$(\pi\pi)$ scattering length measurement

From $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ and $K_{L}\rightarrow 3\pi^{0}$ at NA48-CERN exp.

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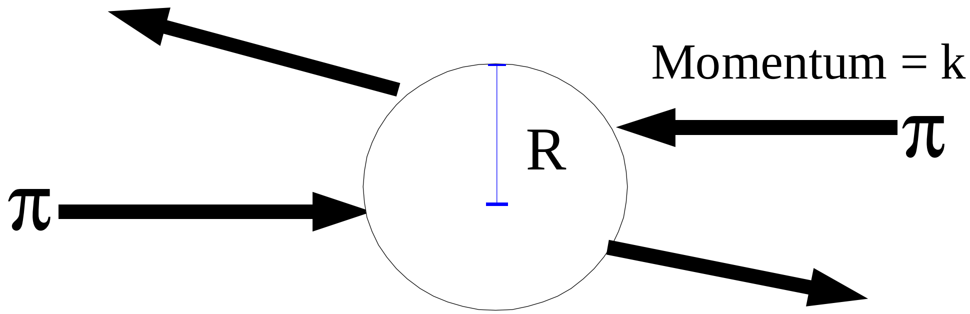
KAON 2005

Scattering length: definition



(ππ) scattering in the center of mass frame

If **kR <<1** (small momentum compared to interaction range) then
S-wave is the dominant component to the total cross section
Bose statistics allows Isospin: I = 0, 2

Scattering matrix

$$S|\pi\pi>_I = \exp(2i\delta_I)|\pi\pi>_I$$

Phases

$\delta_0 = a_0 k$

$\delta_2 = a_2 k$

Parameters **a_0, a_2** are called **Scattering Lengths**

Theoretical predictions

Weinberg (1966)
Effective field theory for
strong interaction at low E

$$a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

Most recently
Colangelo (2001)
χpt -theory two loops
Ref: hep- ph/0103088

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

- **2% level of accuracy:** quite unusual for hadronic physics
  experiments have not yet reached the same level

Experimental status

1977: measurement by Genève/Saclay experiment @ **20% accuracy**

2003: **BNL E865** extracts a_0 at **5% accuracy** by measuring the form factors of the decay K --> ππeν with 400,000 events

$$a_0 m = 0.216 \pm 0.013(stat.) \pm 0.002(syst.) \pm 0.002(theor.)$$

Ref. Pislak et al. (2003) hep-ex/0301040

Present: Cern experiment **DIRAC,** with a sophisticated technique, aims to measure the pionium lifetime @ **10% accuracy**

$$\tau \sim 40 \cdot (a_0 - a_2)^2 \cdot 10^{-15}\,\mathrm{sec}$$

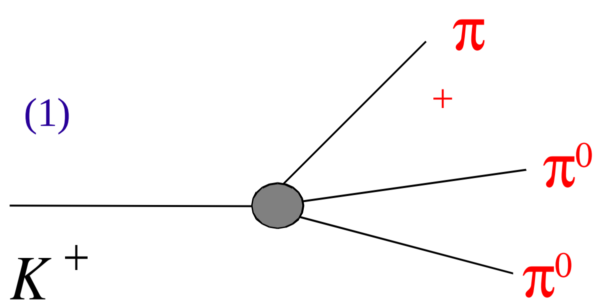
Pionium is the di-mesons atom-like electromagnetic bound state ($\pi^+ \pi^-$)

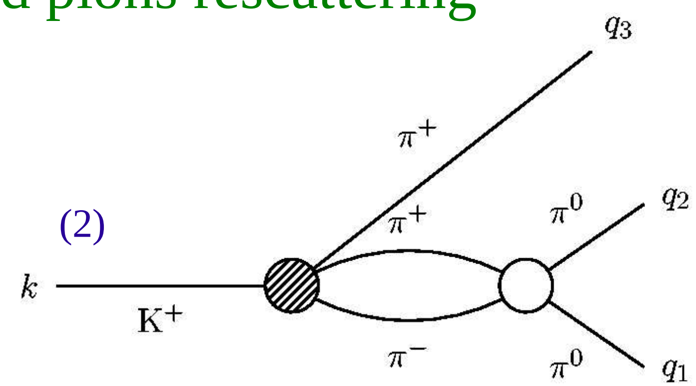
$(a_0 - a_2)$ in $K^+ \rightarrow \pi^+ \pi^0 \pi^0$ decays

Two processes contribute to $K^+ \rightarrow \pi^+ \pi^0 \pi^0$

1) Direct emission of $\pi^+ \pi^0 \pi^0$

2) $\pi^0 \pi^0$ produced in charged pions rescattering





$\mathcal{M}_0 = 1 + gu/2$

$u = 2m_K (m_K/3 - E^*_{odd})/m_\pi^2$

$g = 0.638 \pm 0.020$

(present PDG value)

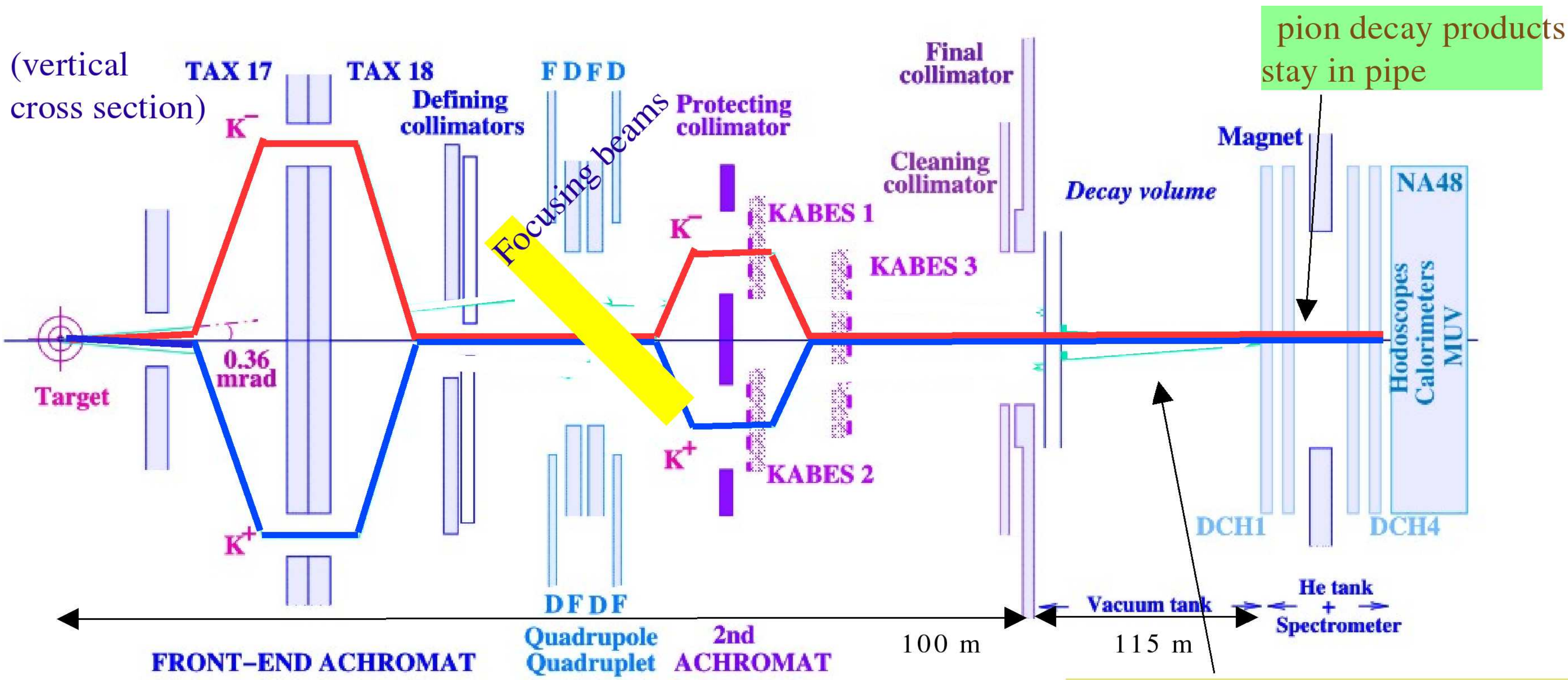
$\mathcal{M}_1 \propto (a_0 - a_2)$

- Small Pionium formation also expected

$$d\Gamma/dm_{\pi\pi} \propto |\mathcal{M}_0 + \mathcal{M}_1|^2$$

<u>Interference is expected</u>

NA48 simultaneous unseparated beams



Beams overlap within ~1mm all along the 115m long decay volume (vacuum 10^{-5} mbar)

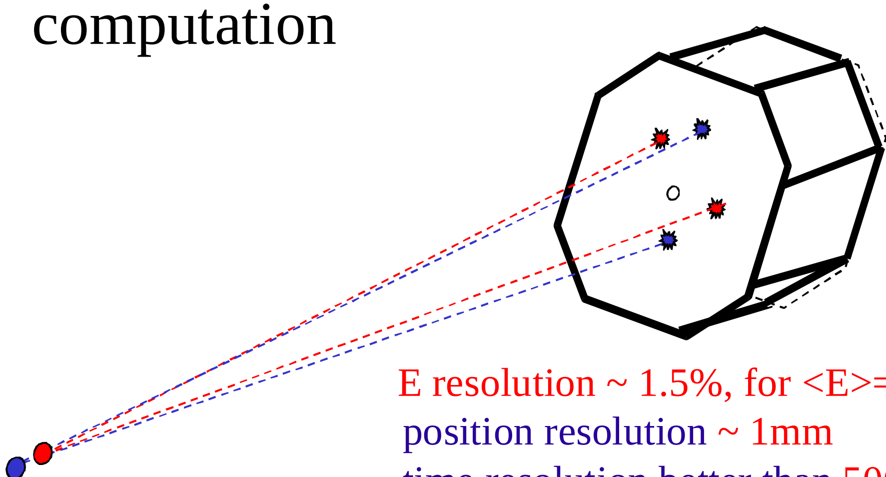
- Split +/-
- Select momentum
- Recombine +/-

- Focusing
- μ sweeping

- Cleaning
- Beam spectrometer (resolution 0.7 %)

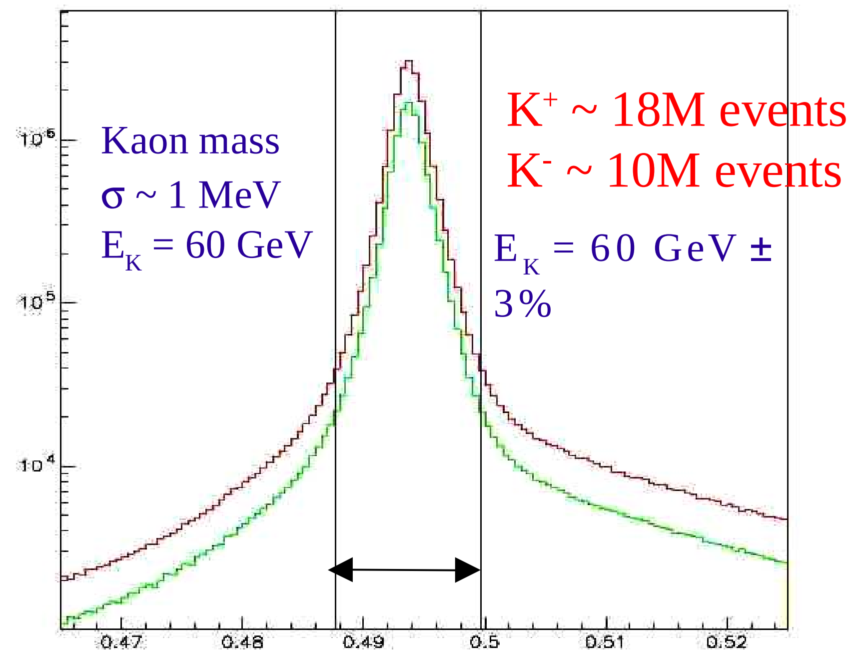
$K^\pm \to \pi^\pm \pi^0 \pi^0$ Reconstruction

2-pi0 invariant mass computation



$\pi^0 \to \gamma\gamma$
$\pi^0 \to \gamma\gamma$

E resolution ~ 1.5%, for <E>=10 GeV
position resolution ~ 1mm
time resolution better than 500ps
non linearity < 0.1%
(very stable over 8 years)



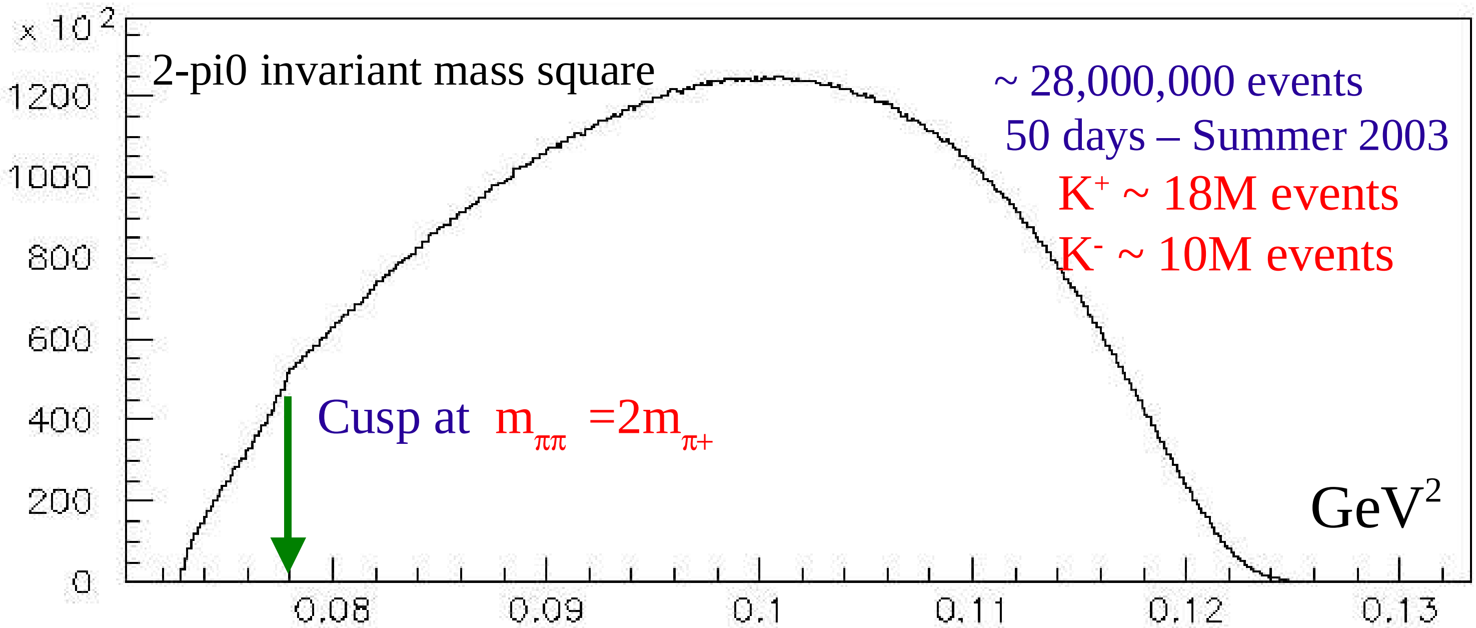
$$\frac{M_{oo}^2}{4m_0^2} = \frac{\sum_{i,j=1,4} E_i E_j d_{ij}^2}{\left(\sqrt{\sum E_1 E_2}\, d_{12} + \sqrt{\sum E_3 E_4}\, d_{34}\right)^2}$$

Decay in flight $\pi^\pm \to \mu^\pm \nu$ **makes no effect** since charged pion track parameters are not involved in $m_{\pi\pi}$

2 π^0 mass constraints
Z from 2 vertices average

Background free

NA48/2 data $K^{\pm} \to \pi^{\pm} \pi^0 \pi^0$



Pionium is not visible at first sight but **CUSP is !!!**

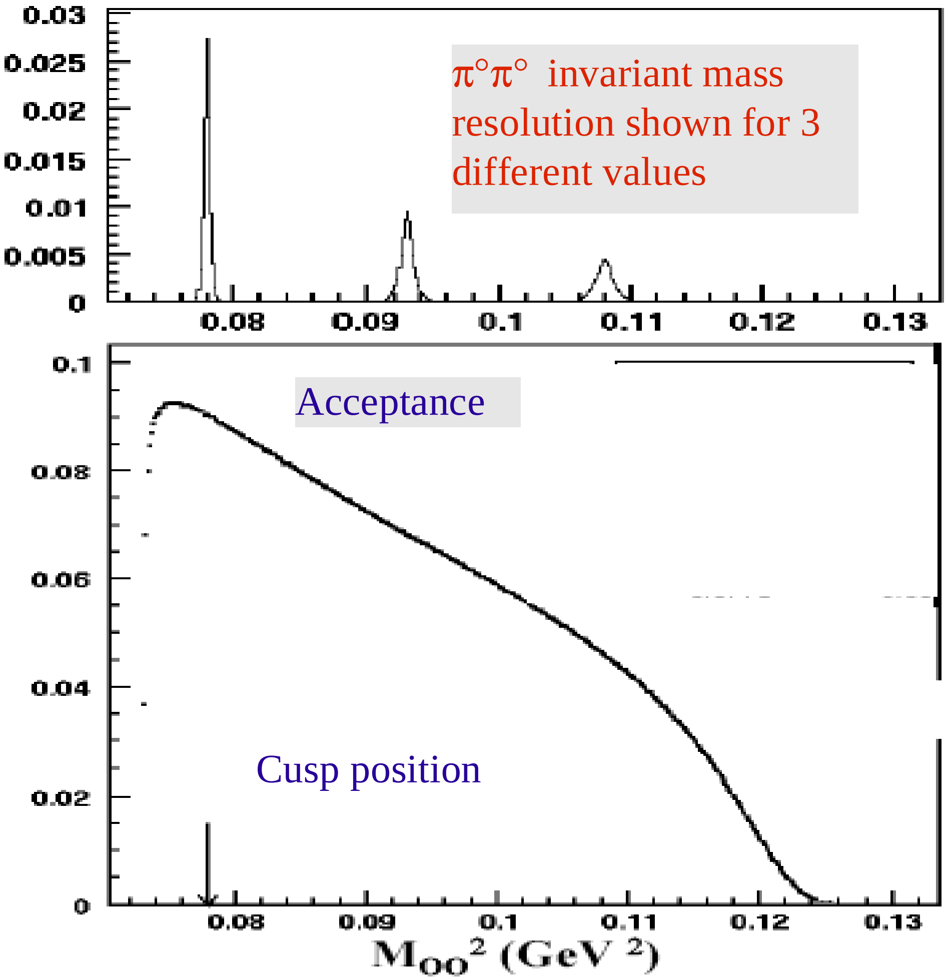
This plot stimulated some theoretical work:

N. Cabibbo realized that it was a clean and beautiful example of a general **cusp-like behaviour of cross sections next to threshold for new channels**

Nicola Cabibbo (2004), hep-ph/0405001 one loop calculation

N. Cabibbo and G. Isidori hep-ph/0502130 two loops calculation

Acceptance and $m_{\pi\pi}$ resolution ($K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \pi^{0}$)



$\pi^0 \pi^0$ mass constraints and LKR resolution

$$\frac{\sigma(E)}{E} = \frac{0.090}{E} + \frac{0.032}{\sqrt{E}} + 0.0042$$

-> $m_{\pi\pi} = 2m_0 + Q$

Q = 9.19 Mev at Cusp

$\sigma(Q) \sim 0.42$ Mev

0.0031 GeV^2 resolution on $(m_{\pi\pi})^2$ @cusp

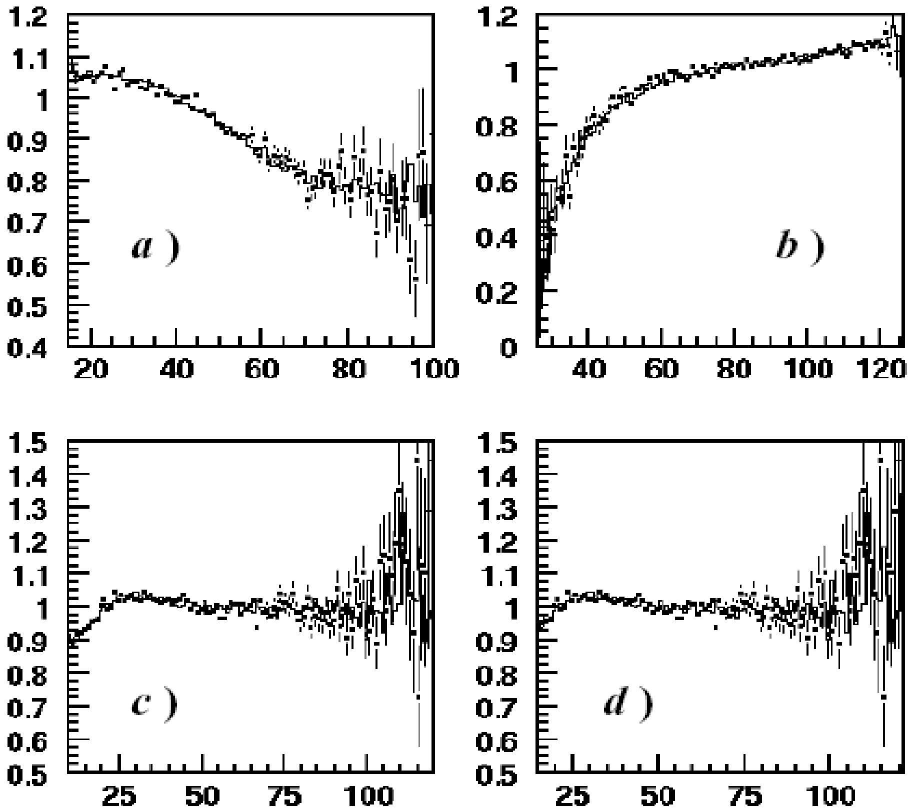
0.0012 ÷ 0.0120 range elsewhere

The acceptance is linearly varying especially around the cusp

The extraction of $(a_0 - a_2)$ is <u>Montecarlo dependent</u> because of the geometrical acceptance

$K^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0$

Is our Montecarlo good enough ?



distance (cm)

Test example:

Fig. (a) shows the ratio of the **min γ distance from axis** @ the calorimeter plane between events above and below the observed cusp.

The Montecarlo well reproduces the data

Fig. (b), (c), (d) show other tests based on other distances

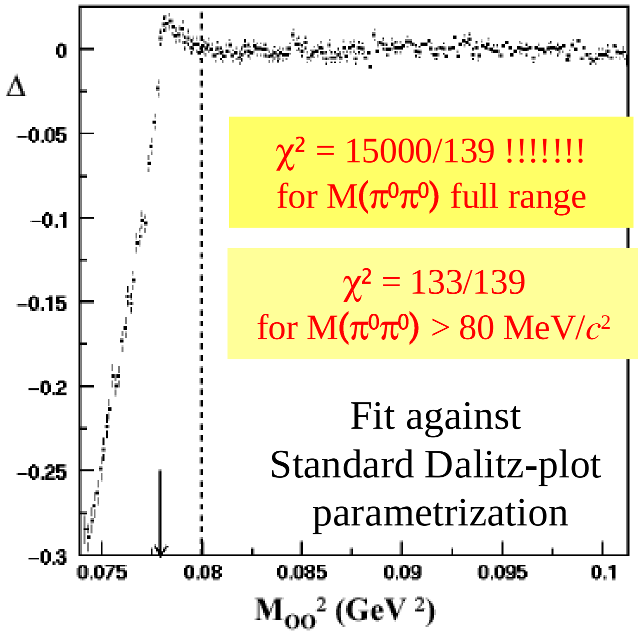
***a*) min γ distance from axis ; *b*) max γ distance from axis;**

***c*) min γ–γ distance; *d*) min γ – $\pi^{\pm}$ distance**

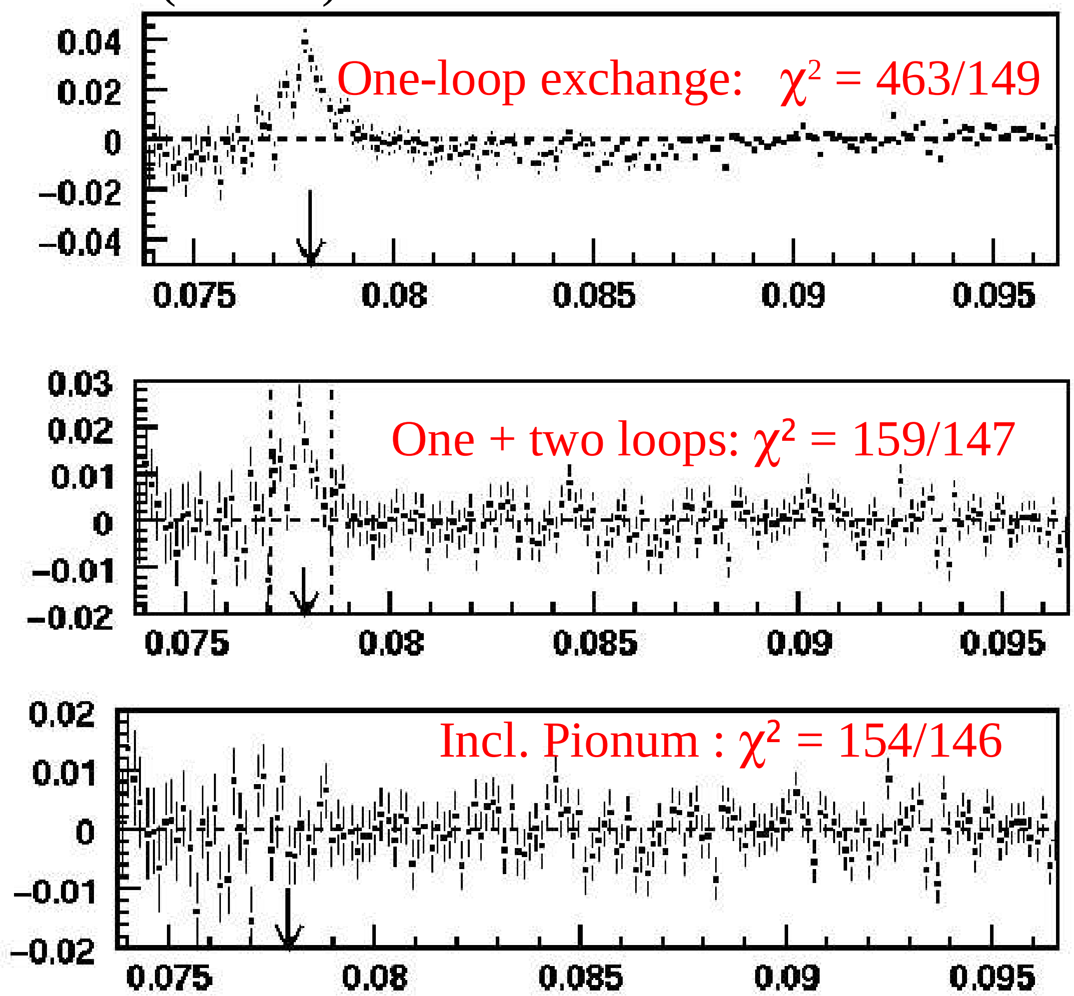
$K^{\pm} \to \pi^{\pm} \pi^{0} \pi^{0}$

Fits against various models

(data-fit)/data

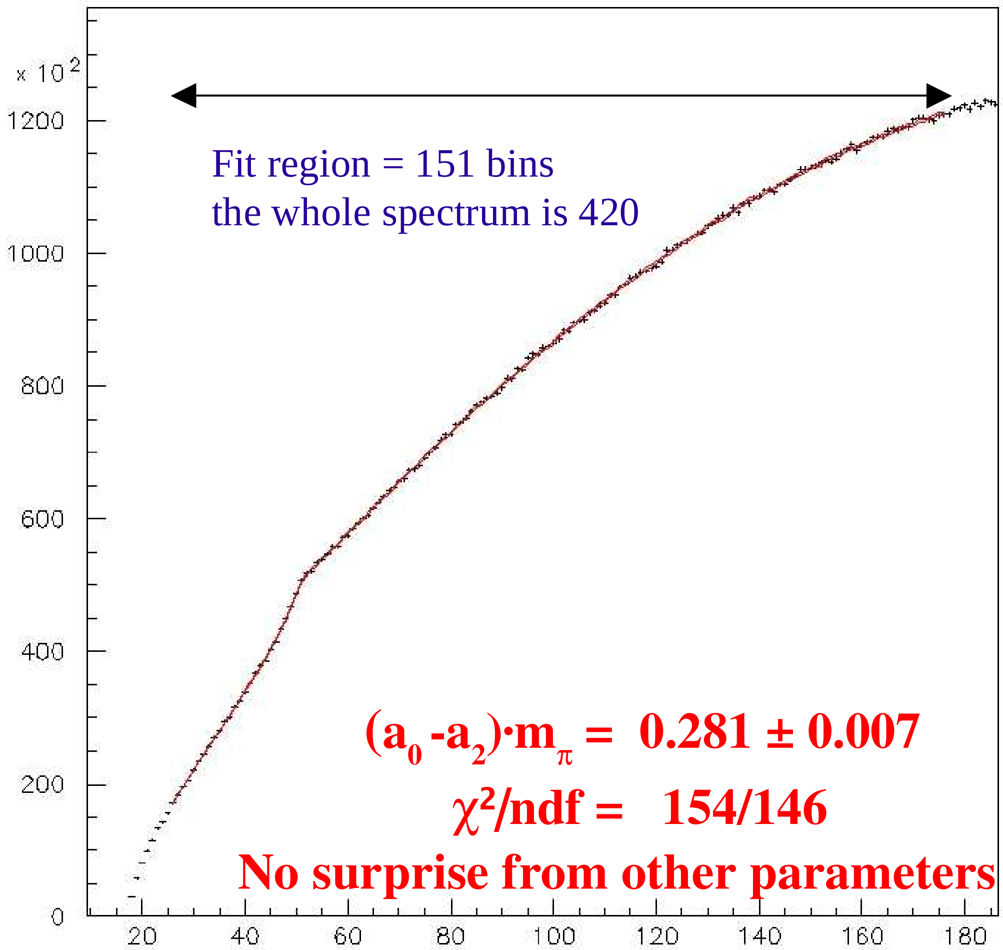


(data-fit)/data



The best fit is obtained with the two loops calculation but a small amount of pionium is to be added in bin 50th to improve the Chi2

Fit results



$(a_0 - a_2) \cdot m_\pi$ = 0.281 ± 0.007

χ^2/ndf = 154/146

No surprise from other parameters

The pionium amount has been fixed according to the prediction

$$\frac{BR(K^+ \to \pi^+ \; pionium)}{BR(K^+ \to 3\pi)} = 0.8 \cdot 10^{-5}$$

Z.K. Silagadze, hep-ph/9411382

$(a_0 - a_2) \cdot m_\pi$ has low sensitivity to pionium

$0.1 \cdot \sigma(BR)/BR$

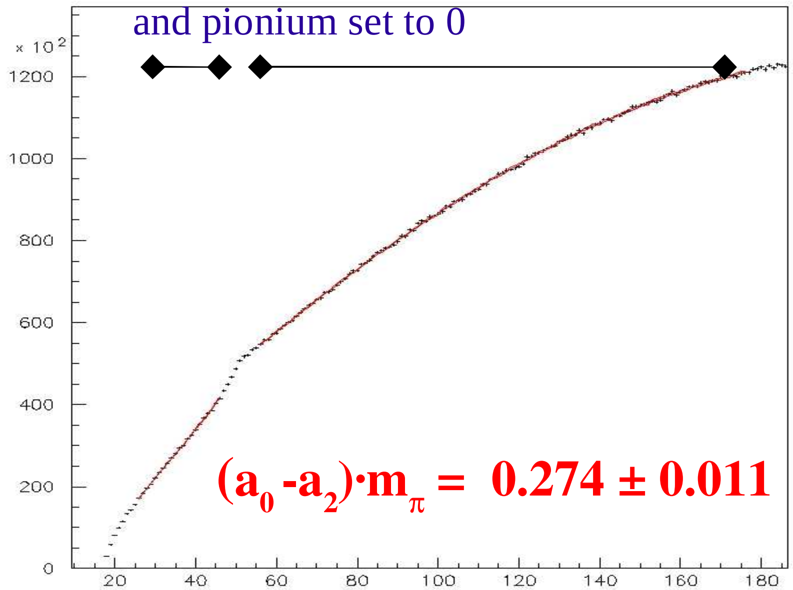
Measured by varying the predicted amount in the range ± 50%

N.B. Cabibbo-Isidori model holds not too far from the cusp --> Not all the spectrum is fitted!

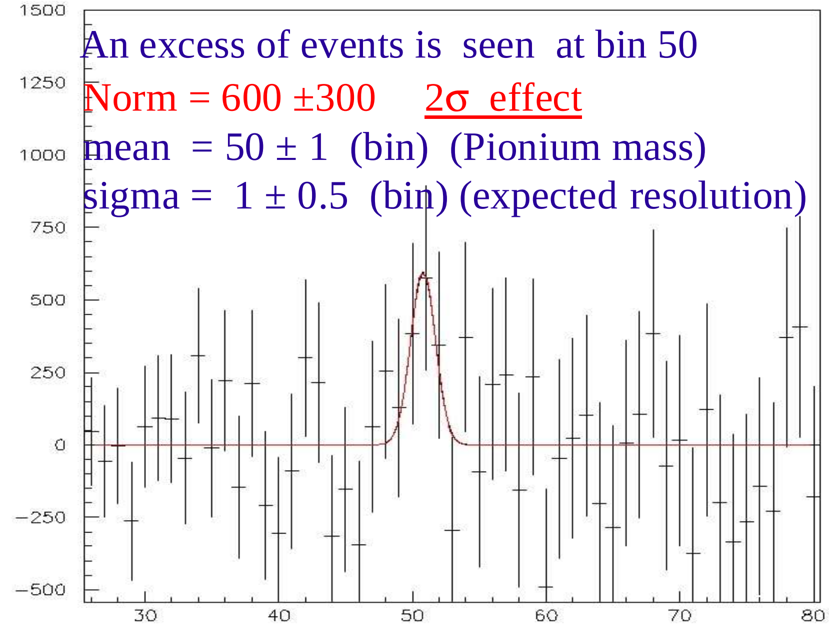
$K^{\pm} \to \pi^{\pm} \pi^0 \pi^0$

Systematics Check: fitting region

Fit excluding 10 bins around the cusp and pionium set to 0



Data – Fit



If the excess is interpreted as Pionium formation data contain more events than predicted by Silagadze

Choice of the fitting region

systematic on $(a_0 - a_2) \cdot m_\pi$ $\pm$ 0.008

Seen/predicted = 1.7 ± 0.8

Check on photon isolation and Z vertex

Cut on transverse distance @ the calorimeter to avoid photon energy mis-measurement

min(γ–γ distance) > 10 cm

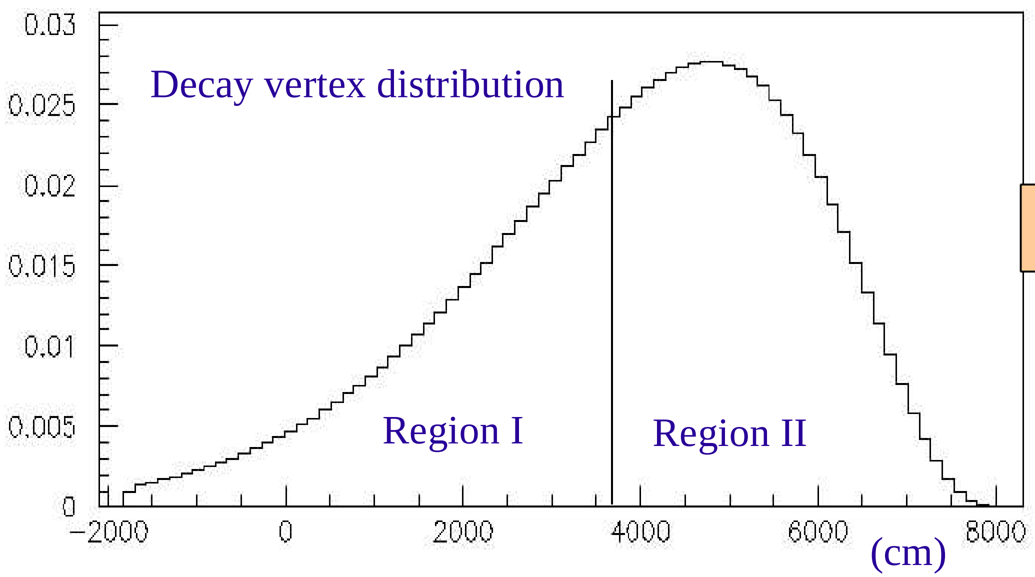
min ($\gamma - \pi^{\pm}$ distance) > (10 + d) cm

If $\pi^{\pm}$ interacts with Kr, energy may be deposited even at large distance from the impact point

Default cut d=5 cm 95% of the extra-shower associated to the $\pi^{\pm}$ is contained

Trying d=5,10,15 cm

systematic on $(a_0 - a_2)\cdot m_{\pi}$ ± 0.004



Measurement from two different decay regions

systematic on $(a_0 - a_2)\cdot m_{\pi}$ ± 0.009

Conclusions on systematics checks
$(a_0\text{-}a_2)m_\pi$ from $K^\pm \to \pi^\pm\pi^0\pi^0$

● Excluding pionium from fit region	0.008
● From Cut on track-photon min. distance	0.004
● From dependence on vertex Z coordinate	0.009
● From K+/K- difference	0.006
TOTAL (adding in quadrature)	**0.014**

Preliminary result on $(a_0 - a_2) \cdot m_\pi$ from $K^\pm \to \pi^\pm \pi^0 \pi^0$

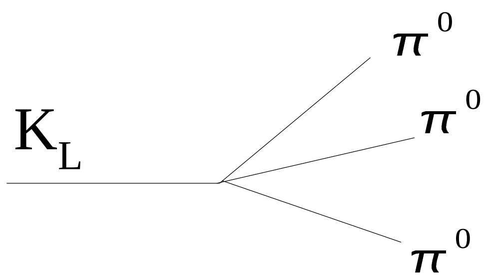
NA48/2 collaboration performed 3 independent analysis on this subject
two based on a toy (...but not so toy!) Montecarlo
the third exploiting a professional GEANT-based Montecarlo

The 3 central values found are compatible within an error of 0.001

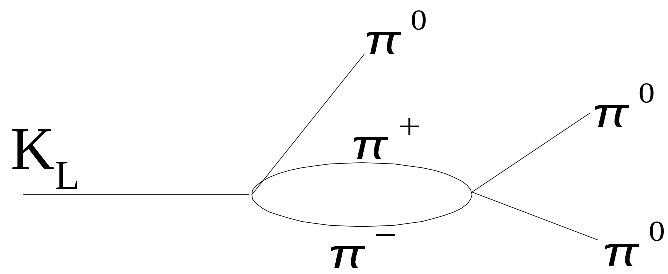
$$(a_0 - a_2) \cdot m_\pi = 0.281 \pm 0.007\ (\text{stat.}) \pm 0.014\ (\text{syst.}) \pm 0.014\ (\text{theor.})$$

The theoretical error quoted refers to the Cabibbo-Isidori model and it has been suggested by the authors

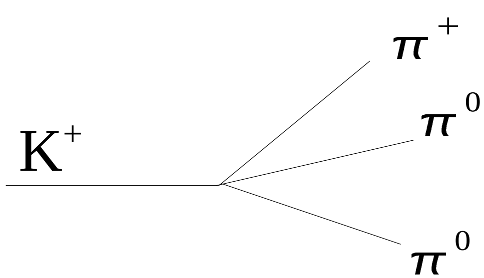
K_L / K^+ comparison



$M_0 \propto 1$



$M_1 \propto (1 + \mathbf{g_{+-0}}\, u)$ (a0-a2)



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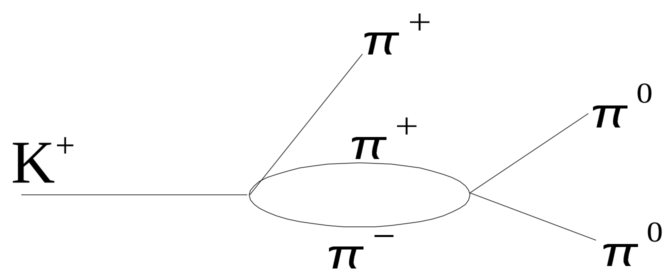


$M_1 \propto (1 + \mathbf{g_{++-}}\, u)$ (a0-a2)

$$\frac{(M_1/M_0)_{K+}}{(M_1/M_0)_{Kl}} = 2\sqrt{2}\,\frac{1+g_{++-}\,u}{1+g_{+00}\,u} \times \frac{1}{1+g_{+-0}\,u} \approx 7$$

The cusp effect for K_L is <u>a factor 7 smaller</u> (at the $2m_\pi$ threshold)

Reconstruction $K_L \to 3\pi^0$

1) **Z vertex** by imposing the **K mass** to the 6 photons

2) Photon pairing by minimizing

$\chi^2 = \Sigma (Z - Z_i)^2 \quad i=1,3$

Z_i = vertices from each π^0 **mass**

3) $m_{\pi\pi}$ computation as in $K^+ \to \pi^+ \pi^0 \pi^0$ analysis

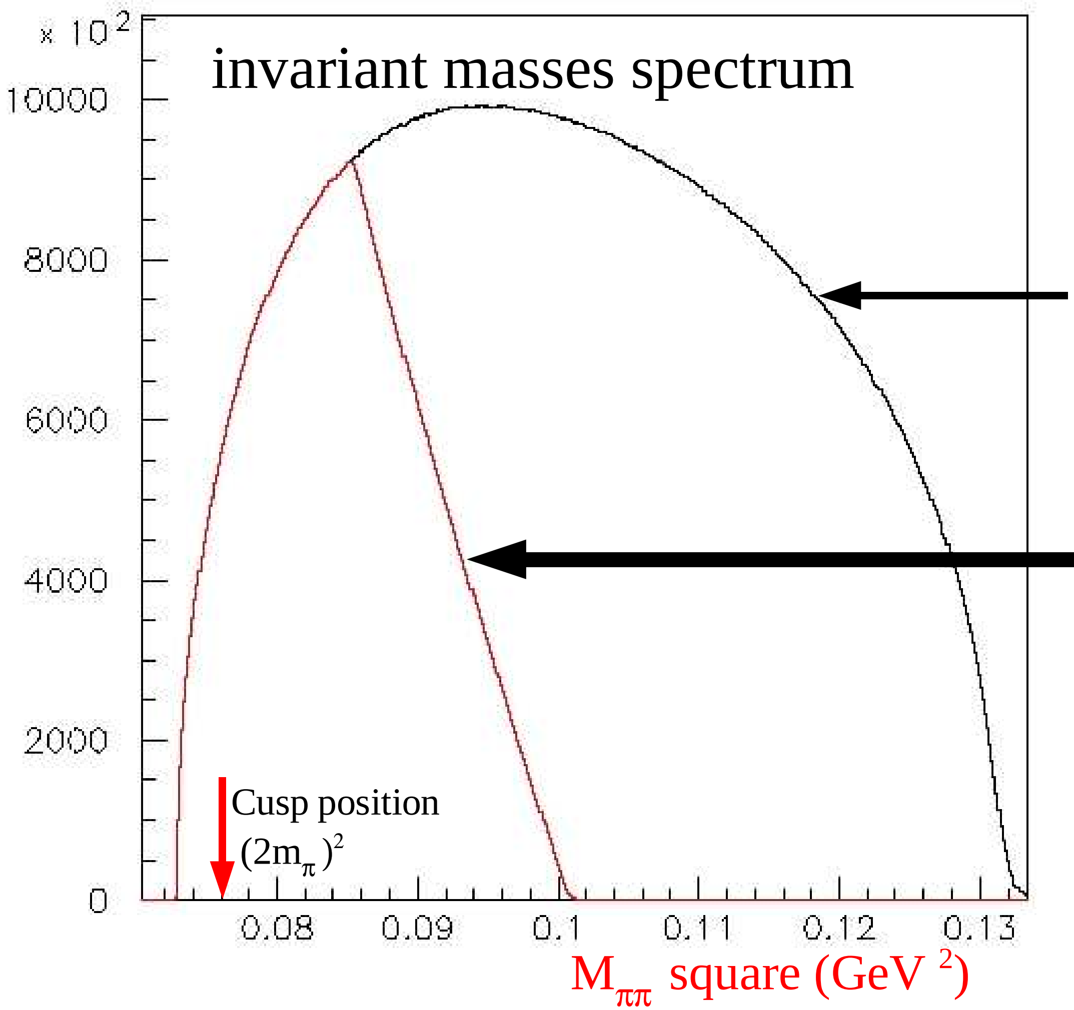
$$\frac{m_{\pi\pi}}{m_\pi} = \frac{\sqrt{\sum_{i,j=1,4} E_i E_j r_{ij}^2}}{\sqrt{E_1 E_2 r_{12}^2} + \sqrt{E_3 E_4 r_{34}^2}}$$

The best resolution is achieved at low mass values (i.e. In proximity of the cusp!)

Statistics: 100 Milion events taken in year 2000

$K_L \to 3\pi^0$

$M_{\pi\pi}$ spectrum



$K_L \to \pi_1\pi_2\pi_3$

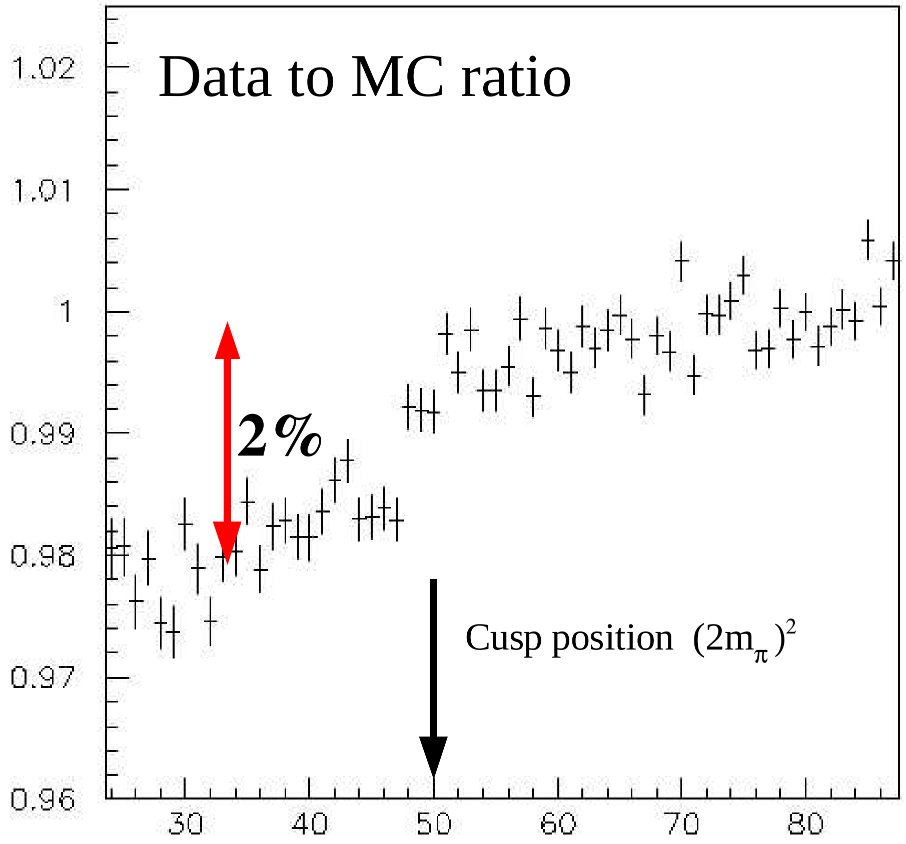
$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M_K^2 + 3m_\pi^2$$

All 3 pairs plotted simultaneously

Lightest mass m_{12}

The cusp is small and cannot be seen by eye

Cusp evidence from $K_L \to 3\pi^0$

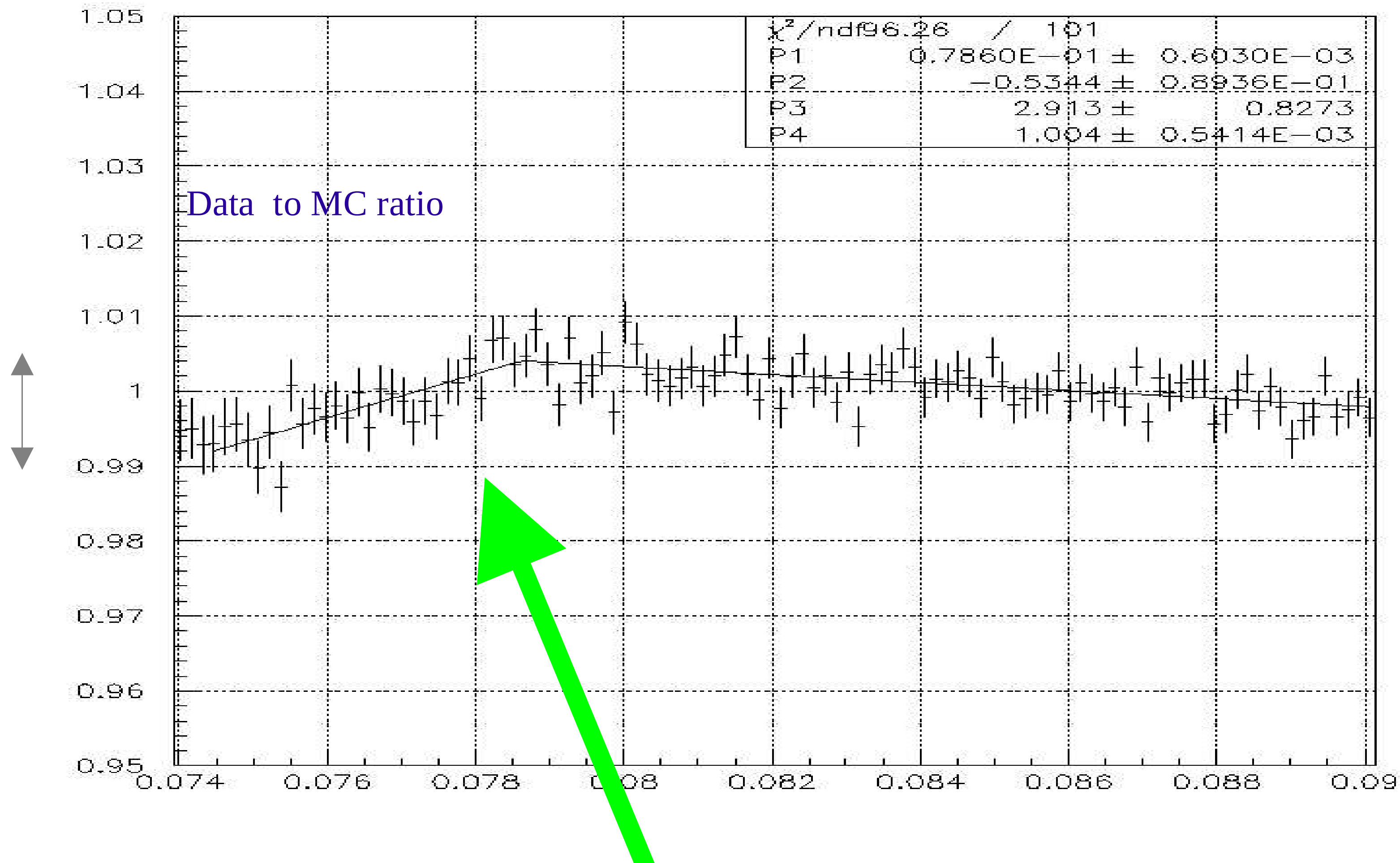


Data to Montecarlo Ratio

$(a_0 - a_2) = 0$ in MC

A **2% depletion** of events below the threshold can be seen

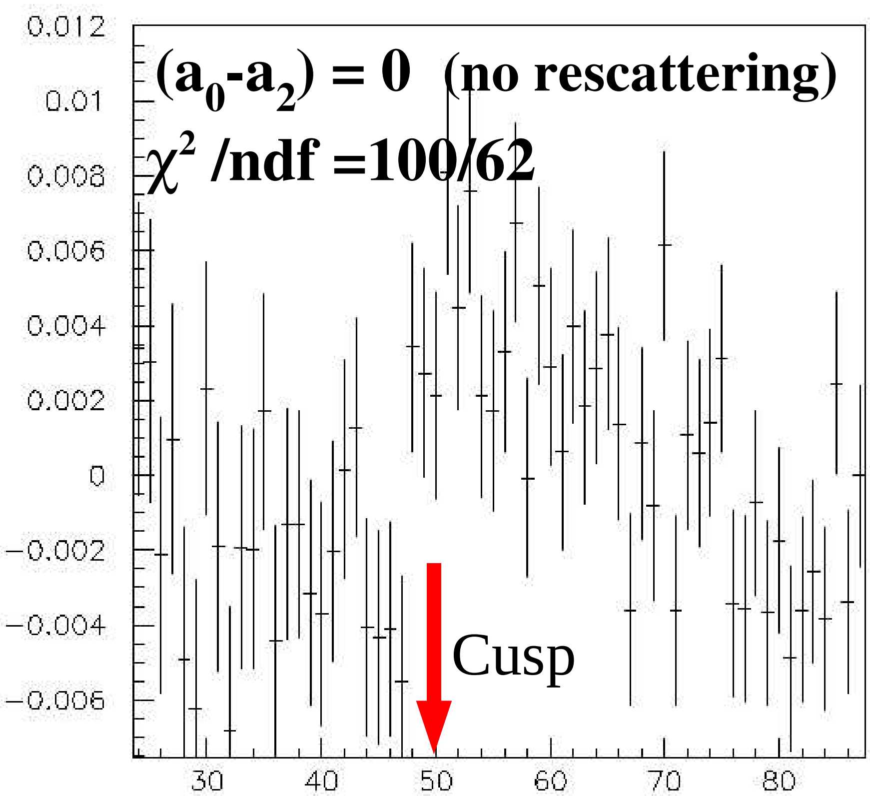
Cusp in $K_L \to 3\pi^0$

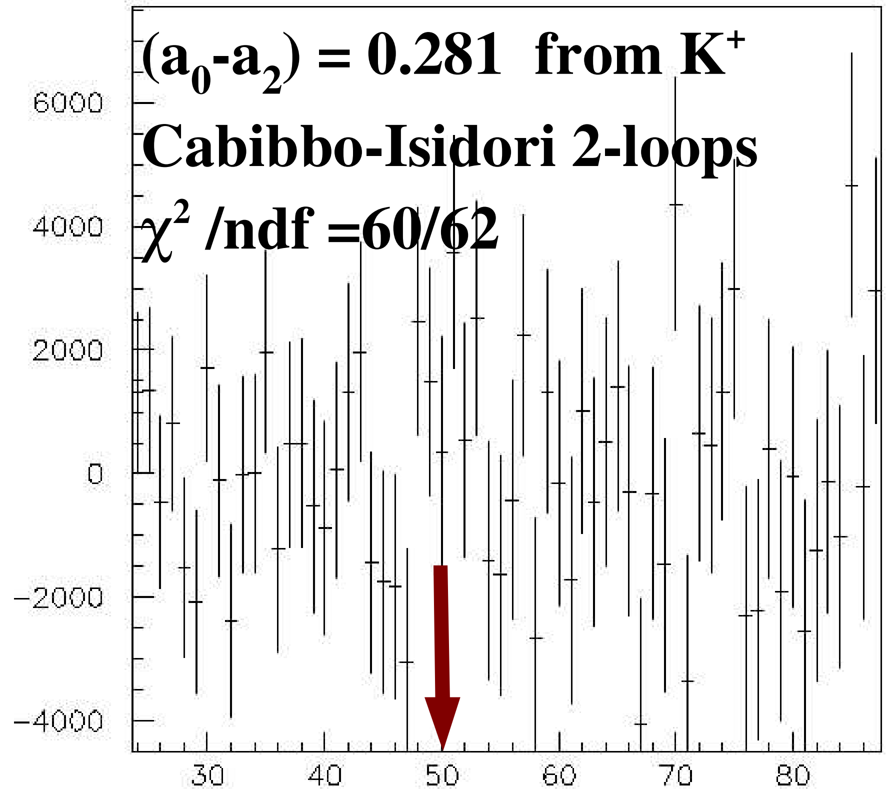


Fitted CUSP position at **(0.0786 +- 0.0006)** GeV^2

1σ from the expected value $\mathbf{4m_\pi^2 = 0.0779}$

The extraction of (a_0-a_2) from $K_L \to 3\pi^0$ is in progress,
no result is ready to be shown now.
However is important to show that **the same CUSP in seen!**





Data – Fit

(a_0-a_2) = ... ± 0.050_{STAT} larger error because the effect is smaller

Conclusions

A new structure in the $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ and $K_L \rightarrow 3\pi^{0}$ Dalitz plot has been seen. If interpreted as charge exchanging rescattering the pion-pion scattering length can be extracted

$$(a_0 - a_2) \cdot m_{\pi} = 0.281 \pm 0.007\ (\text{stat.}) \pm 0.014\ (\text{syst.})$$

$$(a_0 - a_2) \cdot m_{\pi} = 0.274\ \ (\text{excluding pionium region})$$

The data quality calls for additional theoretical effort in order to extract precise values of the scattering parameters (Isopin breaking effect , radiative corrections)

Present analysis involves data taken in 2003, the statistics collected in 2004 is at least a factor 4 larger ---> room to improve stat. and syst. Error