Low energy dynamics and the ππ scattering lengths from the NA48/2 experiment at CERN

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On behalf of the NA48/2 Collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

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Outline

- The NA48/2 experiment
- The decay $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$
- Observation of a "cusp"
- Theoretical interpretation
- Extraction of scattering lengths $(a_0 - a_2)$
- The k term in the matrix element
- The "cusp" in $K_L \rightarrow 3\pi^0$
- Measurements and predictions for a_0 and a_2
- Conclusions and outlook

The NA48/2 experiment

Beam K^+ and K^- narrow band (60 ± 3) GeV simultaneous, coaxial, focused

Detector main components

magnetic spectrometer $\Delta p/p = 1.0\% + 0.044\% * p$ [GeV/c]

liquid krypton e.m. calorimeter $\Delta E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$ [GeV]

Hodoscope, hadron calorimeter, muon veto counters, photon vetoes

Trigger

≥ 1 charged particle, ≥ 4 photons, geometrical cuts, distance γ-γ and γ-track

Two years of data taking: 2003 and 2004

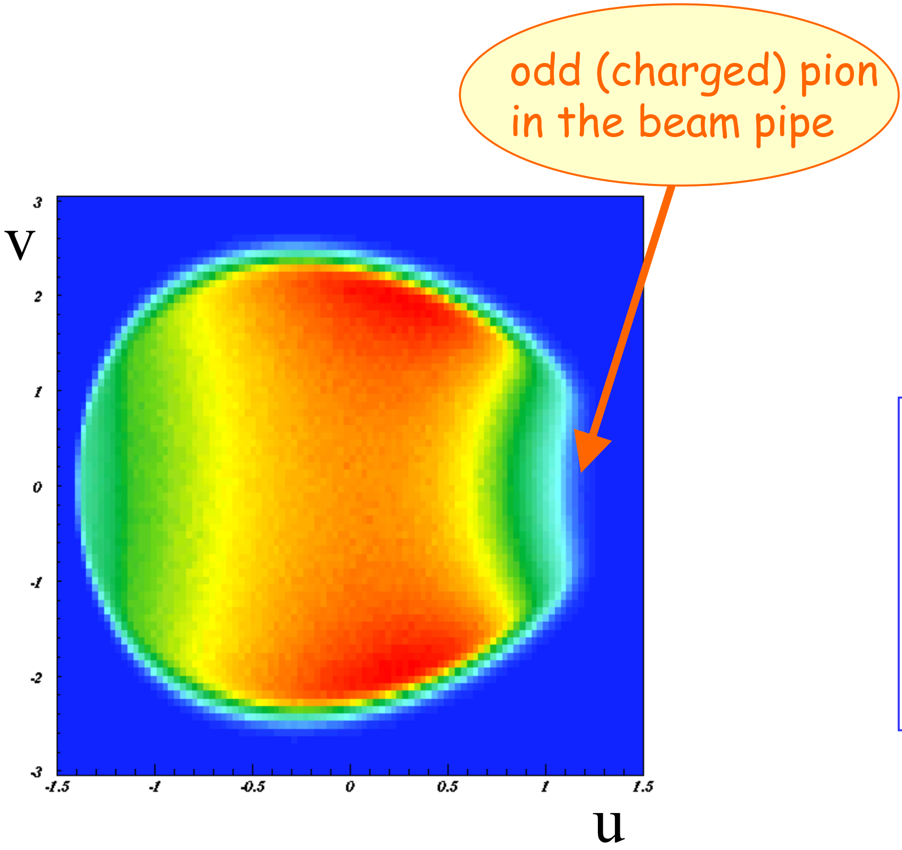
$$K^\pm \to \pi^\pm\pi^0\pi^0 \sim 0.1 \cdot 10^9$$

$$K^\pm \to \pi^\pm\pi^+\pi^- \sim 4 \cdot 10^9$$

Result based on a partial sample of 2003

$\sim 2.3 \cdot 10^7$ $K^\pm \to \pi^\pm\pi^0\pi^0$ decays

The $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ decay



Lorentz-invariants

$s_i = (P_K - P_{\pi i})^2$, i=1,2,3 (3=odd π)

$s_0 = (s_1+s_2+s_3)/3$

$u = (s_3 - s_0)/m_\pi^2 = 2m_K \cdot (m_K/3 - E_{odd})/m_\pi^2$

$v = (s_2 - s_1)/m_\pi^2 = 2m_K \cdot (E_1 - E_2)/m_\pi^2$

Matrix element

$|M(u,v)|^2 \sim 1 + gu + hu^2 + kv^2$

just a polinomial expansion

Linear slope g dominates over quadratic terms h, k ($g = 0.652 \pm 0.031$)

NOTE: symmetry $\pi^0_1 \leftrightarrow \pi^0_2 \Rightarrow$ only even powers of v are allowed (0, 2, ….)

Structure in $K^{\pm}\rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ decay

Search for pionium atoms in the $K^{\pm}\rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ channel as a resonance in the $\pi^0\pi^0$ invariant mass $M_{00}^2 = 4m_{\pi^\pm}^2$ (threshold for $\pi^+\pi^-$ production) exploiting

- ✓Very high statistics,
- ✓very good calorimeter resolution
- ✓proper M_{00} reconstruction strategy

→ Data reveal a **structure** in the $M_{00}^2 = 4m_{\pi^\pm}^2$ region

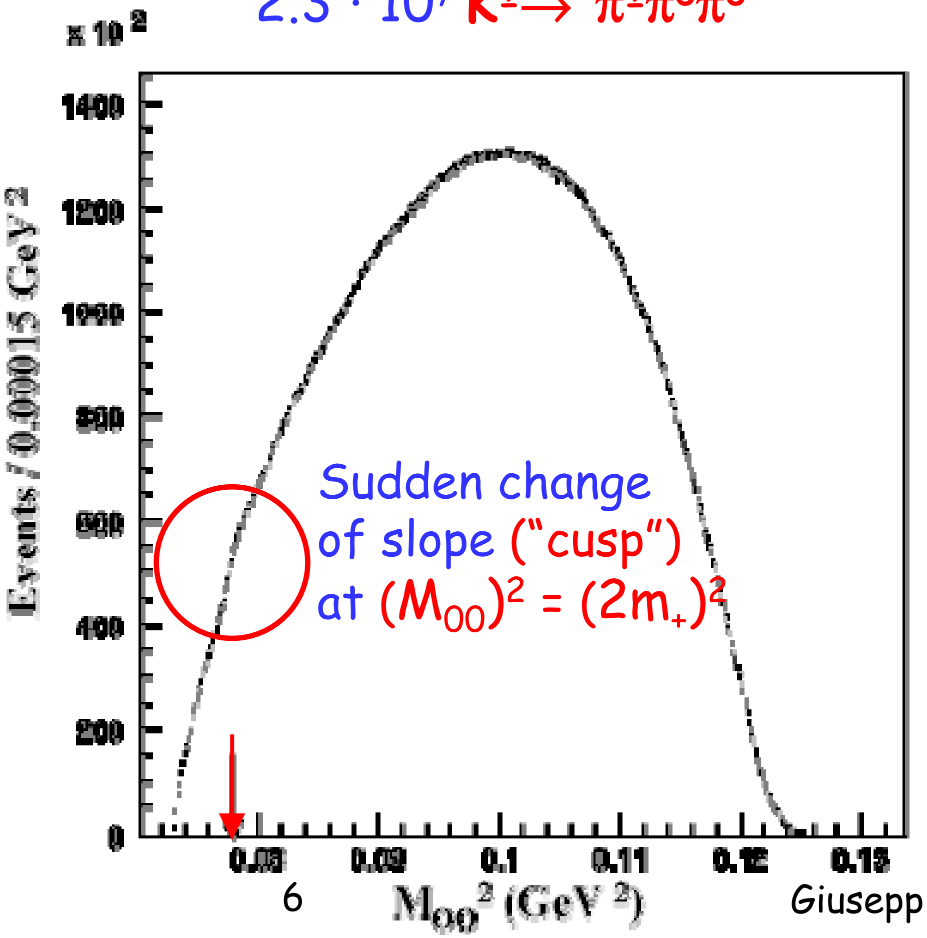
N. Cabibbo: "It is a clean and beautiful example of a general cusp-like behaviour of cross sections next to threshold for new channels"

→ A method based on first principles (unitarity, analiticity) for extracting information on strong interaction at low energy

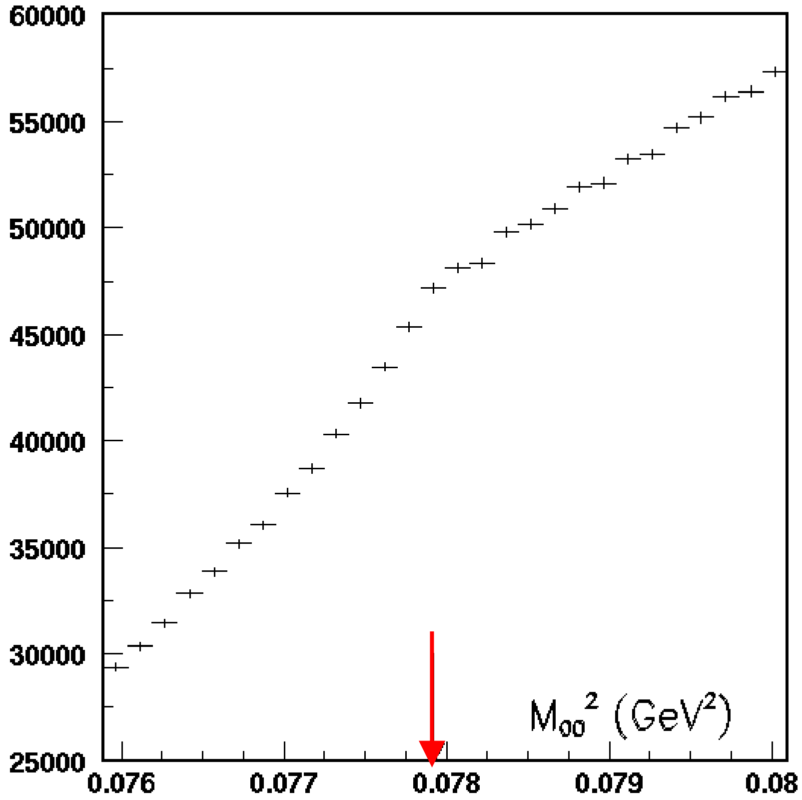
- ❖ First observation of ππ scattering effects in the Dalitz plot
- ❖ Precise and model independent measurement of a_0-a_2 (the difference between ππ scattering lengths in the isospin I=0 and I=2 states)

Observation of a "cusp" in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$

Experimental M_{00}^2 distribution
$2.3 \cdot 10^7$ $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$

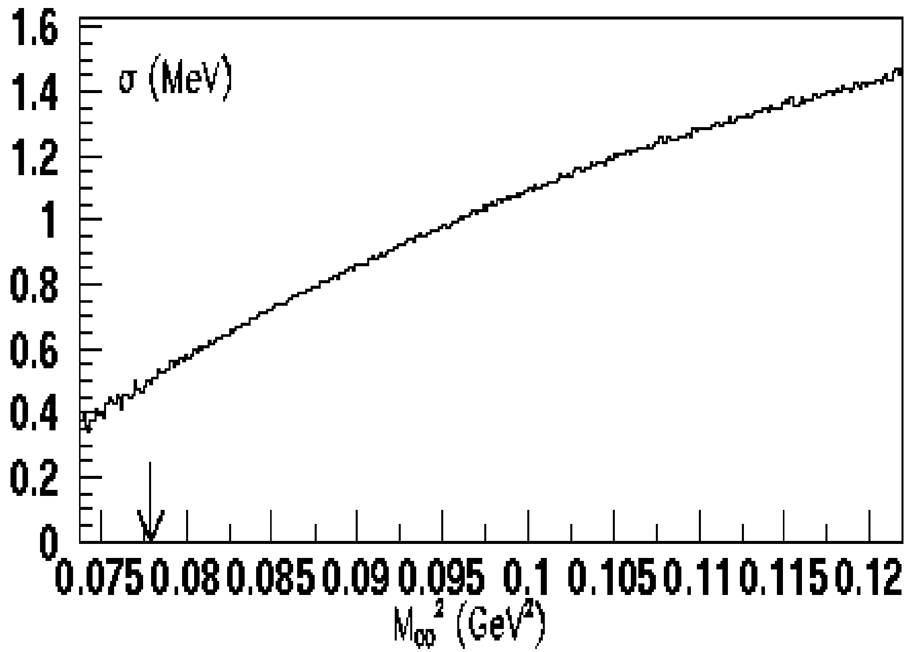


Experimental M_{00}^2 distribution
Zoom on the cusp region

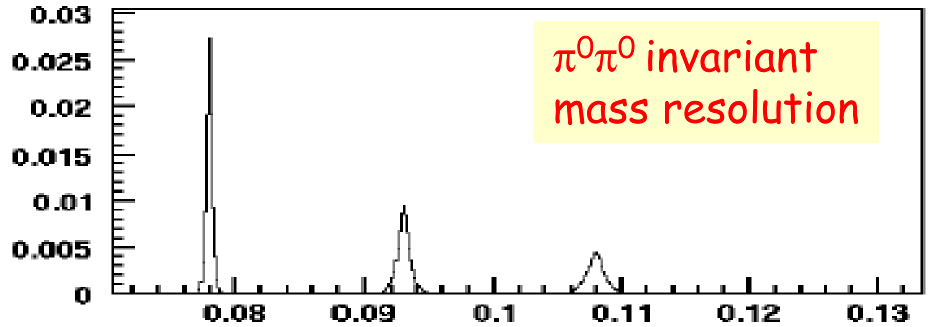


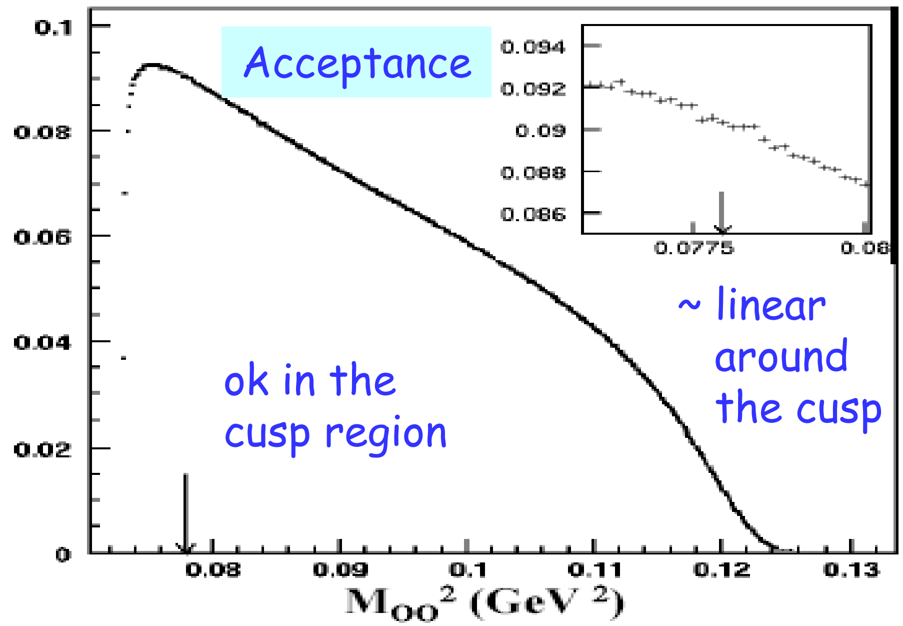
Check against instrumental effects/1
Resolution and acceptance

$\pi^0\pi^0$ invariant mass resolution (σ) versus $(M_{00})^2$ (MonteCarlo)

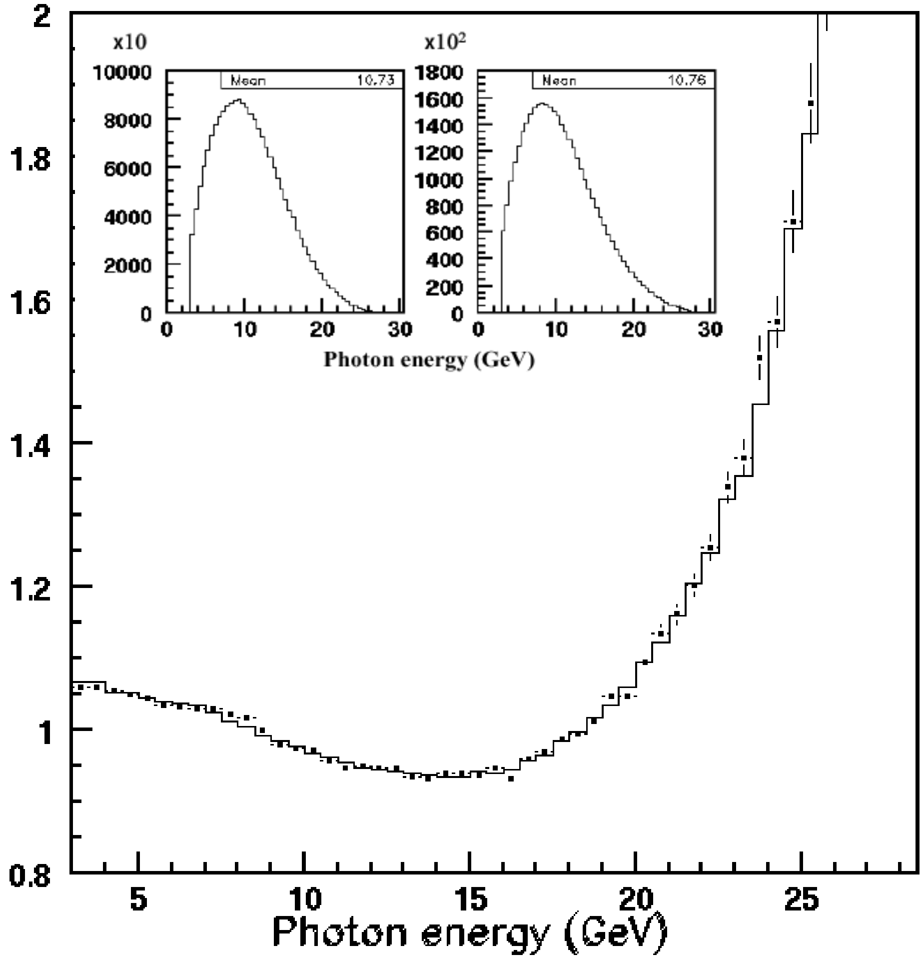


$\sigma \approx 0.5$ MeV at $(M_{00})^2 = (2m_+)^2$





Check against instrumental effects/2 Photon energy above/below threshold

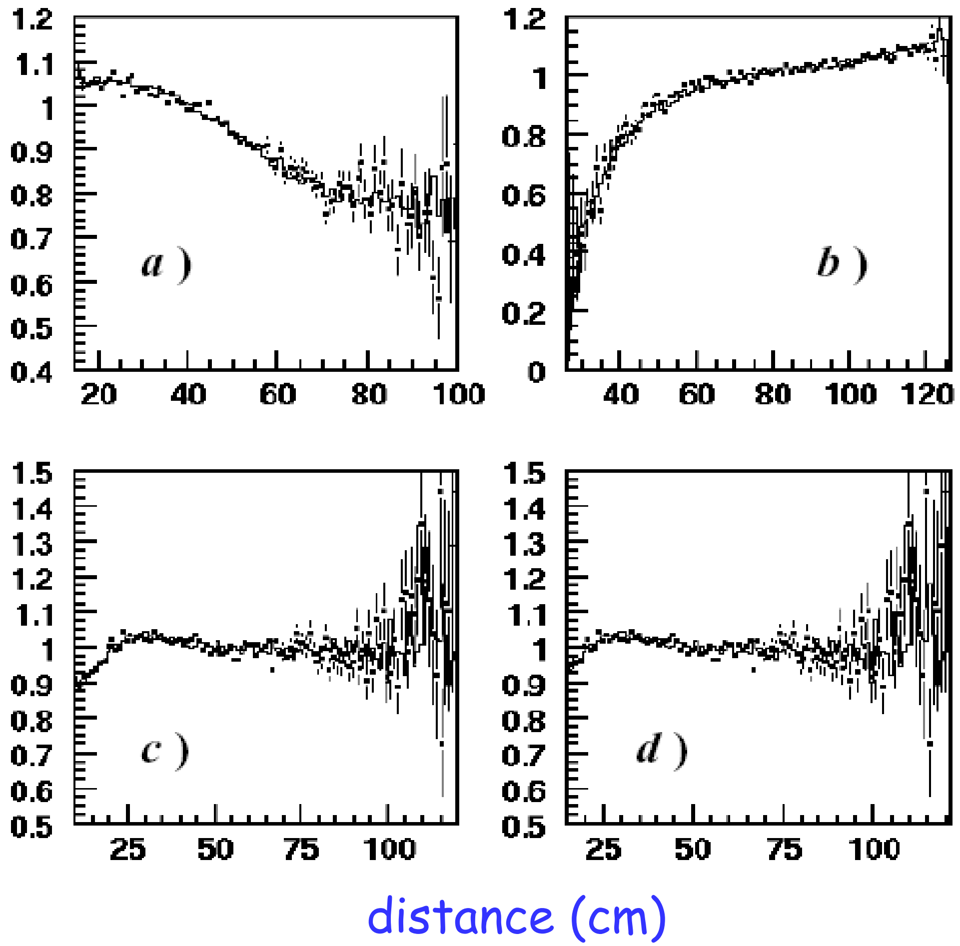


Study data in two small intervals just above and below $2m_+$

Ratio of normalized photon energy distributions between events with $M_{00}^2 > (2m_+)^2$ and $M_{00}^2 < (2m_+)^2$ (data-points, MC-solid line)

Variation of shape of photon energy distribution across the cusp agrees with MC prediction without cusp

Check against instrumental effects/3 Photon distances above/below threshold



Distributions of various photon distances (cm) measured at LKr

a) Min γ distance from LKr axis
b) Max γ distance from LKr axis
c) Min γ–γ distance
d) Min γ–track distance

Good agreement between

- Data – Monte Carlo
- Data above/below

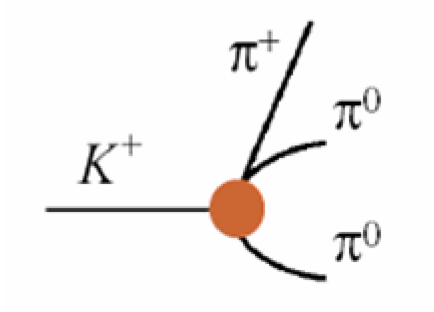
Monte Carlo describes correctly the M_{00} dependence of the detection efficiency

CUSP is a physical effect

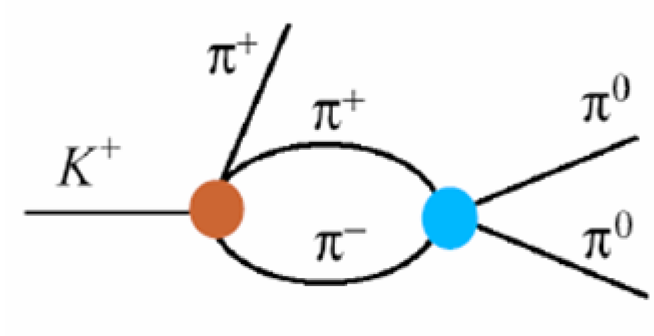
The origin of the "cusp" FSI effects in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$

Two effects contribute to $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$

Direct emission



Charge exchange $\pi^+\pi^- \to \pi^0\pi^0$



Interference (destructive) is the main cause of the singularity in the $\pi^0\pi^0$ invariant mass distribution. The effect of the interference is present (first order) below the threshold and not above.

Cabibbo rescattering model/1

Cabibbo PRL 93 (2004) 121801

$M(K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}) = M = M_0 + M_1$

"Unperturbed" amplitude

$$M_0 = A^0\left(1 + g^0(s_3 - s_0)/2m_{\pi^+}^2\right)$$

$$M_+ = A^+\left(1 + g^+(s_3 - s_0)/2m_{\pi^+}^2\right)$$

contributes to M_1 at threshold

Above threshold (known)

Imaginary for $M_{00}^2 > 4m_{\pi^+}^2$

No interference

$$|M|^2 = (M_0)^2 + (iM_1)^2$$

$$M_1 = i2\frac{(a_0 - a_2)m_{\pi^+}}{3} M_+^{thr} \sqrt{\frac{s_3 - 4m_{\pi^+}^2}{s_3}}$$

Below threshold (analycity)

Real < 0 for $M_{00}^2 < 4m_{\pi^+}^2$

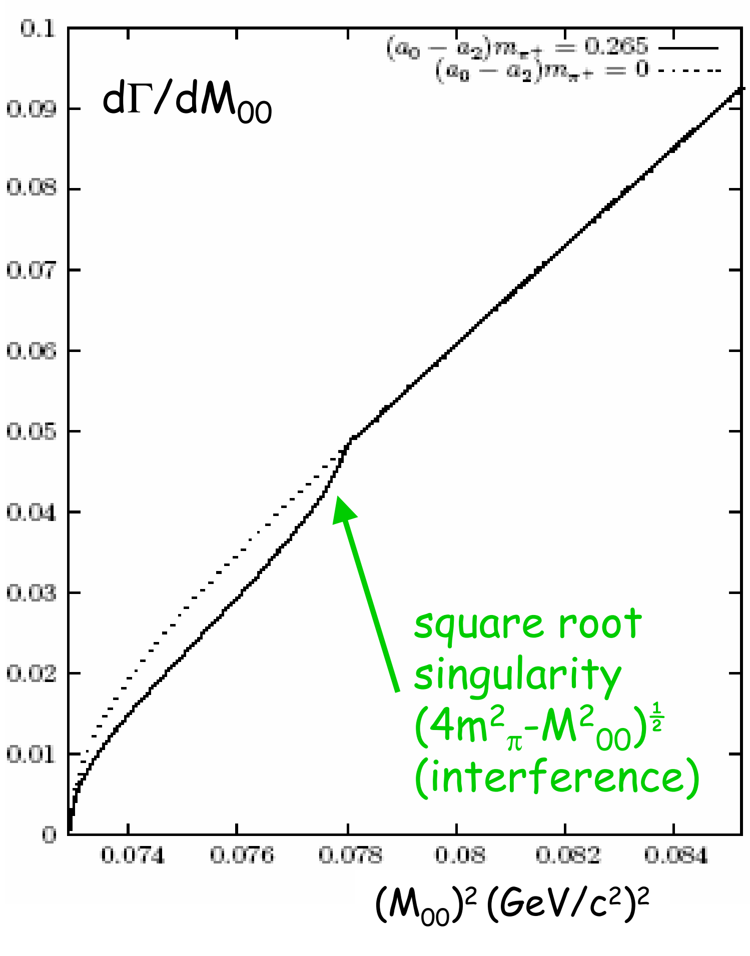
Destructive interference

$$|M|^2 = (M_0)^2 + (M_1)^2 + 2M_0M_1$$

$$M_1 = -2\frac{(a_0 - a_2)m_{\pi^+}}{3} M_+^{thr} \sqrt{\frac{4m_{\pi^+}^2 - s_3}{s_3}}$$

Cabibbo rescattering model/2

The differential decay rate with/without the rescattering corrections (using $A_{AV}^{+} = 2A_{AV}^{0}$, $g^{\pm}$ (PDG), $(a_0\text{-}a_2)m_{\pi+} = 0.265 \pm 0.004$ (CGL+dispersive)



The cusp is proportional to the S-wave $\pi^+\pi^-$ charge exchange scattering length $(a_0 - a_2)$ (in the limit of exact isospin).
Extract $(a_0 - a_2)$ from the $\pi^0\pi^0$ spectrum

In order to deal with experimental measurement of $(a_0\text{-}a_2)$ at few % by NA48, the theory has to be good to few 10^{-3} (cusp is a 10% effect)

Higher order rescattering effects + radiative corrections have to be included

Cabibbo-Isidori approach/1

N. Cabibbo, G. Isidori JHEP 0503 (2005) 21

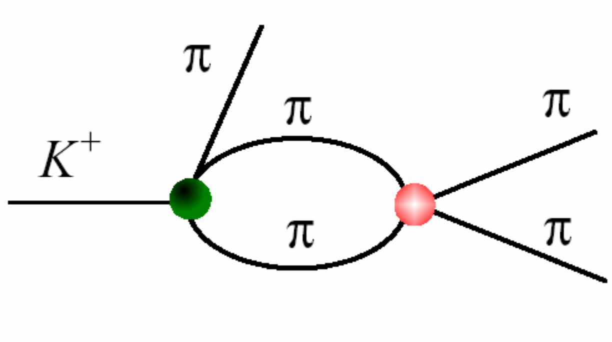
Computation of $O(a_i^2)$ corrections to $K \rightarrow 3\pi$ amplitudes

Including

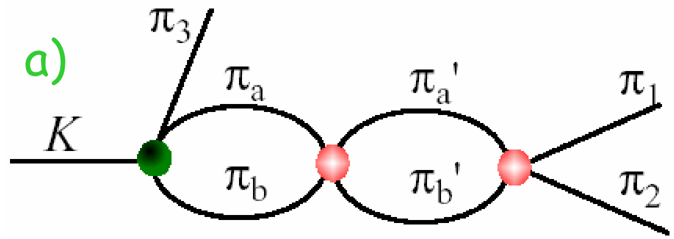
One-loop and Two-loop diagrams

Other re-scattering corrections: $\pi^+\pi^- \rightarrow \pi^0\pi^0$, $\pi^+\pi^0 \rightarrow \pi^+\pi^0$, $\pi^0\pi^0 \rightarrow \pi^0\pi^0$

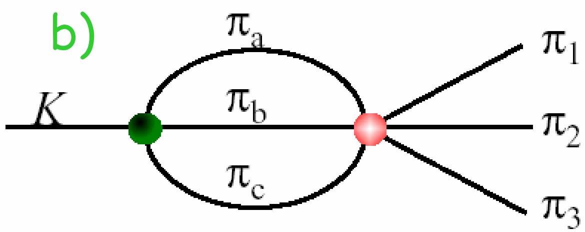
a) Single channel $\pi\pi$ scattering
b) Irreducible $3\pi \rightarrow 3\pi$
c) Multi-channel $\pi\pi$ scattering



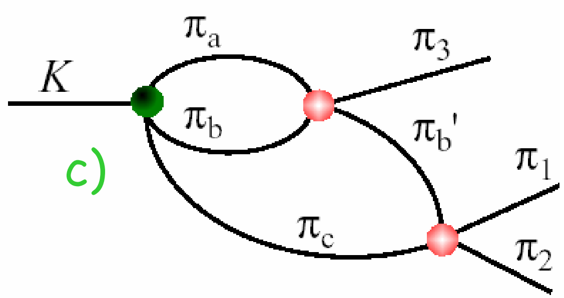
a)



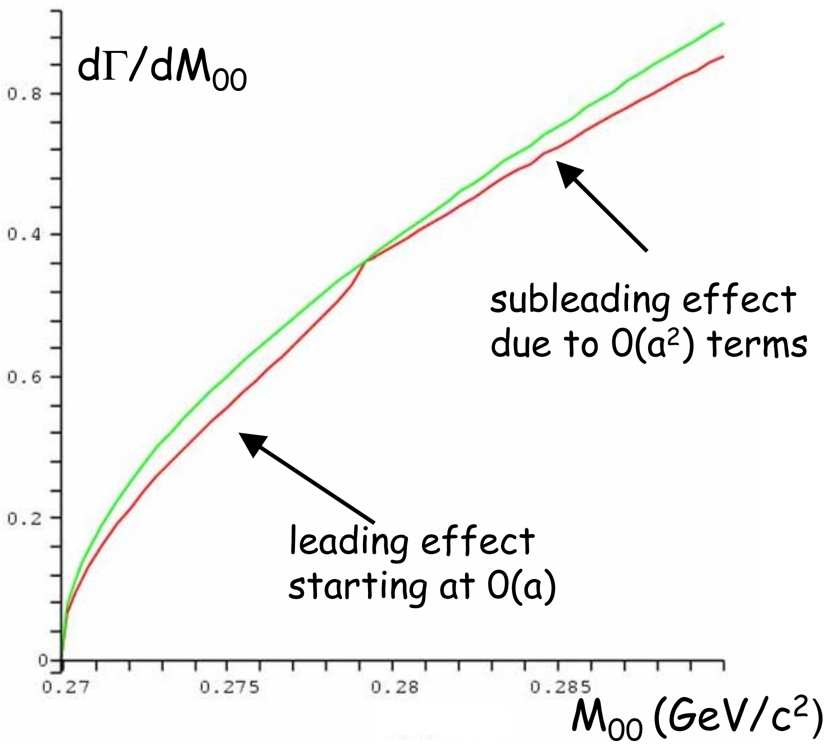
b)

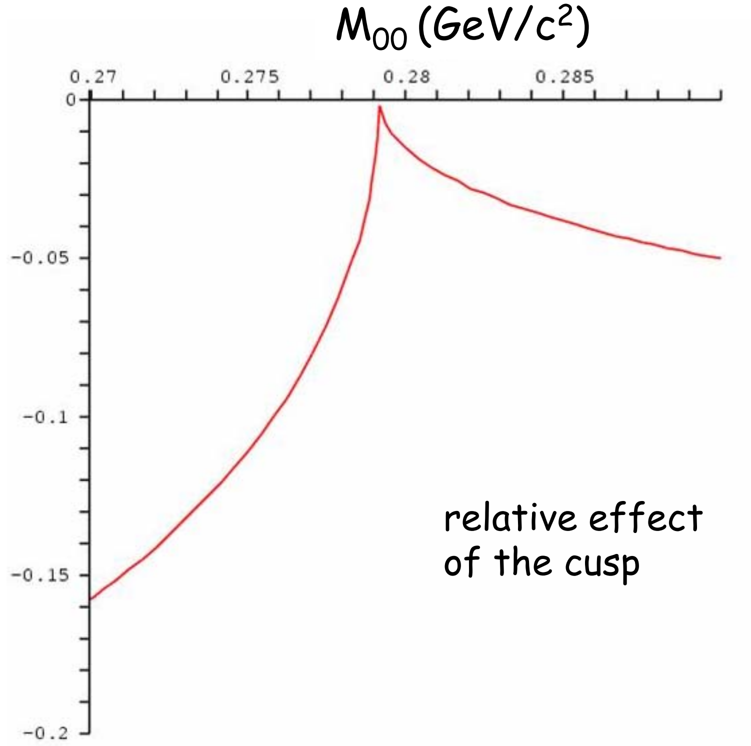


c)



Cabibbo-Isidori approach/2





The development to the second order in powers of a_0 and a_2 produce a small square-root cusp behaviour also above the ππ singularity

➡ Theoretical uncertainty level up to now ~5%

But O(a_i^3) + radiative corrections can be computed
Expected theoretical uncertainty ~1% independent of ChPT

Electromagnetic effects

Radiative corrections

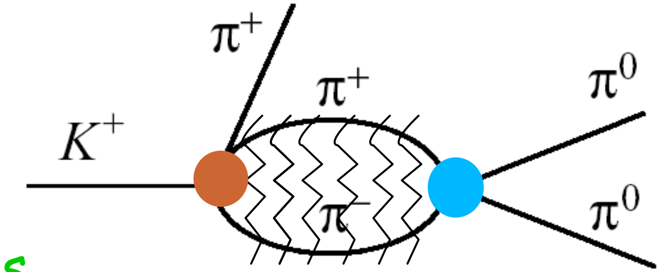
Corrections due to virtual and real photons are expected to be small (% level) except possibly next to the cusp (presence of bound state). Few bins around the cusp are excluded when fitting to extract (a_0-a_2)

Bound state: pionium

A contribution from $\pi^+\pi^-$ bound state is expected (Silagadze, JETP Lett. 60 (1994) 689) with dominant decay mode $\pi^0\pi^0$ proportional to

$$\frac{\Gamma(K^+ \to \pi^+ + pionium)}{\Gamma(K^+ \to \pi^+\pi^0\pi^0)} \approx 2.6 \cdot 10^{-5}$$

recalculated according to the latest PDG BR's



Expected contribution to the $(M_{00})^2$ bin centered at $(2m_+)^2$ is ~2.6%

Effective field theory approach

Colangelo, Gasser, Kubis and Rusetsky hep-ph/0604084

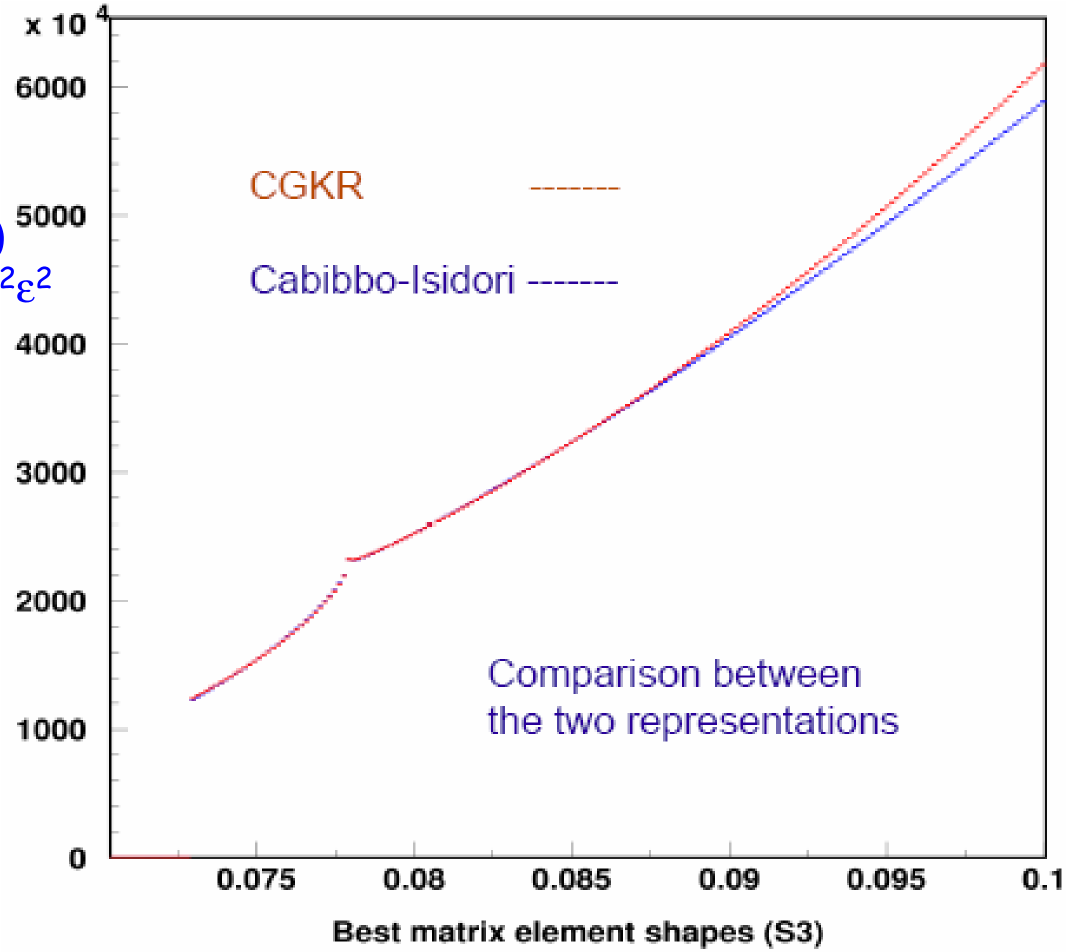
Recently CGKR calculated the $K \rightarrow 3\pi$ amplitudes within a non-relativistic effective Lagrangian framework, by a double expansion in a (scattering lengths) and ε (kinetic energies) at order ε^2, $a\varepsilon^3$, $a^2\varepsilon^2$

CGKR representation is valid in the whole decay region.

Amplitudes agree with CI up to $a\varepsilon^3$, differ away from threshold at order a^2.

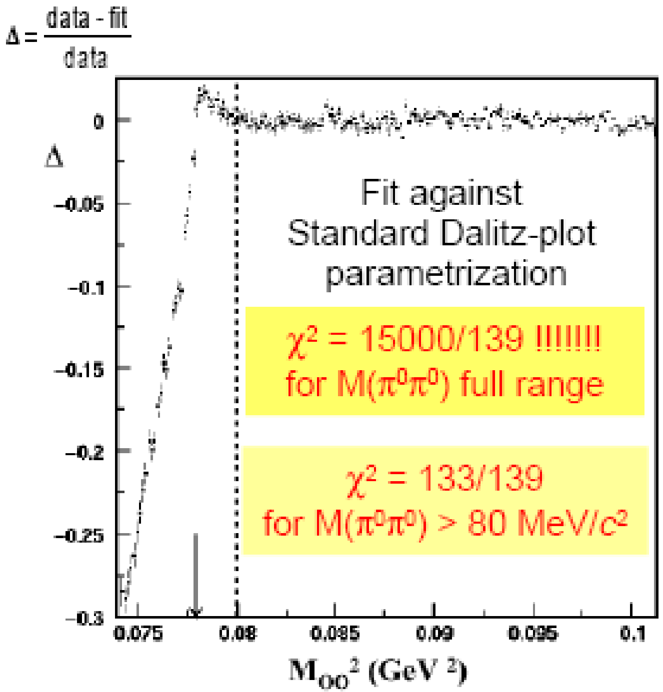
Experimental work is in progress for:

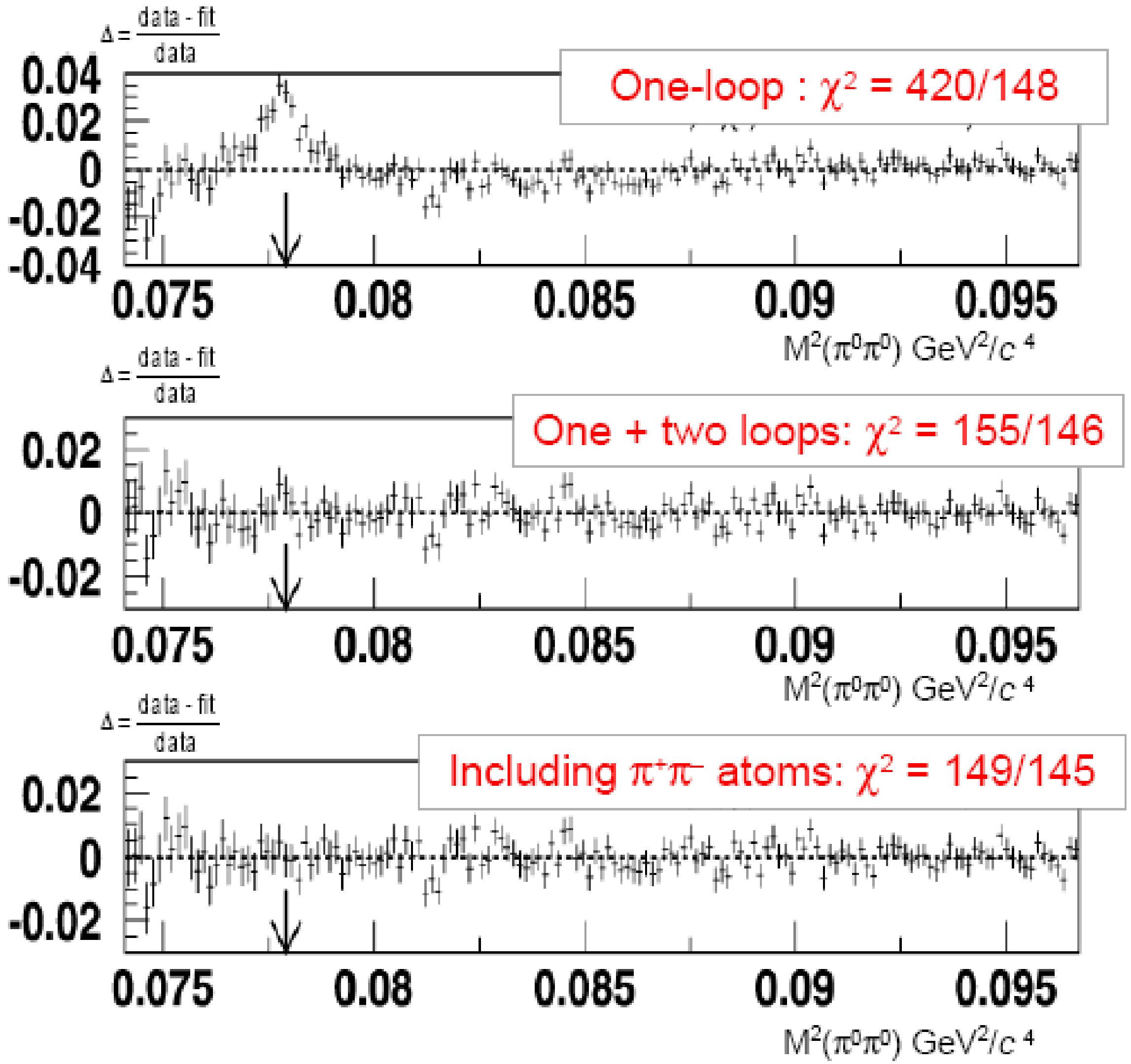
1) analyzing NA48 data with the CGKR representation extending the fit to a wider range;
2) investigating the additional cusps which are present at the border of the Dalitz plots, also for the channel $K^+ \rightarrow \pi^+ \pi^+ \pi^-$



Fit to the experimental invariant mass distribution

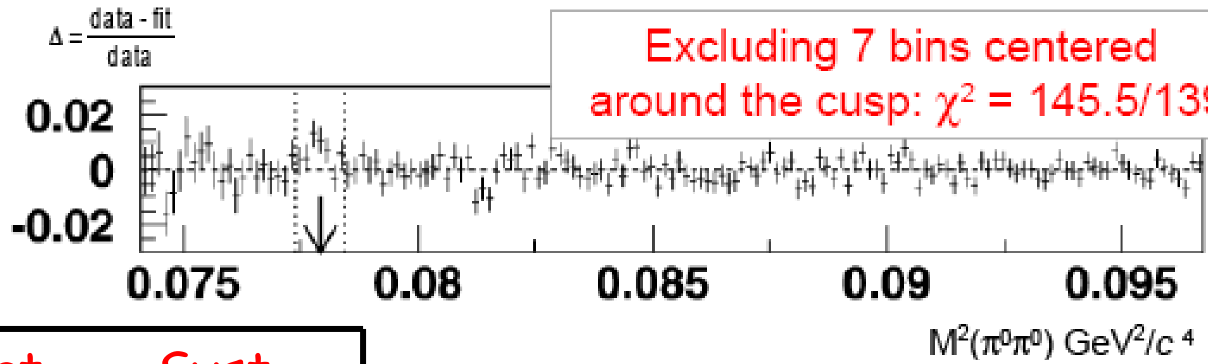
5 fit parameters:
- g, h linear and quadratic Dalitz plot parameters
- $(a_0-a_2)m_+$, $a_2 m_+$
- normalization





Best fit parameters

Three independent analysis: two based on "professional toy" MC and one on full GEANT-based MC



Excluding 7 bins centered around the cusp: χ^2 = 145.5/139

		Stat.	Syst.
$(a_0-a_2)m_+$	0.268	± 0.010	± 0.004
a_2m_+	-0.041	± 0.022	± 0.014
g	0.645	± 0.004	± 0.009
h'	-0.047	± 0.012	± 0.011

Isospin breaking effects included

Systematics

Main systematics

	$(a_0-a_2)m_+$	a_2m_+
Acceptance	± 0.001	± 0.012
Trigger efficiency	± 0.001	± 0.005
Fit interval	± 0.0025	± 0.006
Others	± 0.002	-
Total	± 0.004	± 0.014
External(*)	± 0.013	

(*)main component estimated by Cabibbo-Isidori as the result of neglecting higher order terms and radiative corrections in the rescattering model

Systematic checks

Photon isolation

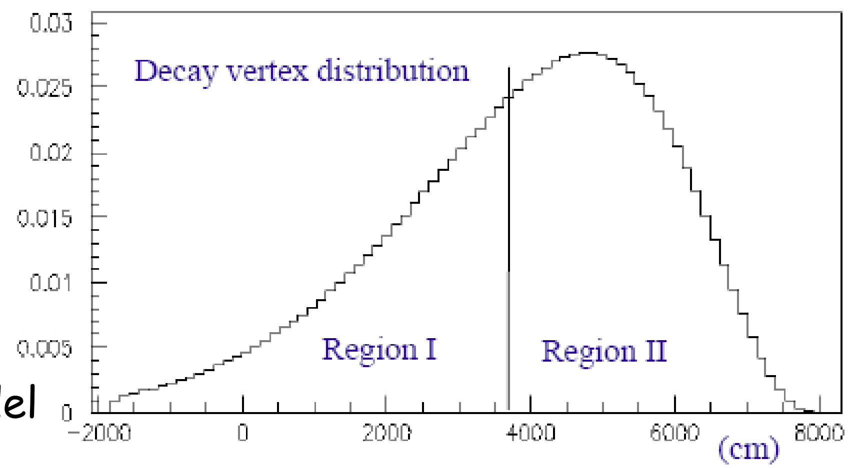
Default cut d = 5 cm, try d = 10,15

Systematic on $(a_0-a_2)m_+$ → ± 0.002

Z vertex

Measurement from two decay regions

No systematic on $(a_0-a_2)m_+$



Results on the scattering lengths

NA48/2 Batley et al., Phys. Lett. B 633 (2006) 173

Final results of the unconstrained fit to the re-scattering model

		Stat.	Syst.	Ext.
$(a_0-a_2)m_+$	0.268	± 0.010	± 0.004	± 0.013
a_2m_+	-0.041	± 0.022	± 0.014	

N.B. The two statistical errors from the fit are strongly correlated (-0.86)

Performing the fit with constraints imposed on a_0 and a_2 by analycity and chiral symmetry (after Colangelo et al. PRL 86 (2001) 5008) leads to the following

		Stat.	Syst.	Ext.
$(a_0-a_2)m_+$	0.264	± 0.006	± 0.004	± 0.013
a_0m_+	-0.220	± 0.006	± 0.004	± 0.011

Effect of the k term

$K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decay matrix depends on two independent variables s_3 and $s_1 - s_2$

$$M_{+00} = 1 + \frac{1}{2} g_0 \frac{(s_3 - s_0)}{m_+^2} + \frac{1}{2} h' \frac{(s_3 - s_0)^2}{m_+^4} + \frac{1}{2} k \frac{(s_1 - s_2)^2}{m_+^4} + \ldots$$

Published cusp analysis → k = 0, g_0 and h free parameters

In PDG 2004 → k = 0.004 ± 0.007

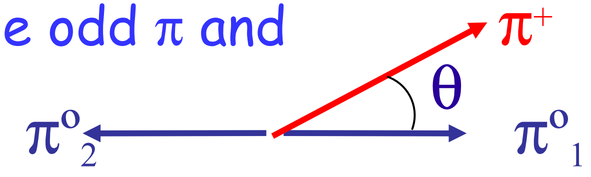
The most precise result → k = 0.001 ± 0.001 ± 0.002 (252K events, ISTRA)

By performing a 2D fit of the Dalitz plot NA48 finds evidence for a non-zero value of the k term (order of magnitude ~ 1%)

Fit of the Dalitz plot with k as additional parameter

STRATEGY

- Instead of (s_3, $s_1 - s_2$) two alternative variables (s_3, cosθ) are used, where cosθ is the angle between the odd π and the direction of the even pions in their CM reference system



- The resulting rectangular shaped Dalitz plot (-1< cosθ <1, for any value of s_3) allows for binning without crossing the physical boundaries
- At the moment we fit only in a region beyond the cusp

1D fit procedure adopted in order not to spoil the excellent resolution in s_3 by a mix with s_1, s_2 or cosθ

Negligible effect of k O(0.01) on the values of a_0-a_2 and a_2

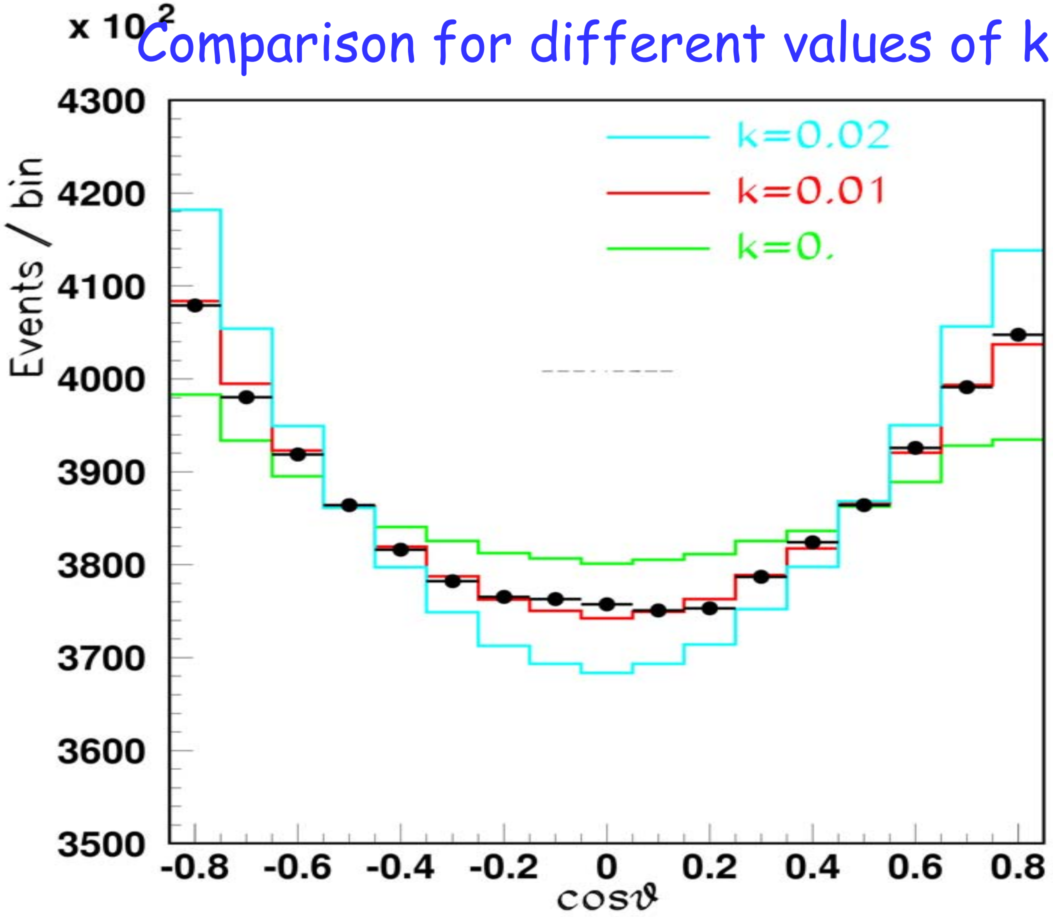
(g and h' move)

...Work in progress...

Different values of the k term

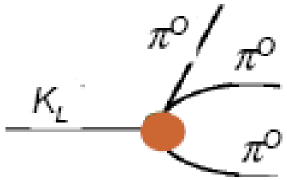
Data – MC

Comparison for different values of k

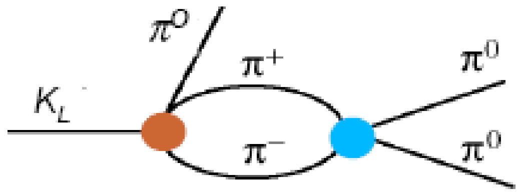


The "cusp" effect in $K_L \to \pi^0\pi^0\pi^0$

K_L

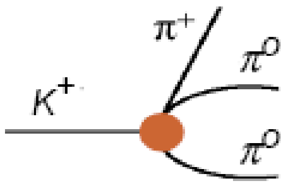


$M_0 \propto 1$

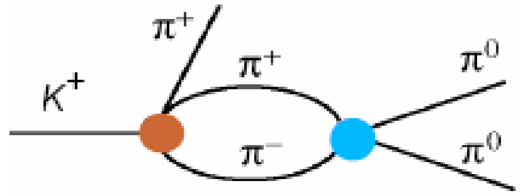


$M_1 \propto (1+g_{+-0}u)(a_0-a_2)$

K^+



$M_0 \propto (1+g_{+00}u)$



$M_1 \propto (1+g_{++-}u)(a_0-a_2)$

$$\frac{(M_1/M_0)_{K^+}}{(M_1/M_0)_{K_L}} = 2\sqrt{2}\,\frac{1+g_{++-}u}{1+g_{+00}u} \times \frac{1}{1+g_{+-0}u} \approx 7$$

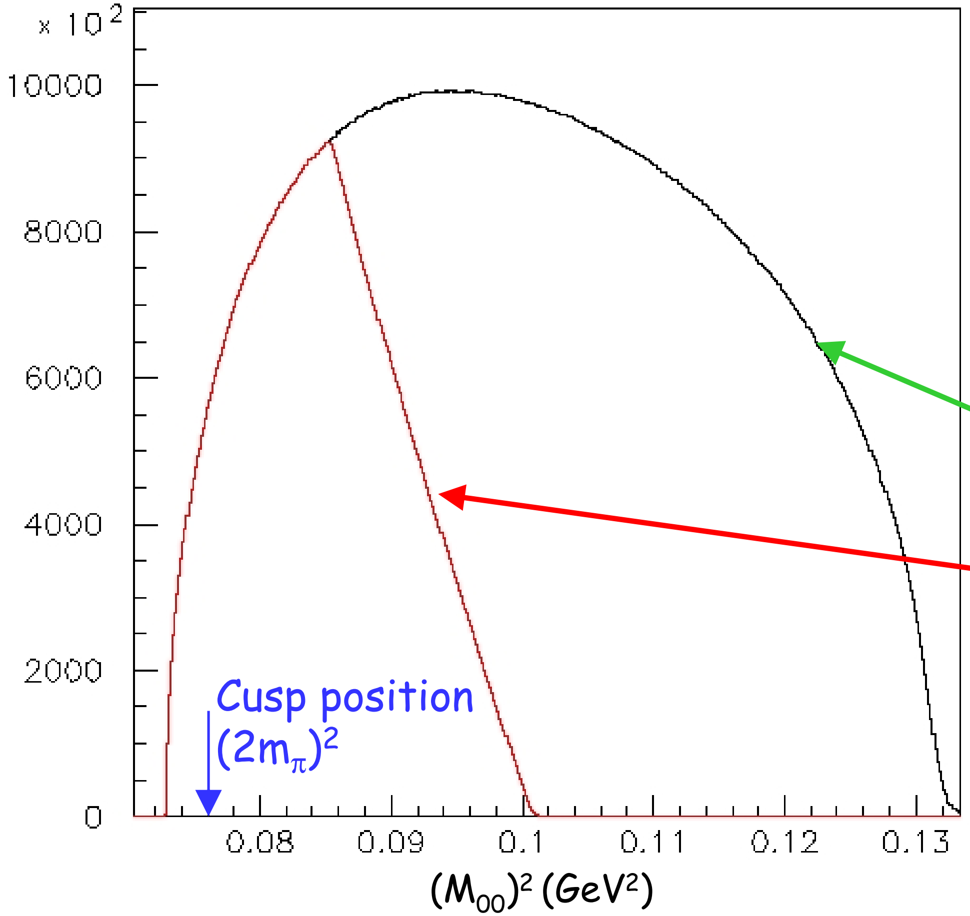
The cusp effect for K_L is a factor 7 smaller (at $2m_\pi$ threshold)

Invariant mass of the $\pi^0 \pi^0$ pairs in $K_L \to 3\pi^0$

Statistics RUN 2000
$0.1 \cdot 10^9$ events

$$K_L \rightarrow \pi_1^0 \pi_2^0 \pi_3^0$$

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M_K^2 + 3m_\pi^2$$



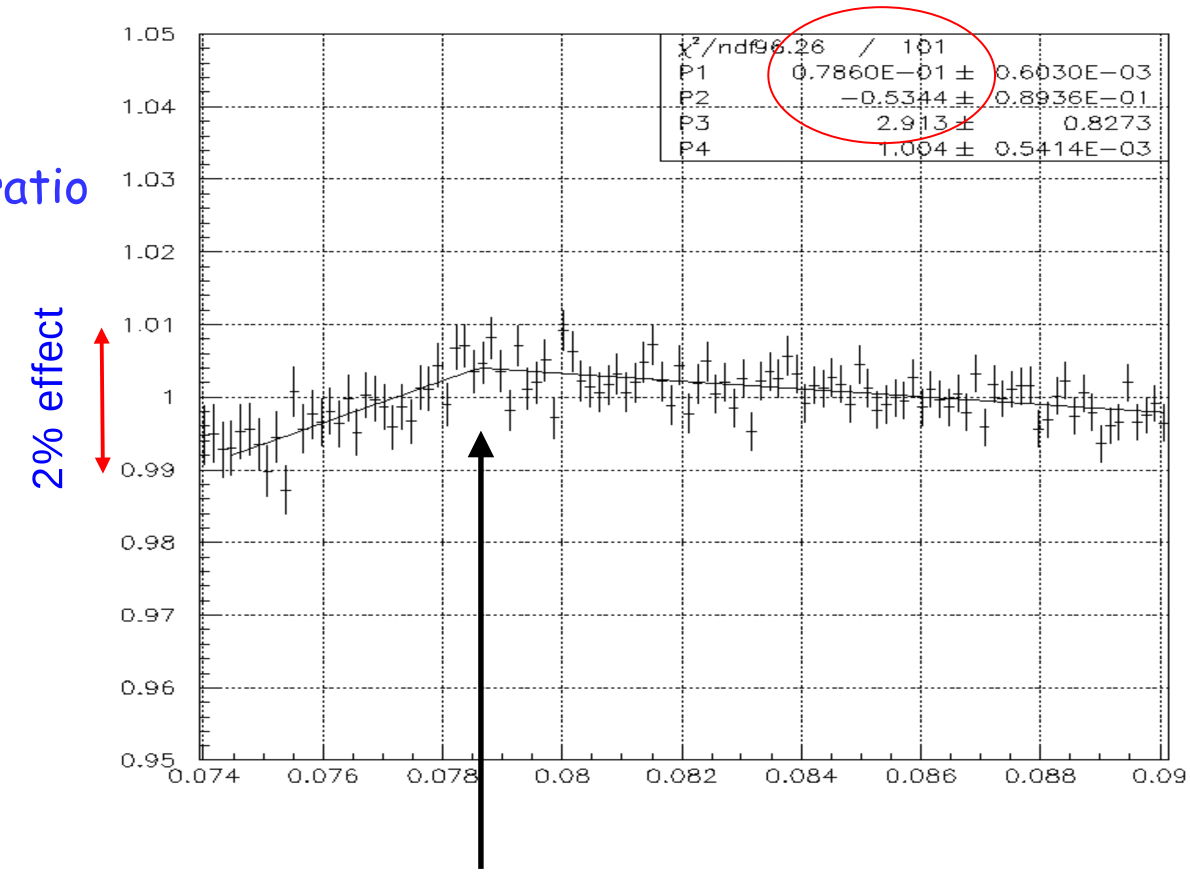
all 3 pairs plotted simultaneously

only the lightest mass m_{12}

Cusp small → hard to see by eye

Fitting the "cusp"

Looking for a cusp:
fit the position on data/MC ratio
(pure phase space in MC and
no rescattering effects)



Cusp position fit
0.0786 GeV^2
close to the expected value
$(2m_\pi)^2$ = 0.07728 GeV^2

Analysis going on

Experimental status on pion scattering lengths

- $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$ (Ke4): FSI of $\pi\pi \rightarrow$ asymmetry of electron direction wrt plane of $\pi\pi$
  - Extraction of form factors and phase shift difference
  - Scattering lengths extracted in a model dependent way (input $a_2 = f(a_0)$)
  - Available c.m. energy range $2m_{\pi+} < M_{\pi\pi} < m_K - m_{\pi+}$ reduced by acceptance
- Pionium lifetime: τ proportional to $(a_0 - a_2)^2$
  - Very short time ($3 \cdot 10^{-15}$) $\rightarrow$ very sophisticated technique: $\pi\pi$ pairs from pionium atoms ionized in the production target
  - Need accurate description of cross section and atom interaction dynamics
  - Insensitive to the sign of $(a_0 - a_2)$
- Scattering $\pi N \rightarrow \pi\pi N$ near threshold: fit of double differential cross section
  - Model dependent
  - Additional hadrons in the final state
- Cusp in $K \rightarrow 3\pi$: direct measurement of scattering lengths at threshold
  - Very accurate, sensitive to the sign of $(a_0 - a_2)$
  - Model independent, only general assumption of unitarity and analycity
  - Radiative + Coulomb corrections needed to enhance the sensitivity to $(a_0 - a_2)$

Experimental results

$K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$ (Ke4)

$a_0 m_+ = 0.280 \pm 0.050$ Rosselet et al., PRD 15(1977) 574 (Geneva-Saclay)

$a_0 m_+ = 0.216 \pm 0.013 \pm 0.003$ Pislak et al., PRD 67(2003) 072004 (E865)

$a_2 m_+ = -0.0454 \pm 0.0031 \pm 0.0013$ using narrow band constraint $a_2=f(a_0)$

NA48/2 result on Ke4 presented at QCD06 (Universal band constraint)

$a_0 m_+ = 0.256 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{theory}$

Pionium lifetime

$|a_0 - a_2| m_+ = 0.264 ^{+0.033}_{-0.020}$ Adeva et al., PLB 619 (2005) 50 DIRAC

improvement in analysis, expected error ~ 3%

Scattering $\pi N \rightarrow \pi\pi N$ near threshold

$a_0 m_+ = 0.260 \pm 0.050$ Froggart et al., NPB 129 (1977) 89

$a_0 m_+ = 0.204 \pm 0.014 \pm 0.008$ Kermani et al., PRC 58 (1998) 3431

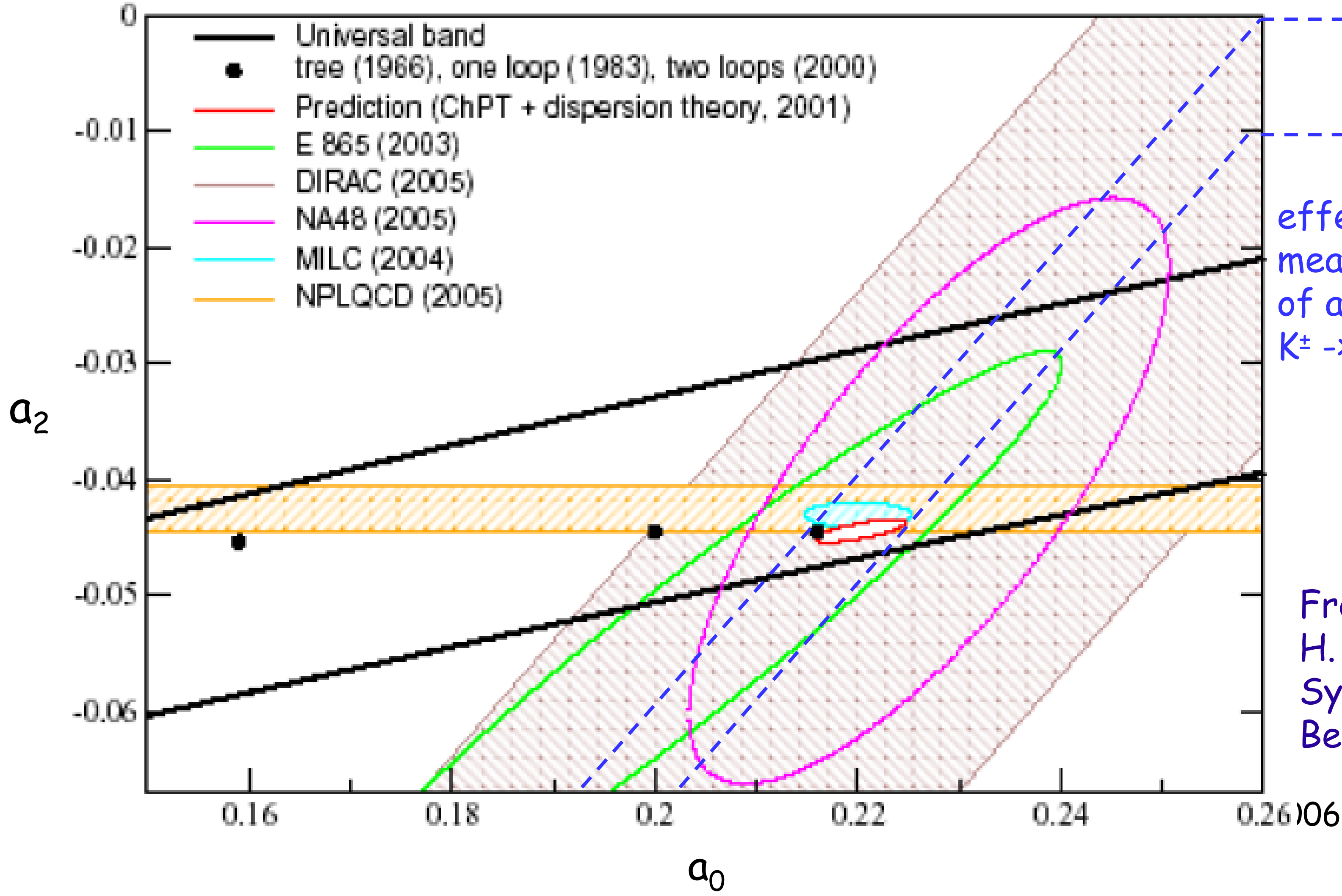
Cusp in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

$(a_0 - a_2) m_+ = 0.268 \pm 0.010 \pm 0.013$ Batley et al., PLB 633 (2006) 173 NA48/2

$a_2 m_+ = -0.041 \pm 0.022 \pm 0.014$ (improvement in analysis expected: statistics x 5)

$(a_0 - a_2) m_+ = 0.264 \pm 0.006 \pm 0.013$
$a_0 m_+ = 0.220 \pm 0.006 \pm 0.012$ } fit including narrow band constraint $a_2=f(a_0)$

Measurements and predictions for a_0 and a_2



effect of a 2% measurement of a_0-a_2 from $K^{\pm} \to 3\pi$

From H. Leutwyler Symposium in Bern 5/5/2006

Conclusions

- A new cusp structure in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ and $K_L \rightarrow 3\pi^{0}$ was observed
- Interpreted by Cabibbo as due to

  $\pi\pi$ final state charge exchange process
- This provides a new method for a precise determination of

  $(a_0 - a_2) = 0.268 \pm 0.010\ (\text{stat.}) \pm 0.013\ (\text{syst.})$

  (systematic error is mainly due to the theoretical uncertainties)
- Measured value in agreement with theory and other measurements
- Parameter a_2 directly measured for the first time, even though with low accuracy
- Pionium bound state also found but further investigation is needed

Outlook

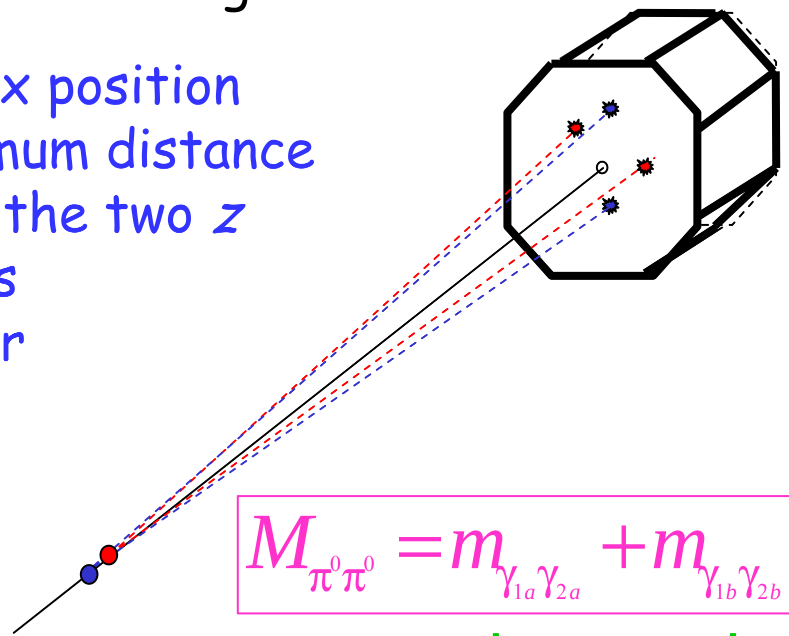
- By analyzing the full data sample NA48/2 expect an increase in statistics by a factor 5
- An experimental error of 1.5% seems not to be out of reach
- Present external uncertainty related to the theoretical method is ~ 5%; the quality of the data calls for additional theoretical effort (higher orders and electromagnetic corrections)
- The fit according to different amplitude representation (CGKR) is in progress
- A study of the cusp effects in $K_L \to 3\pi^0$ using data collected in year 2000 is on the way
- The preliminary NA48 result on a_0 and a_2 from Ke4 will be presented at QCD06

Spares

Invariant mass of the $\pi^0\pi^0$ pair

Event selection quite simple -> no relevant background

1) For each photon pair obtain a vertex position
2) Choose the two pairs with the minimum distance
3) Decay vertex = arithmetic mean of the two z
4) The invariant mass of the 4 photons is the invariant mass of the $\pi^0\pi^0$ pair



$$\frac{M_{00}^2}{4m_{\pi^0}^2} = \frac{\sum\limits_{i<j=1,4;i<j} E_i E_j d_{ij}^2}{(\sqrt{E_1E_2}\,d_{12} + \sqrt{E_3E_4}\,d_{34})^2}$$

$$M_{\pi^0\pi^0} = \underbrace{m_{\gamma_{1a}\gamma_{2a}} + m_{\gamma_{1b}\gamma_{2b}}}_{2m_{\pi^\circ}} + Q$$

Q = 9.19 MeV at $\pi^+\pi^-$ threshold

Fits to the experimental M_{oo}^2 distribution METHOD

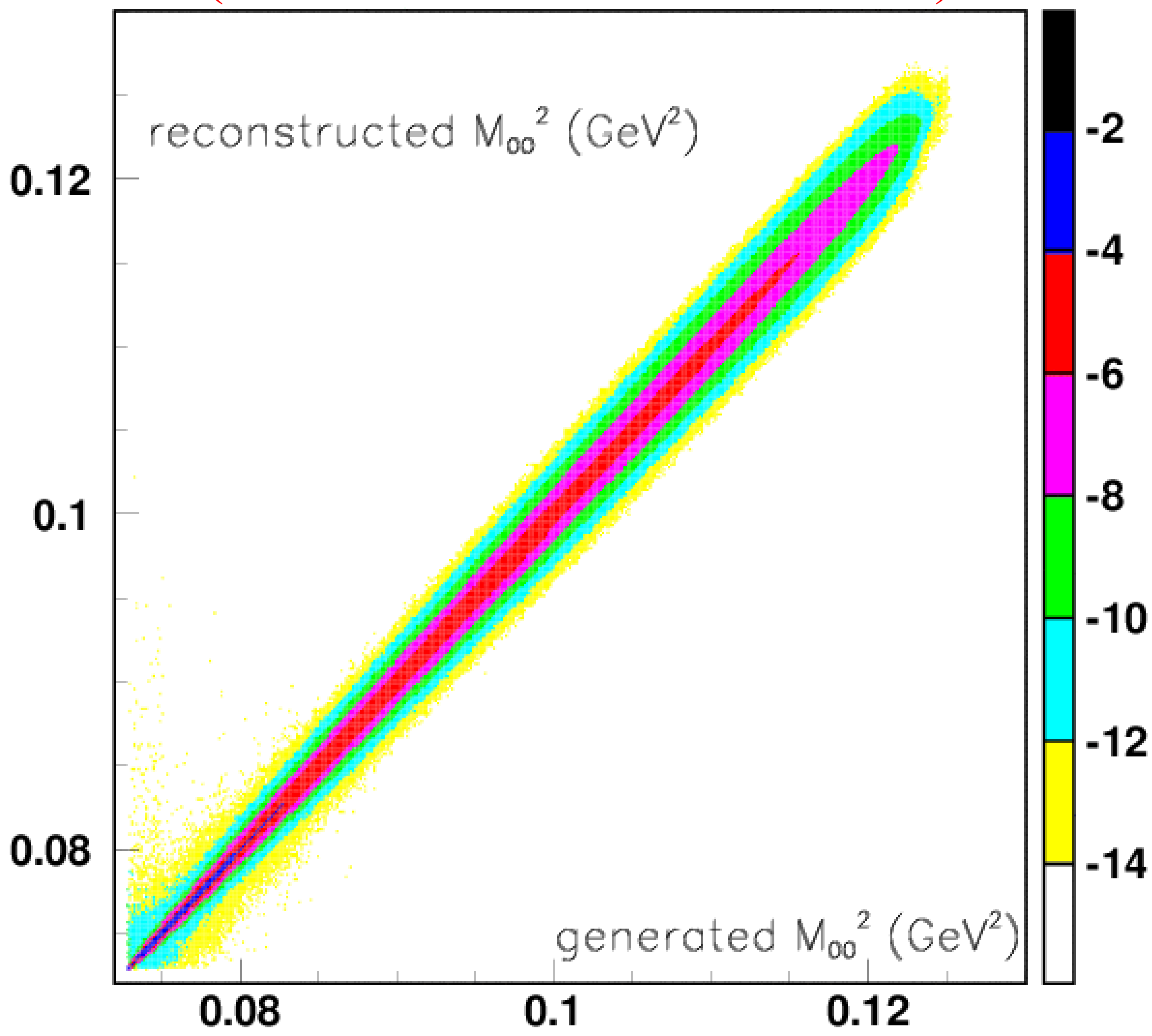
- Generate theoretical M_{oo}^2 distribution G_i (420 bins of 0.00015 GeV^2)
- From MonteCarlo simulation derive 420 x 420 matrix T_{ik} where:
  i : bin number of generated M_{oo}^2 value ;
  k : bin number of reconstructed M_{oo}^2 value
- Produce "reconstructed" M_{oo}^2 distribution R_k :

$$\mathbf{R}_k = \sum_i \mathbf{T}_{ik} \mathbf{G}_i$$

- Fit distribution R_k to experimental M_{oo}^2 distribution

Log(T_{ik})

(from MonteCarlo simulation)



Pionium contribution

Fit including $\pi^+\pi^-$ atoms: χ^2 = 149/145 d.f.

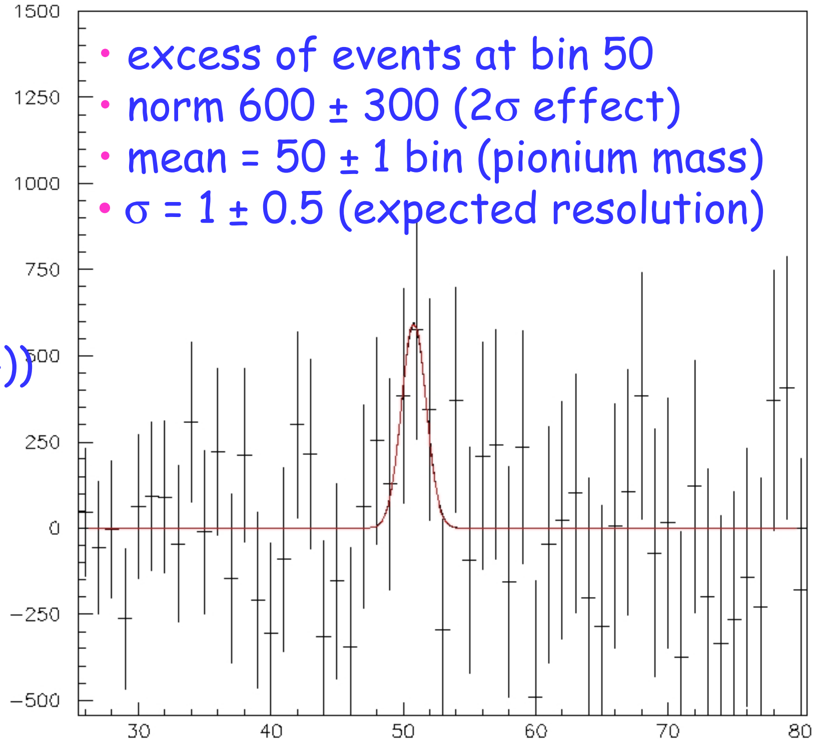
$$\frac{\Gamma(K^+ \to \pi^+ + pionium)}{\Gamma(K^+ \to \pi^+\pi^+\pi^-)} \approx (1.6 \pm 0.7)\cdot 10^{-5}$$

in reasonable agreement with the predicted value ~ 0.8 · 10^{-5} (Silagadze JETP Lett.60 (1994))

Pionium

- delta function (convoluted with detector resolution)
- sits on top of two cusps
- very difficult to measure BR

data - fit



No radiative corrections included → 7 bins centered at $s_3 = 2m_+$ excluded from the final fit

Pion scattering lengths

Weinberg (1966)
Effective field theory for
strong interaction at low E

$$a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

Colangelo et al. (2001)
ChPT + dispersion relations

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

Pelaez and Yndurain (2005)
phase shift analysis of data (no ChPT)

$$(a_0 - a_2)\, m_\pi = 0.278 \pm 0.016$$

High precision (1.5%) is quite unusual for hadronic physics predictions
Experiments have not yet reached the same level of accuracy……
……but they are on the way