$\pi\pi$ scattering lengths from Kaon Ke4 and 3π decays by NA48/2

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On behalf of the NA48 collaboration

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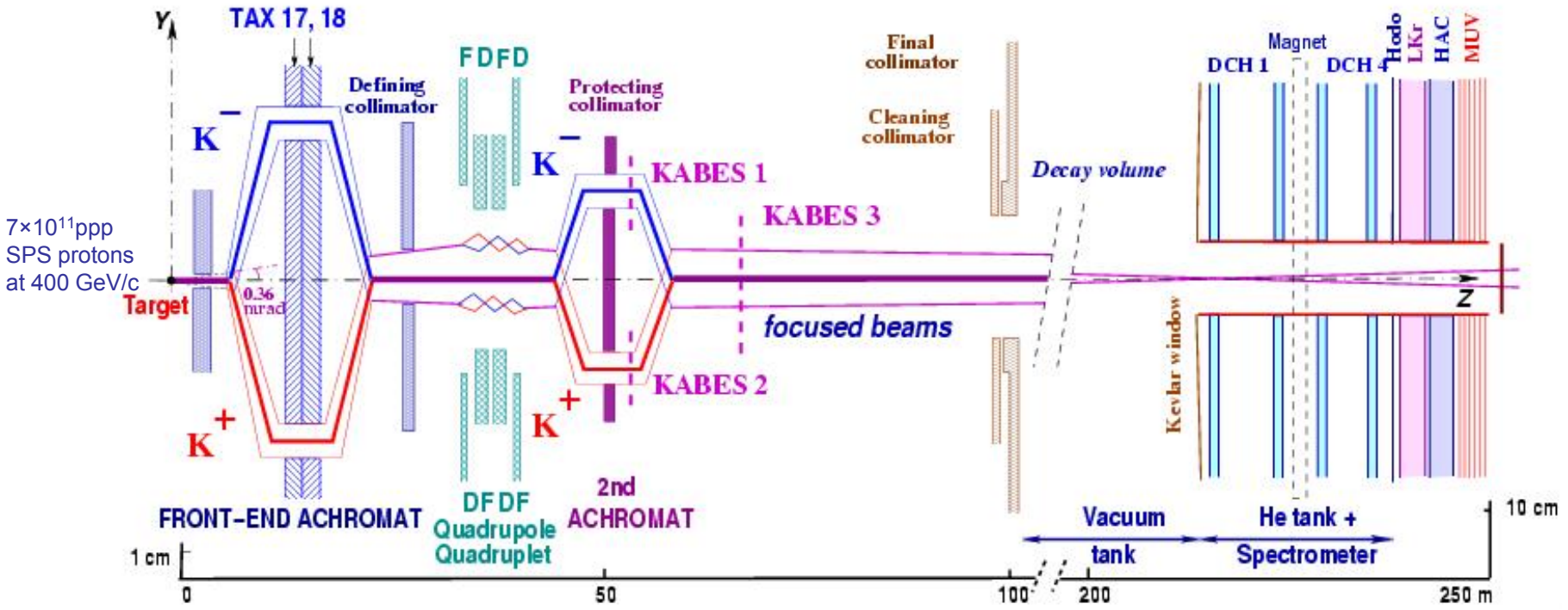
Plan of the presentation

- The NA48/2 experiment
- The Cusp ($K^{\pm} \to \pi^{\pm}\pi^0\pi^0$)
  - Data 2003+2004: preliminary results
  - $\pi^+\pi^- \to \pi^0\pi^0$ rescattering
- Ke4 ($K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$)
  - Data 2003: preliminary results
  - Form factors fits
- Conclusions

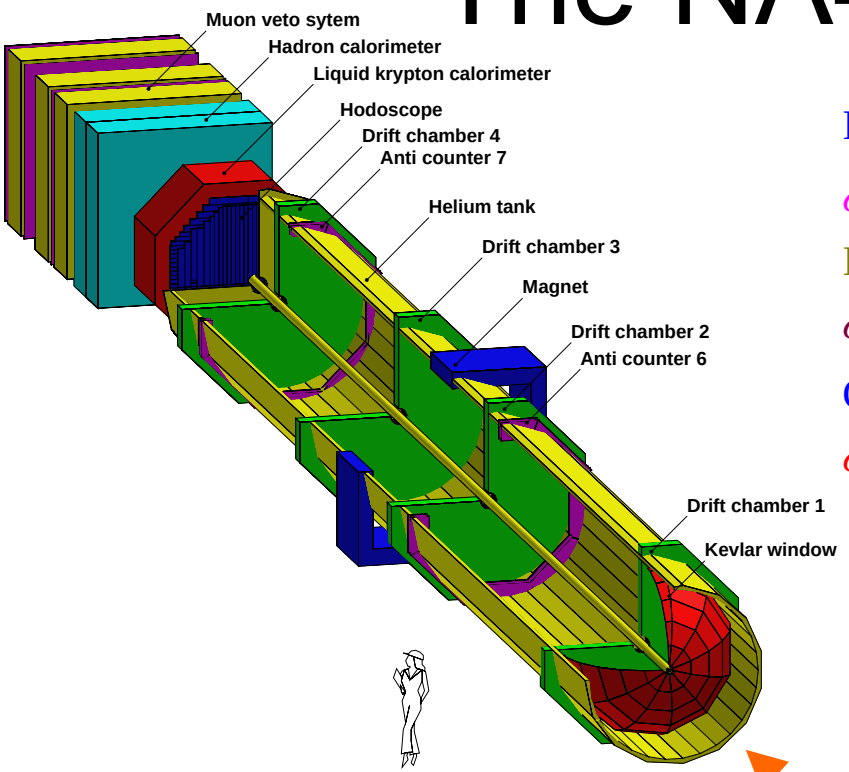
The CERN NA48/2 experiment



Kaon beam at 60±3 GeV/c momentum

K^+/K^-~1.8

The NA48 detectors



Liquid Krypton Calorimeter:

$\sigma_E / E = 3.2\% / \sqrt{E} \oplus 9\% / E \oplus 0.42\%$ [E in GeV]

Magnetic Spectrometer:

$\sigma_p / p = 1.0\% \oplus 0.044\% \times p$ [p in GeV/c]

Charged Hodoscope:

$\sigma_t = 150$ ps

- 2003 run: 50 days
- 2004 run: 60 days

- 1997-2001: NA48 $K_L K_S$ (ε'/ε)
- 2002: NA48/1 K_S
- 2003-2004: NA48/2 $K^\pm$ (Δg)
- 2007: NA62 $K^\pm$ (Ke2/Kμ2)
- 2011: NA62 $K^+ \to \pi^+ \nu\nu$

$K^\pm$

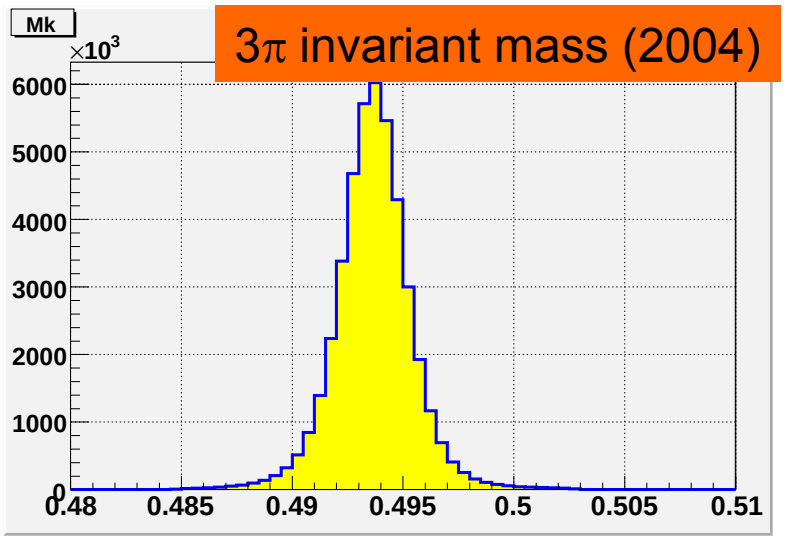
Total Statistics 2003+2004:

$$K^\pm \to \pi^+\pi^-\pi^\pm \approx 4\times10^9;$$

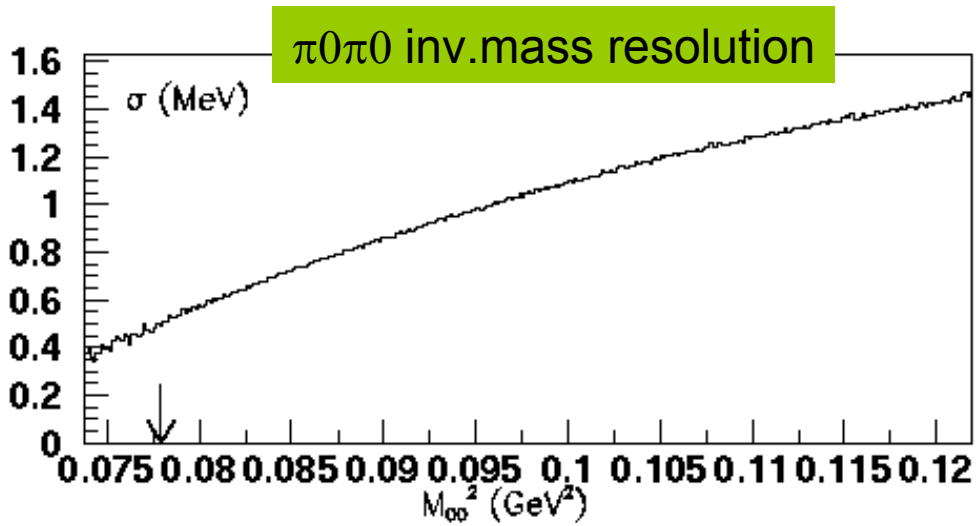
$$K^\pm \to \pi^0\pi^0\pi^\pm \approx 1\times10^8.$$

$K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$:selection

- 1 tracks, 4 e.m. Clusters
- clusters pairing: all possible combinations (3)
- $\pi^0 \rightarrow \gamma\gamma$ vertex by imposing π^0 mass
- choose combination with closest vertices
- K decay vertex: average of π^0 vertices
- Calculate $\pi^0\pi^0$ invariant mass



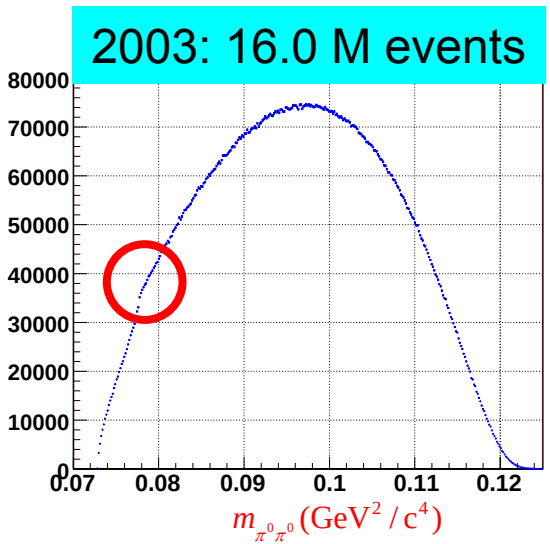
3π invariant mass (2004)

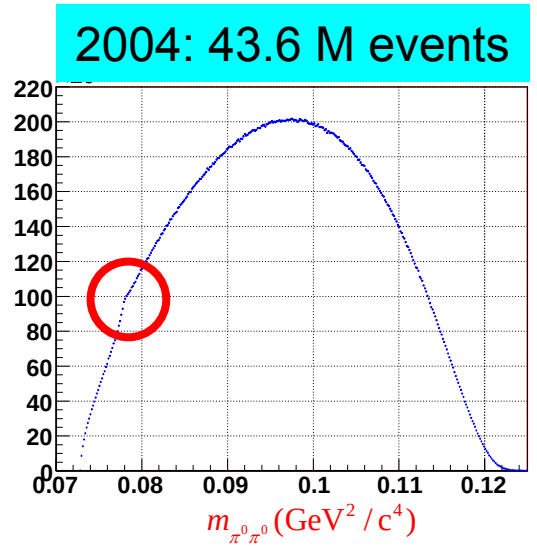


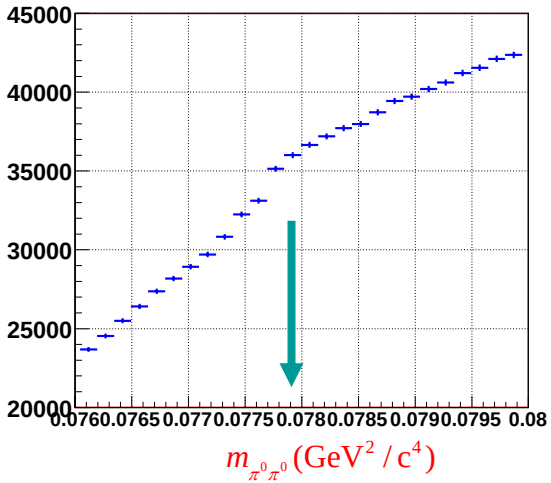
π0π0 inv.mass resolution

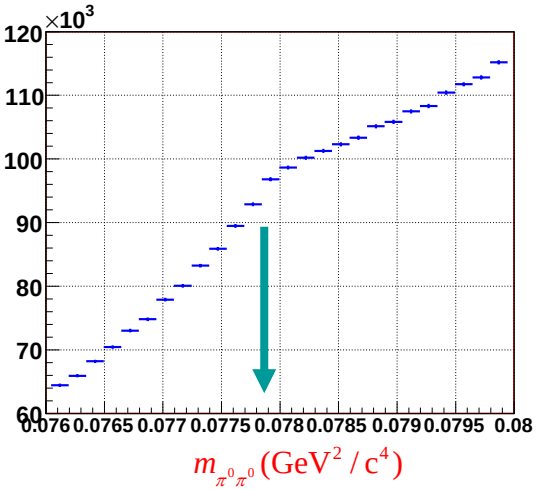
Cusp evidence

Change of slope
at the $\pi^+\pi^-$ threshold





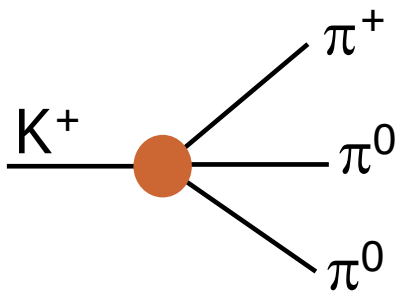




N.Cabibbo, PRL 93 (2004) 121801

Cusp interpretation: 1 Loop

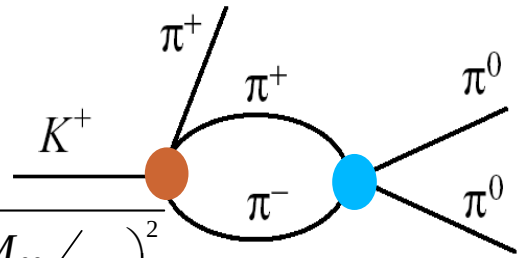
$$M\left(K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}\right)=M_0+M_1$$



Direct Emission:

$$M_0=A_0\left(1+g_0\frac{u}{2}+h'\frac{u^2}{2}+k'\frac{v^2}{2}\right);$$

Rescattering:



$$M_1=-\frac{2}{3}(a_0-a_2)\,m_+M_+\sqrt{1-\left(M_{00}/2m_+\right)^2};$$

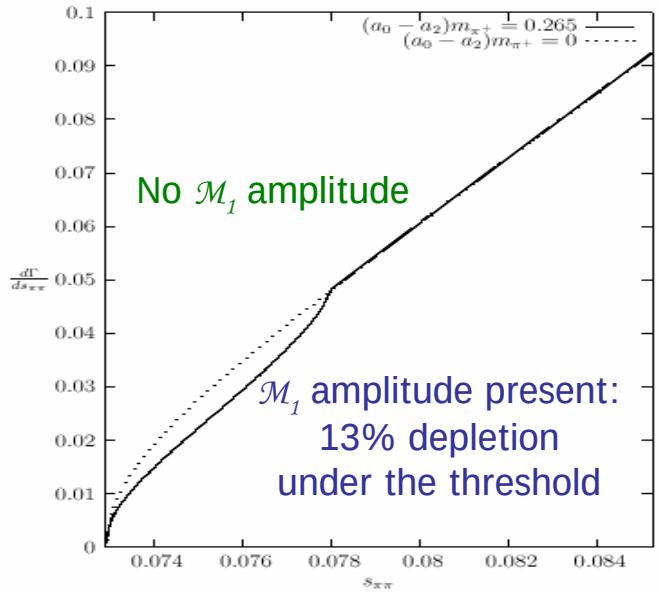
a_0, a_2 : S-wave $\pi\pi$ scattering length

(isospin symmetry assumed)

Kaon rest frame

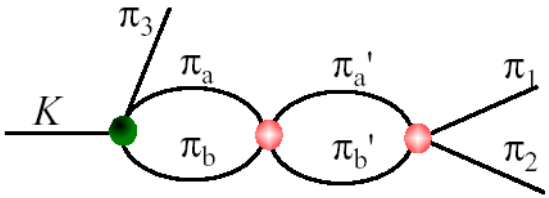
$$u=2m_K\left(m_K/3-E_{odd}\right)/m_\pi^2$$

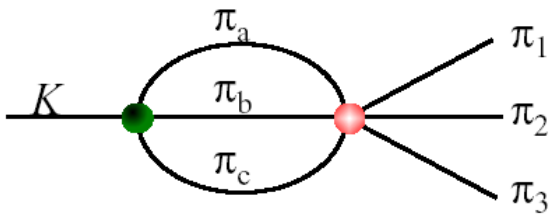
$$v=2m_K\left(E_1-E_2\right)/m_\pi^2$$

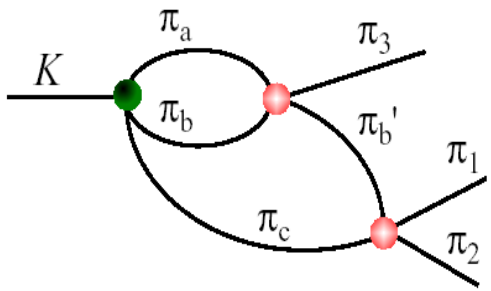


N.Cabibbo and G.Isidori, JHEP 503 (2005) 21

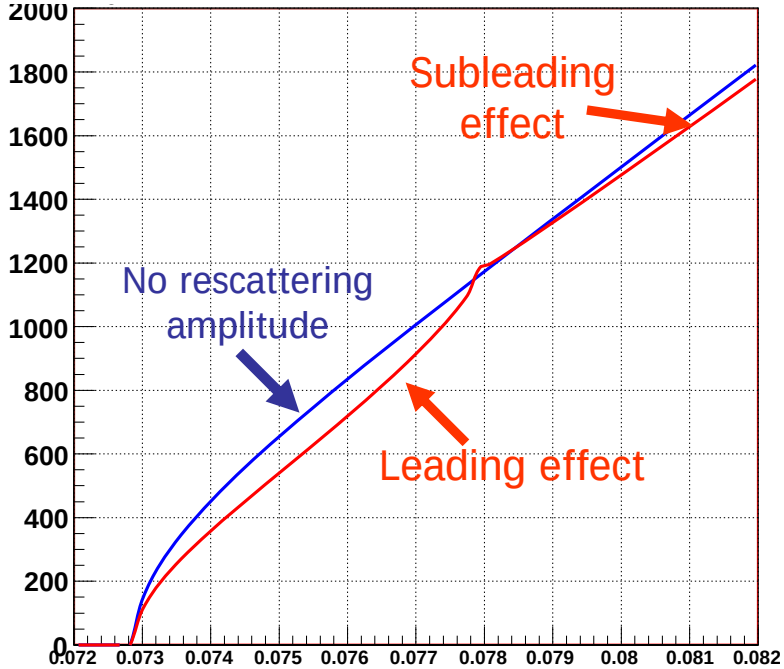
Cusp interpretation: 2 Loops







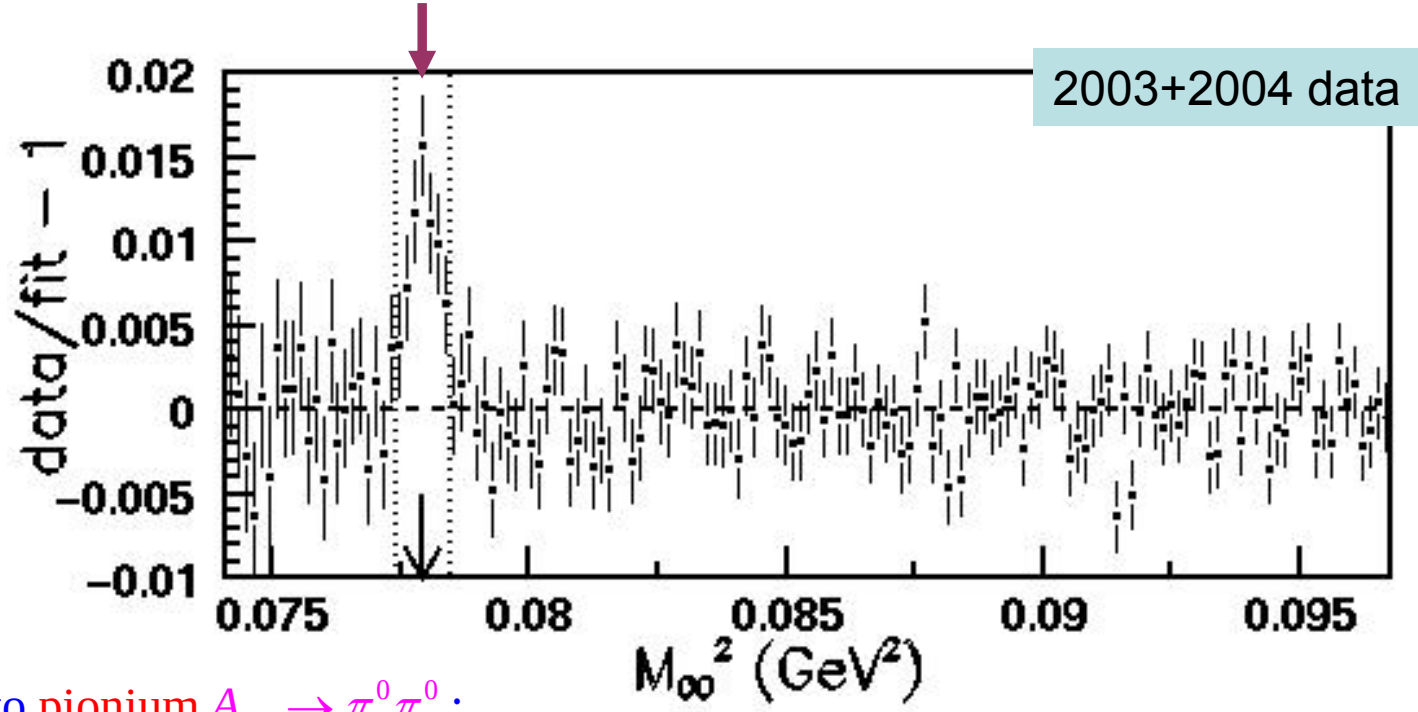
- All 1- and 2-loops rescattering processes
- 5 S-wave scattering lengths expressed as linear comb. of a_0 and a_2
- Isospin symmetry breaking effect in
- No radiative corr. : 5% precision on a_0-a_2



Cusp: fit to the data

5 parameters fit (g, h', $m_+(a_0-a_2)$, m_+a_2, N)

7 bins excluded from the fit



If excess due to pionium $A_{2\pi} \to \pi^0\pi^0$:

$$R = \Gamma\left(K^\pm \to \pi^\pm A_{2\pi}\right)/\Gamma\left(K^\pm \to \pi^\pm\pi^+\pi^-\right) = (1.82 \pm 0.21)\times 10^{-5}$$

[Theory: Z.K.Silagadze, JETP Lett. 60 (1994) 689: $R = 0.8\times 10^{-5}$]

Cusp: results (1)

Preliminary NA48/2 result (2003+2004):

$$(a_0 - a_2)m_+ = 0.261 \pm 0.006_{stat} \pm 0.003_{syst} \pm 0.0013_{ext.}$$
$$a_2 m_+ = -0.037 \pm 0.013_{stat} \pm 0.009_{syst} \pm 0.0018_{ext.}$$

External uncert.:

$$\left(A_{+\,+\,-} / A_{+00}\right)\big|_{threshold} = 1.975 \pm 0.015;$$

Theory uncert.(rad.corr and more loops):

$$\delta(a_0 - a_2)m_+ = 0.013$$

Systematic effect	$(a_0-a_2)\times10^2$	$a_2\times10^2$
Analysis technique	±0.10	±0.20
Trigger inefficiency	negl.	±0.50
Description of resolution	±0.06	±0.11
LKr non-linearity	±0.06	±0.26
Geometric acceptance	±0.02	±0.01
MC sample	±0.03	±0.21
Simulation of LKr showers	±0.17	±0.50
V-dependence of amplitude	±0.17	±0.38
Total	**±0.28**	**±0.90**

Cusp: result (2)

Imposing chiral symm.constraint [Colangelo et al., PRL 86 (2001) 5008]:

$$a_2 m_+ = -0.0444 + 0.236(a_0 m_+ - 0.22) - 0.61(a_0 m_+ - 0.22)^2 - 9.9(a_0 m_+ - 0.22)^3$$

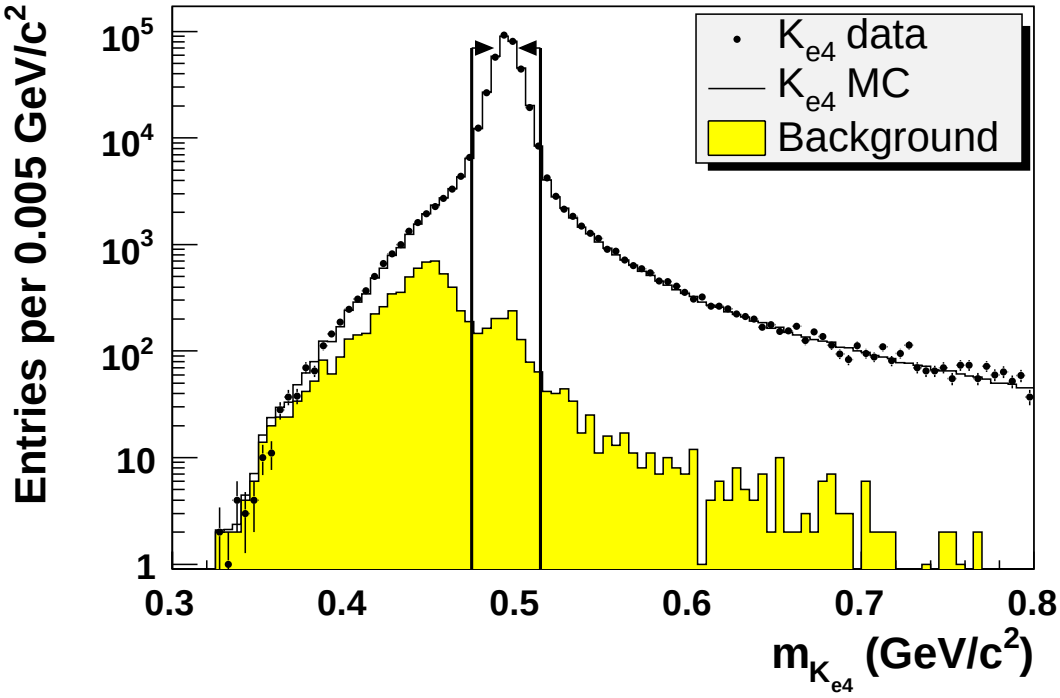
$$(a_0 - a_2) m_+ = 0.263 \pm 0.003_{stat} \pm 0.0014_{syst} \pm 0.0013_{ext.} \text{[Preliminary]}$$

Theory uncert.(rad.corr and more loops):

$$\delta(a_0 - a_2) m_+ = 0.013$$

Systematic effect	$(a_0-a_2)\times 10^2$
Analysis technique	±0.08
Trigger inefficiency	negl.
Description of resolution	±0.06
LKr non-linearity	±0.05
Geometric acceptance	±0.02
MC sample	±0.06
Simulation of LKr showers	±0.04
V-dependence of amplitude	±0.02
Total	±0.14

Ke4 ($K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$): selection



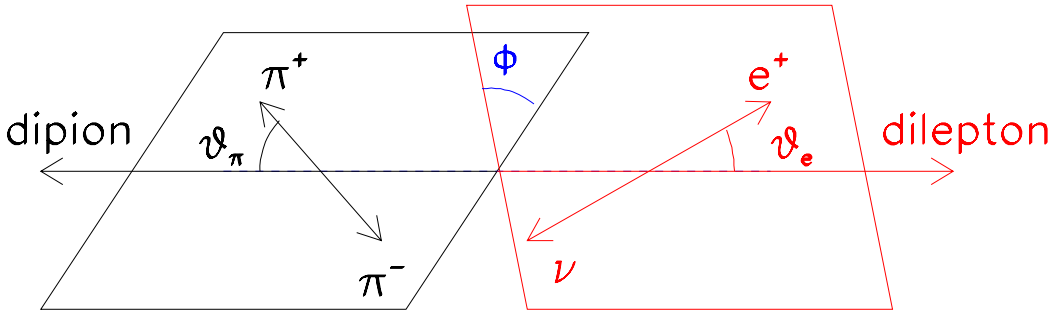
- 3 tracks
- missing energy and pt
- E/p for electron PID

Background estimated from
Wrong sign events: 0.5% level

440000 from K^{+}
240000 from K^{-}

2003 data set

Ke4: kinematics variables



Cabibbo-Maksymovicz variables:

$$M_{\pi\pi}^2, M_{e\nu}^2, \cos\vartheta_\pi, \cos\vartheta_e, \varphi$$

Decay amplitude described by two axial (F, G) and one vector (H) form factors which can be partial wave expanded (s and p wave; d neglected):

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\vartheta_\pi$$

$$G = G_p e^{i\delta_p}$$

$$H = H_p e^{i\delta_p}$$

$$\delta = \delta_s - \delta_p$$

and further expanded in powers of q^2:

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e' \left(M_{e\nu}^2 / 4m_\pi^2 \right) + \ldots$$

$$F_p = f_p + f_p' q^2 + \ldots$$

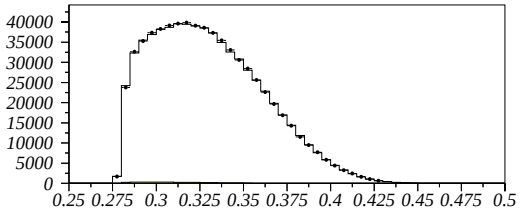
$$G_p = g_p + g_p' q^2 + \ldots$$

$$H_p = h_p + h_p' q^2 + \ldots$$

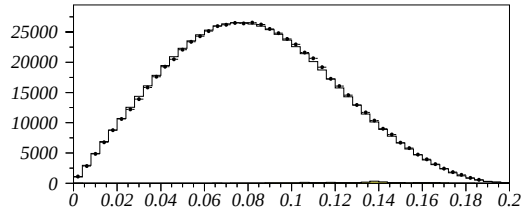
$$q^2 = (M_{\pi\pi}^2 / 4m_\pi^2) - 1$$

Ke4: form factors fit

F_s, F_p, G_p, H_p and δ are extracted from a fit to
$10(M_{\pi\pi})\times5(M_{e\nu})\times5(\cos\theta_e)\times5(\cos\theta_\pi)\times12(\phi)$ = 15000 equi-populated bins

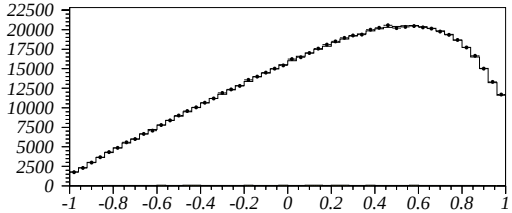


$M_{\pi\pi}(GeV/c^2)$

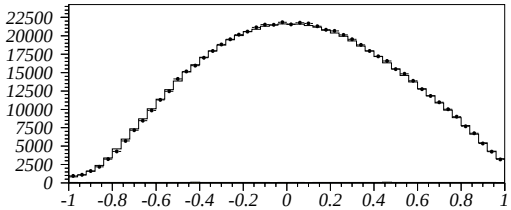


$M_{e\nu}(GeV/c^2)$

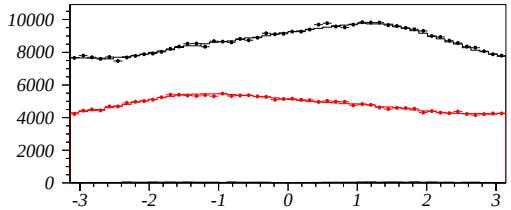
• Data
– MC



$\cos\theta_\pi$



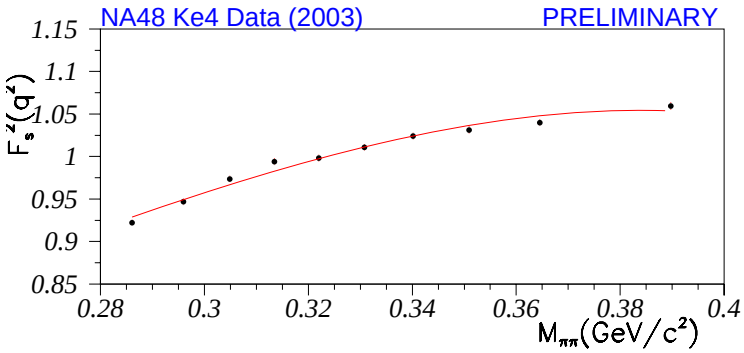
$\cos\theta_e$

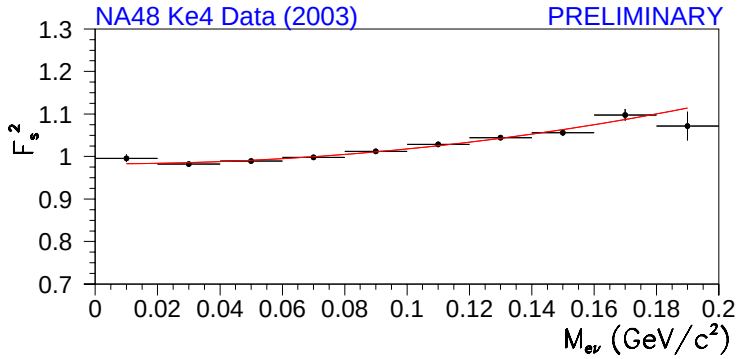


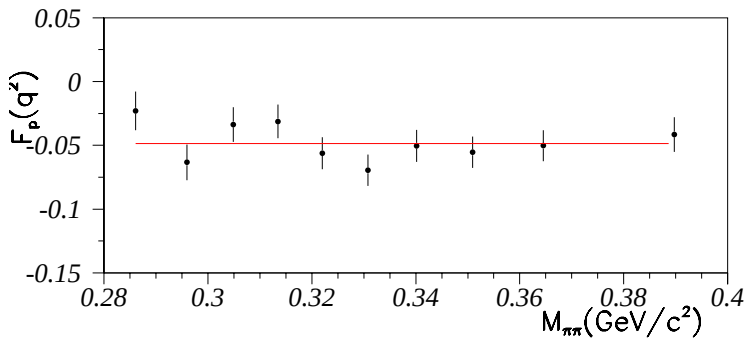
ϕ(rad)

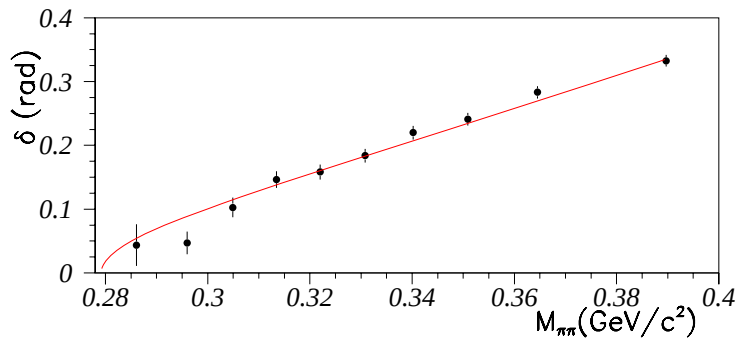
• K^+
• K^-

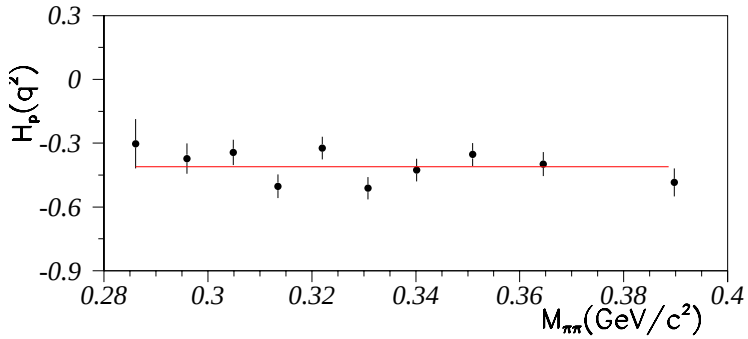
Ke4: form factors fit

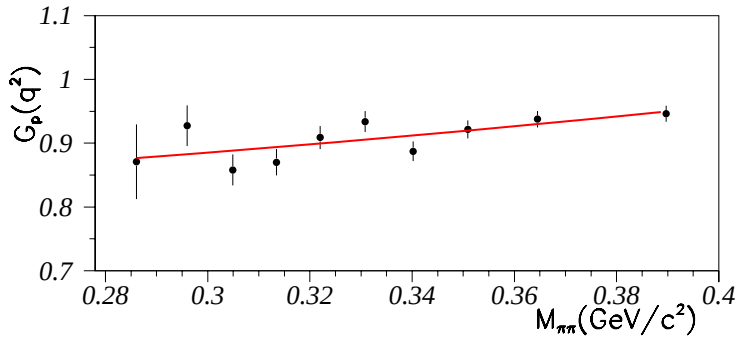












Ke4: fit results

Preliminary (2003 data)

$$f_s'/f_s = 0.172 \pm 0.009_{stat} \pm 0.006_{syst}$$

$$f_s''/f_s = -0.090 \pm 0.009_{stat} \pm 0.007_{syst}$$

$$f_e'/f_s = 0.081 \pm 0.008_{stat} \pm 0.008_{syst}$$

$$f_p/f_s = -0.048 \pm 0.004_{stat} \pm 0.004_{syst}$$

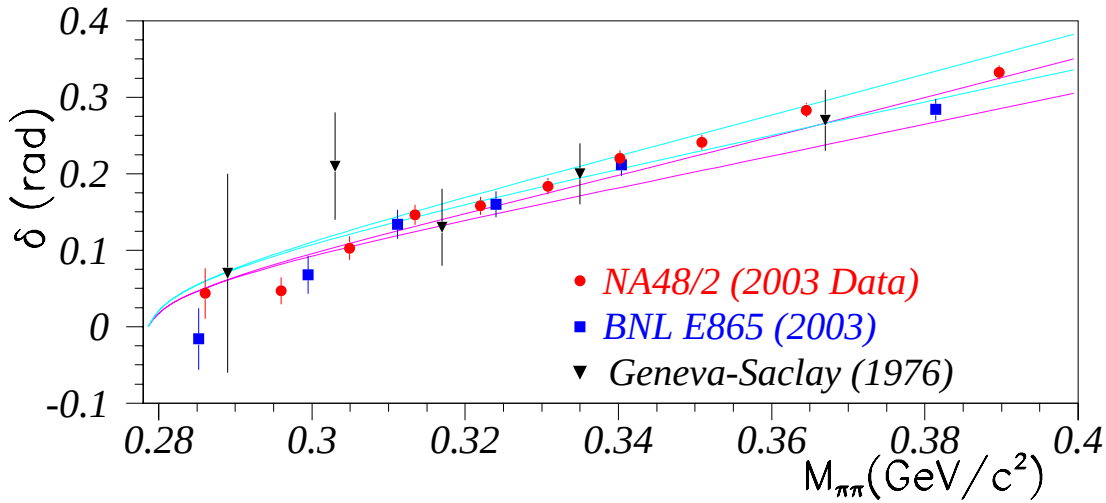
$$g_p/f_s = 0.873 \pm 0.013_{stat} \pm 0.012_{syst}$$

$$g_p'/f_s = 0.081 \pm 0.022_{stat} \pm 0.014_{syst}$$

$$h_p/f_s = -0.411 \pm 0.019_{stat} \pm 0.007_{syst}$$

Ke4: δ dependence

Use Roy equations in order to relate the δ variation and the ππ scattering lengths



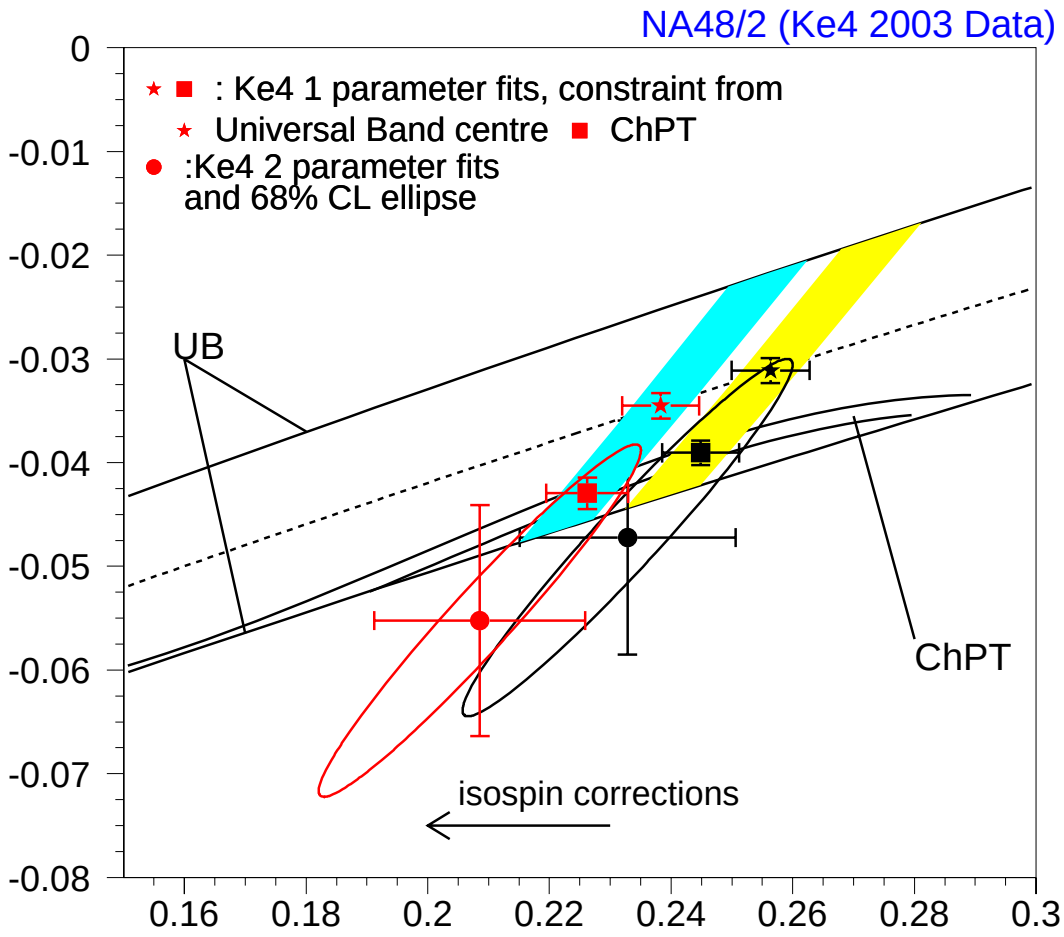
The Universal band center line parametrization corresponds to a 1-parameter fit with a fixed relation between a_2 and a_0

$$a_0\,(\text{1p fit}) = 0.256 \pm 0.006_{stat} \pm 0.005_{syst}$$

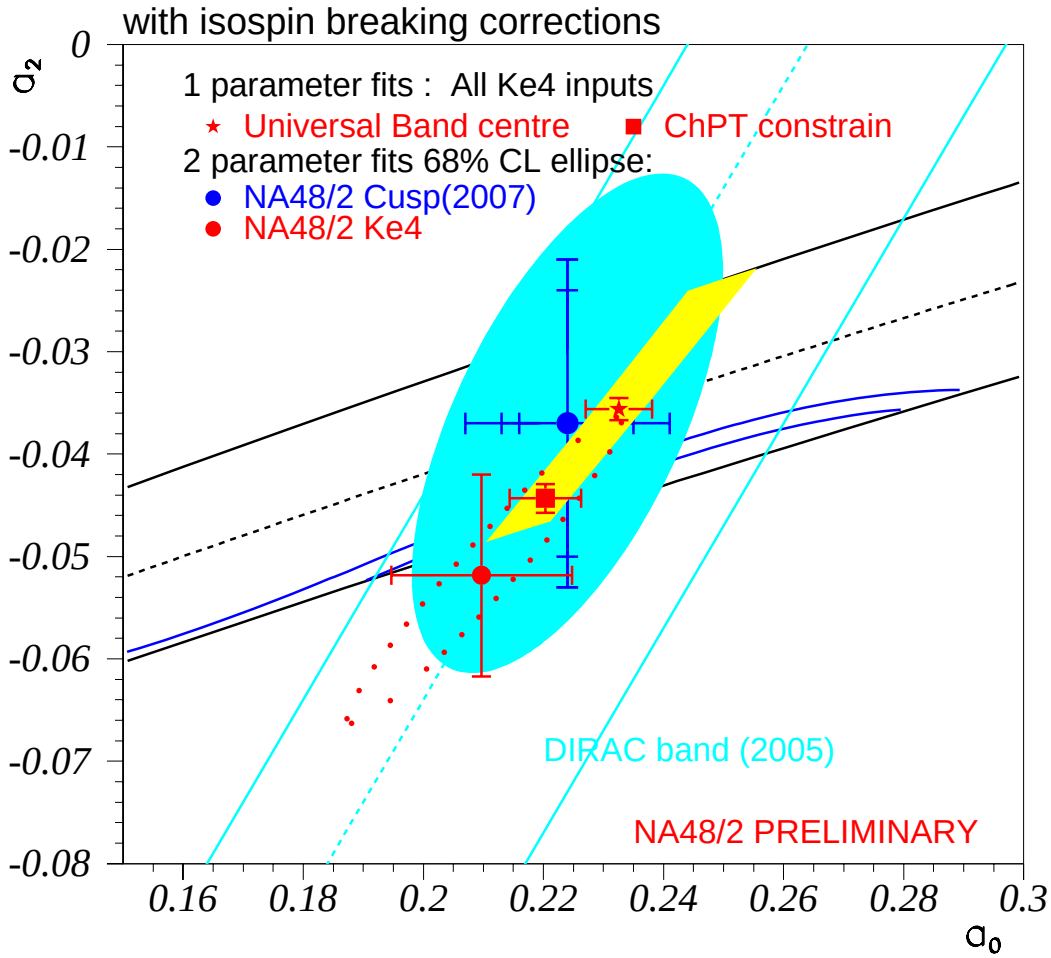
$$a_0\,(\text{2p fit}) = 0.233 \pm 0.016_{stat} \pm 0.007_{syst}; \quad a_2\,(\text{2p fit}) = -0.047 \pm 0.011_{stat} \pm 0.004_{syst}$$

No isospin corr. applied above

Ke4: $\pi\pi$ scattering lengths



Cusp and Ke4: comparison

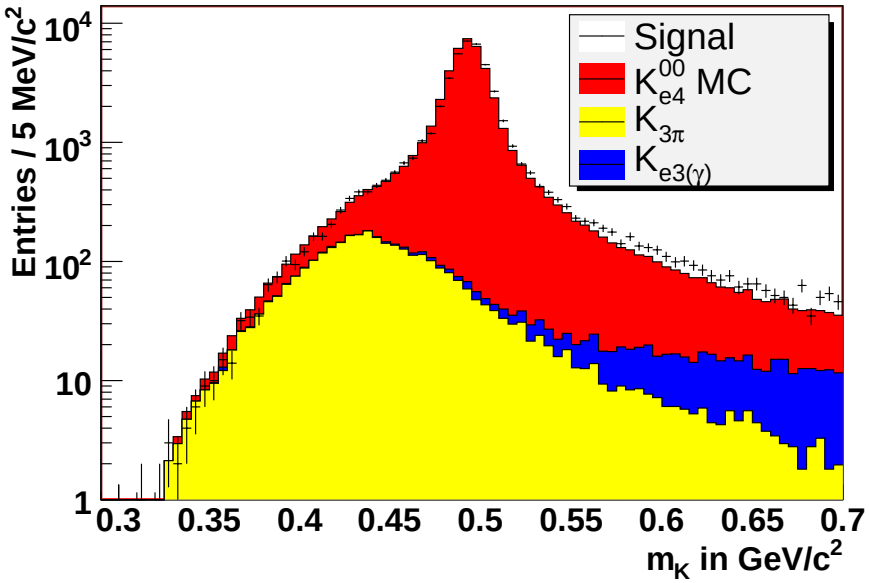


Conclusions

- Preliminary 2003+2004 measurement of a_2-a_0 in 3π decay presented
- Preliminary 2003 measurement of a_2, a_0 and form factors in Ke4 presented
- General agreement between the NA48/2 results above

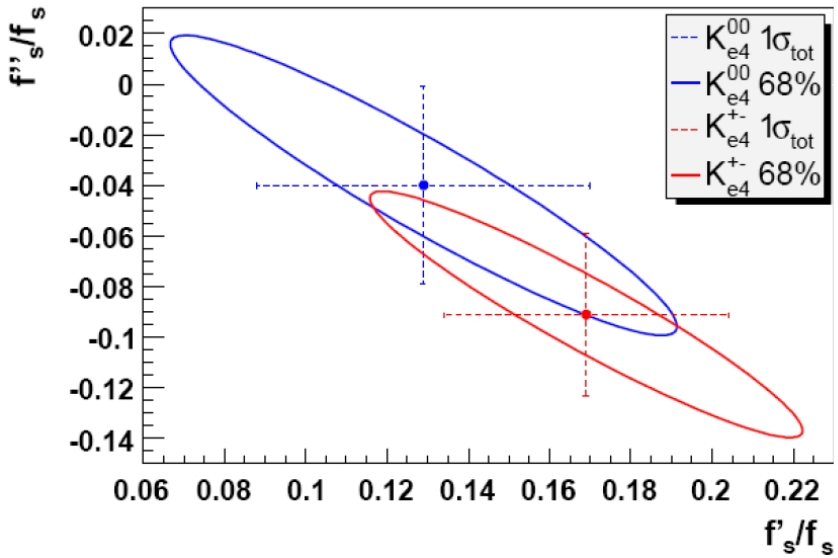
Spare Slides

$K^{\pm} \to \pi^0 \pi^0 e^{\pm} \upsilon$



Only s-wave possible

Preliminary 2003+2004
40000 evts, bck 2-3%



$$f_s'/f_s = 0.129 \pm 0.036_{stat} \pm 0.020_{syst}$$

$$f_s''/f_s = -0.040 \pm 0.034_{stat} \pm 0.020_{syst}$$

Dalitz plot slopes of $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$

$$M\left(K^{\pm} \to \pi^{\pm}\pi^0\pi^0\right) = M_0 + M_1$$

$$M_0 \approx \left(1 + g_0 \mathrm{u}/2 + h'\mathrm{u}^2/2 + k'\mathrm{v}^2/2\right)$$

$$\left|M_0\right|^2_{(PDG)} \approx \left(1 + g\mathrm{u} + h\mathrm{u}^2 + k\mathrm{v}^2\right) \qquad \left[g_0 \approx g, h' \approx h - g^2/4, k' \approx k\right]$$

Preliminary NA48/2 result:

$$g_0 = \left(+0.649 \pm 0.003_{stat} \pm 0.004_{syst}\right)$$

$$h' = \left(-0.047 \pm 0.007_{stat} \pm 0.005_{syst}\right)$$

$$k' = \left(-0.0097 \pm 0.0003_{stat} \pm 0.0008_{syst}\right)$$

Dalitz plot slopes of $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$

$$\left|M\left(K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}\right)\right|^{2}_{(PDG)} \approx \left(1 + g\mathrm{u} + h\mathrm{u}^{2} + k\mathrm{v}^{2}\right)$$

Final NA48/2 result (4.71×10^{8} events)

$$g = \left(-0.21134 \pm 0.00013_{stat} \pm 0.00010_{syst}\right)$$

$$h = \left(+0.01848 \pm 0.00022_{stat} \pm 0.00033_{syst}\right)$$

$$k = \left(-0.00463 \pm 0.00007_{stat} \pm 0.00012_{syst}\right)$$

[J.R.Batley et al, Phys.Lett.B649(2007) 349.]

Ke4: Transition Amplitude

$$\frac{G_w}{\sqrt{2}} V_{us}^{*} \left\langle \pi^+\pi^- \left| V^\lambda - A^\lambda \right| K^+ \right\rangle \bar{u}_\nu \gamma_\lambda \left(1-\gamma_5\right) \mathrm{v}_e \quad \text{where}$$

$$\left\langle \pi^+\pi^- \left| A^\lambda \right| K^+ \right\rangle = \frac{1}{m_K}\left(F\left(p_{\pi^+} + p_{\pi^-} \right)^\lambda + G\left(p_{\pi^+} - p_{\pi^-} \right)^\lambda + R\left(p_e + p_\nu \right)^\lambda \right)$$

$$\left\langle \pi^+\pi^- \left| V^\lambda \right| K^+ \right\rangle = \frac{H}{m_K} \varepsilon^{\lambda\mu\nu\sigma} \left(p_{\pi^+} + p_{\pi^-} + p_e + p_\nu \right)_\mu \left(p_{\pi^+} + p_{\pi^-} \right)_\rho \left(p_{\pi^+} - p_{\pi^-} \right)_\sigma$$