry [6] we obtain

$$a_0\mathrm{m}_+ = 0.220 \pm 0.006(\mathrm{stat}) \pm 0.004(\mathrm{syst}) \pm 0.011(ext)$$

which corresponds to

$$(a_0 - a_2)\mathrm{m}_+ = 0.264 \pm 0.006(\mathrm{stat}) \pm 0.004(\mathrm{syst}) \pm 0.013(ext)$$

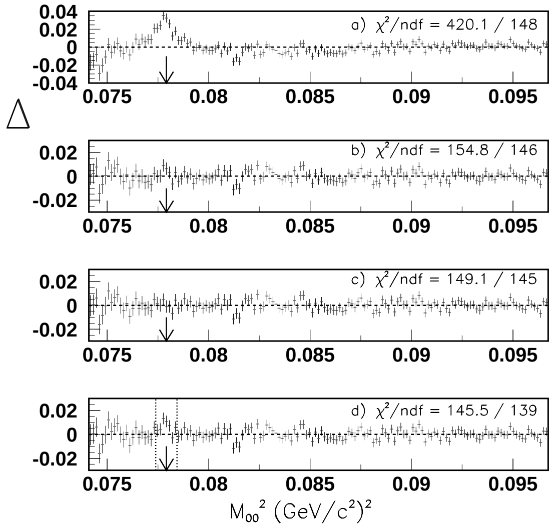

Figure 4. Δ=(data-fit)/data versus M_{00}^2 for various theoretical models. See text for explanation.

It is worth to note that this analysis gives the first direct measurement of a_2, though not as precise as that of $(a_0 - a_2)$. This result is compatible, within the errors, with the results obtained by the BNL E865 [7] and the CERN DIRAC experiments [8]. It is also in very good agreement with theoretical calculations performed in the framework of Chiral Perturbation Theory (ChPT) [10], which predict $(a_0 - a_2)\mathrm{m}_+ = 0.265 \pm 0.004$. Another theoretical calculation based on direct analysis of $\pi\pi$ scattering data without using chiral symmetry [11] leads to the result ($(a_0 - a_2)\mathrm{m}_+ = 0.278 \pm 0.016$), slightly different and with a larger uncertainty, which also agrees with our result.

Recently Colangelo et al. (CGKR) [12] calculated the $\mathrm{K} \to 3\pi$ amplitudes within a non-relativistic effective Lagrangian framework, by a double expansion in a (scattering lengths) and ϵ (kinetic energies) at order ϵ^2, $a\epsilon^3$, $a^2\epsilon^2$; CGKR representation is valid in the whole decay region. The amplitudes agree with Cabibbo-Isidori calculation up to $a\epsilon^3$ and differ away from threshold at order a^2.

7. Conclusions

The cusp anomaly observed in the $\pi^0\pi^0$ invariant mass distribution at $\mathrm{M}_{00} = \mathrm{m}_+$ can be interpreted in terms of an additional contribution to the decay amplitude from the charge exchange process $\pi^+\pi^- \to \pi^0\pi^0$. From the spectrum we extracted $(a_0 - a_2)$, the difference between the I=0 and I=2 S-wave $\pi\pi$ scattering lengths.

The determination of $\pi\pi$ scattering lengths relies on a variety of methods, such as the measurement of the $\mathrm{K}^\pm \to \pi^+\pi^-\mathrm{e}^\pm\nu_\mathrm{e}$ decay, the measurement of the pionium lifetime as well as the pion-nucleon scattering near threshold. Our collaboration has studied the $\mathrm{K}^\pm \to \pi^+\pi^-\mathrm{e}^\pm\nu_\mathrm{e}$ decay and recently has reported a preliminary result based on a sample of 3.7×10^5 events; using the universal band constraint a value of $a_0\mathrm{m}_+ = 0.256 \pm 0.008_{\mathrm{stat}} \pm 0.007_{\mathrm{syst}} \pm 0.018_{\mathrm{theory}}$ has been measured [13]. It is important to note that care must be taken in comparing results obtained with different techniques. They may be compared only in the same theoretical framework.

The cusp method is very accurate, sensitive to the sign of $(a_0 - a_2)$ and model independent, since it is based only on general assumption of unitarity and analyticity. We note that a similar effect arises in the interference between $K_L \to \pi^0\pi^0\pi^0$ and $K_L \to \pi^+\pi^-\pi^0$, followed by a $\pi^+\pi^- \to \pi^0\pi^0$ process. The effect is smaller than in the charged kaon decay, but could also lead to a determination of $(a_0 - a_2)$. We are studying this effect using a set of 0.1×10^9 events collected in 2000 by NA48, when the experiment was taking data to measure the direct CP violation parameter ε'/ε.

To conclude, the study of a large sample of $\mathrm{K}^\pm \to \pi^\pm\pi^0\pi^0$ decays with excellent resolution on the $\pi^0\pi^0$ invariant mass has provided a novel, precise determination of $(a_0 - a_2)$, independent of other methods. In the near future, the full data sample collected in 2003 and 2004 by NA48/2, which represents an increase of a factor 5 in statistics, will be analyzed resulting in a further reduction of the statistical error of the measurement. An improvement of the rescattering model to include higher order terms and radiative correction is expected on the theoretical part.

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